J.J. SMULSKY

THE THEORY OF INTERACTION

TO THE XX CENTURY SEARCHERS OF SCIENTIFIC TRUTH IS DEVOTED

The inside of the cover is on page 303.

It is corrected at September 26, 2018.

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J.J. SMULSKY

THE THEORY of INTERACTION

The responsible editor
Prof. S. P. Gabuda,
The doctor of physics-mathematical sciences

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In the monograph it is justified the calculation approach of electromagnetic and gravitational interactions of moving bodies, in which the space, time and mass do not depend on the movement. The main concepts of mechanics are analysed and determined. From the experimental laws the main electrodynamic equations are derived. Having solved them the force of interaction of two moving bodies is founded and the trajectories of their movement are obtained. The forces of action of the different form bodies are derived and the methods of account of the elementary particles accelerators are given. The applicability boundaries of the Theory of Relativity are considered and the light phenomena are represented at movement of the receiver and source. The interactions of two bodies at nearluminal velocities are investigated. The results of modern observations of superluminal movements are analysed, the methods of superluminal particles production are represented and the perspectives of their use at the unasteroidal defense of the Earth and for cosmic flights to stars are shown. The problems of the gravity velocity propagation, precession of the Mercury perihelion, the substance accretion and the Sun energy are considered. The problem of many bodies is represented and for special cases the accurate solution is obtained.

The book will be useful for inquisitive senior pupils, students, teachers of physics and physicists.

Figures. 59. References: 128.

The referees:

The doctor of physics-mathematical sciences, professor V.F. Novikov
The doctor of physics-mathematical sciences A.V. Shavlov

Scientific Council of the Institute of the Earth's Cryosphere, Siberian Branch of Russian Academy of Sciences authorizes it for printing.

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CONTENTS

FOREWORD

All happening round us is caused by bodies interactions. We perceive the world in the way we understand interactions. For example, the ancient people considered that gods controlled the winds and a rain, gave us the Sun and the Moon light, directed people and supervised their activities. Now we consider these ideas naive. We know what air is and what its structure is, what are the reasons of air velocity and when it becomes wind or even a terrible hurricane. Our knowledge is so reliable, that we undoubtedly consider mistaken the ancient people's vision of the world.

In this connection there are some questions. Are true our conceptions of the world? Which of them will not be found naive and therefore rejected by our descendants? Are there some true conceptions among our ones, which will never be rejected? Can we find out our mistaken conceptions and reject by ourselves?

There is a surrounding world round us. It is the sky, the stars, the trees, our house, the subjects in it and so on. These objects of the external world change and influence each other. We investigate them, explain the reasons of objects change, in this way we create the world understanding. If the ancient people involved the gods' power or demons' ones in the world explanations, we explain everything with the help of forces, fields, ether, energy, space-time etc. As we can see, the explanations and the understanding of the world constantly change, but the world practically remains changeless.

Apparently, any inquisitive person has the questions, similar to above-mentioned one. Since young years I searched the answers to these questions. As a result, I have found them and presented my ideas in this work.

This book is issued in Russian in 1999. Since the readers and I have revealed the lapsuses and typing errors. For example, Prof. Janusz Laski from Krakow has seen defect in substantiation of equation (3.22). Dr. Zinoviy B. Gaydukov from Novosibirsk has detected a error in the formula (8.82). All the lacks are corrected in this edition. Everyone who has drawn my attention to the lacks I express my gratitude.

We have translated the book on English with the help of Stylus program. Thus, Olga I. Zumareva has done a great amount of work. Tatyana S. Olesova has edited the text. Konstantin E. Sechenov has edited this book. So I am very grateful to all of them.

Even the serious ideas while being translated in other language may look ridiculous. I expect that in this case the reader will smile and continue the way in difficult mathematical proofs. Nevertheless, I ask the English-speaking reader to excuse me for my English.

The book is issued due to the grant of the Integration program of Presidium, Russian Academy of Science No. 13, direction 3.

All consists of two:
world around and its understanding.

The author *1998.*

PREFACE TO RUSSIAN EDITION

The modern understanding of micro- and macrocosm is based on the TR. From the moment of its creation and up to present the critical consideration to TR has not disappeared. Such known scientists as academicians: A.N.Krylov [25], S.I.Vavilov [9], V.F.Mitkevich [37], Hungarian academician L.Yanoshi [78,79]; professors: O.D.Hvolson, N.P.Kasterin, K.N.Shaposhnikov [74], A.K.Timiriazev, T.A.Lebedev, S.A.Bazilevsky, A.A.Tiapkin [67] and many others in the former USSR and others countries did not accept TR, because it offered the abstract world with paradoxical properties, which contradicted human logic.

The works with criticism of the TR practically were not published and the so more so were not reprinted. On being available rare information it is possible to make the following submission about their development. At the end of the 50-th years Australian G.Builder [87] considered the connection of the theories of an ether with a TR. In the 60-th the critical works of the Minsk's philosopher A.K.Maneev [34] and Chelyabinsk physician G.D.Lomakin [29, 30] were published. These scientists, and also S.A.Basilevsky, A.K.Shurupov, A.G.Zamiatin and others organized the Lomonosov scientific-physical coterie, issued the handwritten journal of the same title. It is necessary to mark it as the first stage.

In an extremity of 60-th years the American Bryan G.Wallece [122] shown a possibility of summation of radiowaves speed with a velocity of the Earth relatively a Venus. The B.G.Wallece's work, based on observational data, might be marked as the second important stage in the development of the unrelativistic direction in physics. It rendered the consequent opponents of TR moral support.

The third stage is the publication of the prof. V.V.Cheshev's book in 1984 [69], where with a large evidence the incompatibility of the theory of relativity with the actual reality is shown.

The end of 80-th – the beginning of 90-th years of the present century was marked by the intensive development of works with criticism of TR as special and general. By decades the ripening of antagonism to TR by many representatives of a science and engineering, whose activity was not broken with practice, it was gradually accumulated in the unpublished articles, treatises and books. The reorganization in the USSR has become the trigger mechanism promulgation of these

works in popular printing, in the new scientific journals and at many national and international conferences, carried in different countries. They have begun from the publication of V.I. Sekerin's brochure [47] and also proceeded the popular scientific journals [4, 11], in the cooperative-issues brochures [14] and even in the newspapers [17, 41, 42, 48, 57, 61, 70].

The V.I.Sekerin's brochure is to be considered as the fourth stage of development of antirelativistic movement. In it with a large conviction and expressiveness the defects and inconsistencies of the TR were shown. The large effect on the public opinion was also induced by the brochure of the professor and national deputy USSR A.A.Denisov [14]. Due to it the broad public has known, that there are scientists, who do not recognize the TR as top of human wisdom.

Every day we learn names of scientists from different countries, standing in an opposition to the Theory of Relativity, who were earlier unknown to the scientific community. The different sides of a problem are considered. The works of [41, 43, 79, 87, 92, 103] are devoted to the analysis of the spatially-temporary transformations, the paradoxes of the TR are considered in other works [6, 51, 66]. The serious critical analysis of the Theory of Relativity is represented in works the Lie Koe (USA) [100], Xu Shaozhi and Xu Xiangqun (Peoples Republic of China) [127, 128], Xowusu S.X.K. (Nigeria) [126] and many others [8, 11, 13, 14, 44, 89, 93, 95, 104, 105, 109]. Being based to the classical laws of mechanics and physics, Thomas E. Phipps, Jr. [107] and J.P Wesley [125] consider the interactions of the bodies and phenomenon of light propagation in moving bodies; D.L. Bergman [85] develops the models of elementary particles; Ph.M. Kanarev [20] derives the postulates of a quantum mechanics and axioms of spatially-temporary relations; C.W. Lucas, Jr. [102] determines the periodic properties of the elements; V.I. Suhorukov with the colleagues analytically derives the spectra of first ten elements [63] etc. In these works the classical conceptions of space, time and mass, which do not depend on movement, are used.

In 1994 our monograph [59] was published, where we analysed the fundamentals of the Theory of Relativity, its main defects were revealed and the new method of the interactions description, based on classical submissions about space, time and mass, was offered. Dissidents-physicists met this work with unanimous approval. Many orthodox scientists of TR paid attention on it. It could not be discarded by referring to unprofessionalism of the author. Therefore academic issuing of the book is possible to estimate as the fifth stage of new physics becoming.

In 1997 there was printed a monograph of the West Virginia University professor O.D. Jefimenko [98], where he derived all relativistic formula within the framework of a classical mechanics and electrodynamics from fields delay at relative movement of interacting objects. This method ascends to works of Oliver Heaviside[94].

At the end of the 19th century it became evidently, that the interaction of the charged bodies depends on their movements. G.A.Lorentz proposed a hypothesis, according to which the variation of interaction force occurs because of change of the sizes of a body with its movement in imagined ether. However the interaction of two bodies depends on their velocity in relation to each other, but do not depend on any absolute velocity. Therefore A.Einstein had to alter G.A.Lorentz's hypothesis and as a result the description of interactions of moving bodies was constructed as the description of interactions of motionless bodies. At this case the bodies parameters (space, time and mass) change on Lorentz's transformations. It is the first line of development Electrodynamics, which the modern physics is founded.

The second line ascended to Oliver Heaviside's works from 1888 and was based on final velocity of propagating of interaction. Owing to it, the interaction of two bodies, which relatively moved each other, is not equal to interaction of motionless bodies with the same distances between them, as the fields, expressing this interaction, are retarded. If to move the fields to on interval of time, which is necessary for propagating of interaction between bodies, they will express the interaction of relatively moving bodies. In 1997 the book of the professor of Western Virginia University Oleg D. Jefimenko was issued, where he deduced all relativistic results within the framework of the classical Mechanics and Electrodynamics, grounded on retardation of fields. This method can completely replace the special and general TR.

Unlike the first two methods, the author of the monograph developed a method, which is completely within the framework of mechanics. As a result of the analysis of development of electromagnetism he has come to a conclusion, that force of interactions of moving each other two charged or of magnetized bodies depends not only on distance between them, but also depends on heir relative velocity. The author has derived the force expression basing on the experimental laws of electromagnetism. He has spread this method developed to various cases. He has solved a number of new tasks and the new results are received. They show that our world is not arranged in the way following from modern physics, based on TR. This method is ready to use and the present book is devoted to its statement.

The Theory of Relativity influenced deeply on our world outlook and on the bases of our knowledge. In repeated controversies with the TR supporters you could hear many times that the relativistic law of addition of velocities is perfect, and the simple summation of velocities in a classical mechanics is approximate and fair for small velocities. When we showed that the classical addition of velocities is identical to an arithmetical operation $2 + 2 = 4$, and relativistic is equivalent $2 + 2 = 5$, we were answered approximately in this way. The first law of addition and the second one are hypothetical, but the relativistic law of addition of velocities is confirmed experimentally, therefore it is correct.

The space is distorted and becomes isolated per it. Mass turns to energy, and time pass into a matter in the entrails of stars. There is a trip on time in the future and past. All this is not from fantastic novels. These are themes of fundamental researches. The theoretical physics has resulted to mystical perception of the world. Therefore for the author there was a difficult task to understand the bases of our knowledge. It was necessary to find out, whether it is possible with space, time, velocity, mass, force, energy etc. to make above-stated manipulations? What is random hypothesis in our knowledge? Also whether there is, what is firm foundation and eternal truth?

Therefore in the first chapter the bases of our knowledge are considered. All, what the person is connected, with the author has shared into two parts, first is the world around, and second is perception, understanding, i.e. the world around description. The person cannot change the world around by his meditation, but he constantly changes and improves the description of the world around. It is shown what space and time are. It appears, the result of comparing properties of subjects: their sizes and variability, with properties of the standard bodies. The author comes to a conclusion, that the only reliable knowledge is one we receive as a result of comprising the different properties of bodies with the appropriate properties of the standard bodies. Representations, based on the assumptions are inevitably rejected at occurrence of the new facts. Therefore it is necessary aspired not to enter hypotheses into the description of world around.

In the second chapter the concepts of the Mechanics are considered: velocity, acceleration, action, force. It is shown that the laws of the Mechanics can be considered as our method of describing the interactions. In nature objectively there is an interaction between bodies. It is displayed in acceleration of body. We describe the interaction by means of force. Therefore the acceleration and force are equal to accuracy of coefficient, which we call mass. The author proves that there cannot be other dependence of force on acceleration. It is established, space, time and mass cannot depend on velocity of a movement, and therefore the introduction of such dependence in the TR was a mistake.

In the third chapter it is shown that the interaction between moving charged bodies depends on velocity. It is grounded the definition of the basic equations of electrodynamics as results of measurement of forces of interaction between bodies. Their mathematical derivation is given. The differential equations for force are deduced from the experimental bases.

In the fourth chapter these equations are solved for interaction of two point bodies and the expressions for force of interaction of two moving bodies are received. The law of force is checked up in different limiting cases.

In the fifth chapter the movement of two interacting bodies is considered. The velocities, time of movement and trajectory of bodies are calculated. In the micro- and macrocosm the bodies move on these trajectories. They give a new explanation to the observable phenomena.

In the sixth chapter it is considered the interaction of the charged and magnetized different form bodies on a moving particle. It is shown how to apply the approach if the electrical and magnetic field strength is only given.

In the seventh chapter the method of forces is applied for calculation of different interactions, including design of accelerators of elementary particles. Here are also shown wide opportunities of the method, allowing solving a number of tasks that cannot be accomplished by other methods.

In the eighth chapter the mutual relation of the method of forces with the TR method is considered. The light phenomena between moving bodies including the phenomenon aberration and Doppler's effect are calculated. The expression for velocity of light is received at relative movement of a source and receiver.

In the ninth chapter the nearluminal velocity trajectories of movements are considered. The attracting from infinity stone will fall on the Earth with velocity of 11 km/sec, on the Sun - 500 km/sec. And there are such massive and dense stars, at which velocity of fall will achieve velocity of light. Their names are "black holes". The objects of the macrocosm can give each other such velocities.

The tenth chapter is devoted to superluminal movements. They are observed in space, when the space particles enter atmosphere of the Earth and on the Earth. The process of observation of superluminal object is analysed and it is investigated its interaction. A lot of attention is given to acceleration of elementary particles up to superluminal velocity and prospects of applying superluminal movements are also considered for anti-asteroid protection of the Earth and for interstar flights.

In the eleventh chapter the gravitational interactions are investigated. It is considered the Mercury perihelion precession problem and is also shown, that there is no ground of considering the velocity of gravity propagation equal to light of velocity. The numerical algorithm of solving many bodies problem is stated. The axisymmetric many bodies problem is precisely solved and its results are compared to the numerical decisions with interaction three and four bodies. The task center-symmetric accretion of substance is solved and the problems of the Sun energy are considered.

In the end of the monograph the definitions of main concepts, computer programs of problem solving and some results of numerical solutions are given.

The formulated above stages of development of unrelativistic physics are subjective and rather conditional. It is necessary to mention the influence of the conferences, the new scientific journals, originating societies and individual scientists in different countries. For example, per 80 years the large response has received the statement of group scientists of Pulkovo’s observatory against a TR. Their activity, including S.A.Tolchelnikova, and also other scientists from St.-Petersburg: P.F. Parshin, M.P. Varin etc. in organization and realizations of International Scientific Conferences plays a large role in antirelativistic movement. Last decade the unrelativistic works are published due to the publisher of the journal "Apeiron" C. Roy Keys, the founder of the journal "Galilean Electrodynamics" Prof. Petr Beckmann, editor of "Physics Essays" doctor E. Panarella.

Since 1994 the Alliance of Natural Philosophy headed by the doctor John E. Chappell together with Southwest Branch of American Society of Development of a Science carries out the annual conferences of the dissidents-physicists. It is necessary to mention the argued criticism of the TR by a Chinese physicist Xu Shaozhi, on pages of the some international journals per the last years. On being available little information the articles and books with TR criticism of some foreign scientists, so as unknown and known, including prof. L. Essen, are published.

Many scientists, including Bryan G. Wallace, Petr Beckmann, Stefan Marinov, V.I. Sekerin etc., underwent oppressions and persecutions for their antirelativistic ideas. During the last years some of them have died: Prof. S.A. Bassilevsky, Prof. Richard A. Waldron, Lee Coe, Prof. Petr Beckmann, candidate of the physics-mathematical sciences G.D. Lomakin, candidate of technical sciences A.G.

Zamiatin, Dr. Toyvo Jaakkola, Bryan G. Wallace, Stefan Marinov. Not each from them managed to comprehend truth, but they searched it, being returned this up to an extremity. To them, researchers of truth, we devote the present book.

Finally I express my gratitude to Prof. L. Essen, Roy Keys, Dr. E. Panarella, Prof. Oleg D. Jefimenko, Dr. David L. Bergman, Prof. Umberto Bartocci for improving my articles promoting their publication in English. The correspondence with Prof. Andre K.T. Assis, David L. Bergman, Prof. A.D. Vlasov, Dr. George Galeczki, Oleg D. Jefimenko, Millenium Twain, candidate of the physics-mathematical sciences L.A. Pobedonoscev, candidate of technical sciences G.I. Suhorukov, Robert J. Hannon, Xu Shaozhi and many others strengthened reliance of the my conclusions and was a source of an information and new ideas, what I am grateful for to the listed scientists.

Especially I am very much obliged to my colleagues Pavel.A. Apasev and Oleg I. Krotov for their help in my work at the book, Ol'ga S. Gul' for the preparation of the manuscript, Leonid J. Smulsky for the help in the development of the computer programs and in computer service.

The work at the book would be impossible without the benevolent relation to it of the director of Institute, academician Vladimir P. Melnikov, what I am cordially grateful for to him.

All notes and wishes to send direct to the address:

Address: Prof. Joseph J. Smulsky: Inst. of Earth's Cryosphere, P.O. Box 1230, 625000, Tyumen, RUSSIA.

NOTATIONS

A - work;
a - semiaxis of elliptic orbit;
$c, c_1 = c/\sqrt{\mu\varepsilon}$; - speed of light in vacuum and medium;
E - Electrical strength;
E_f - full energy;
E_c - kinetic energy;
E_t - thermal energy;
e - charge of an electron;
F - force of action on a body of other body;
G - Gravitational constant;
H - magnetic strength;
$h = v_t R$ - kinematic angular momentum;
I - magnitude of a current;
m - mass of a particle or body;
m_0 - initial mass of object with a variable mass;
$m_{re} = 2\dfrac{m_1 m_2}{m_1 + m_2}$ - arranged mass of two interacting objects;
M - magnetic charge;
q - electrical charge;
$\vec{r} = i\vec{x} + j\vec{y} + k\vec{z}$ - position vector of a point or particle;
$\vec{r}_1, \vec{r}_2$ - position vectors of two interacting objects;
$\vec{R} = \vec{r}_2 - \vec{r}_1$ - relative position vector between two interacting objects;
$R_v = \sqrt{R^2 - \left[\vec{\beta} \times \vec{R}\right]^2}$;
$R_g = -2\mu_1 / c_1^2$ - "gravitational" or light radius;
R_a - radius of an apocentre: maximum distance between interacting objects;
R_p - radius of a pericentre: minimum distance between interacting objects;
$\overline{R} = R/R_p$ or $\overline{R} = R/R_0$ - dimensionless distance between particles;
T - temporary period of movement on a trajectory;
$T_{0.5}$ - temporary halfcycle of movement on a trajectory;

$\vec{v} = \vec{i}\,v_x + \vec{j}v_y + \vec{k}v_z$; v - velocity of movement of a particle;

v_r, v_t - radial and transversal velocity of a particle;

v_{r0}, v_{t0} - radial and transversal velocity of a particle in a point R_0;

v_p - transversal velocity of a particle in pericentre;

$\bar{v}_r = v_r/v_p$; $\bar{v}_r^0 = v_r/v_{t0}$ - dimensionless radial velocities;

v_R - velocity of the receiver of a radiation;

v_S - velocity of a stimulus source;

w - acceleration of a particle or body;

$w_p = \dfrac{4q_1q_2}{\varepsilon m S}$;

x_{ijk}, y_{ijk}, z_{ijk}, u_{ijk}, v_{ijk}, w_{ijk}, $\dot{u}_{ijk}$, $\dot{v}_{ijk}$, $\dot{w}_{ijk}$ - projection of coordinates, velocity and acceleration of a particle with number i, j, k.

$\alpha = \dfrac{2\mu_1}{R_p c_1^2} = -\dfrac{R_g}{R_p}$ - parameter of interaction;

$\alpha_1 = \mu_1/\left(R_p v_p^2\right)$, - parameter of a trajectory;

$\alpha_1^0 = \mu_1/\left(R_0 v_{t0}^2\right)$ - parameter of a trajectory concerning any point with a radius R_0;

$\vec{\beta}_o = \vec{v}/c$; $\vec{\beta} = \vec{v}/c_1$; $\beta_x = v_x/c_1$; $\beta_y = v_y/c_1$; $\beta_z = v_z/c_1$; $\beta_{t0} = v_{t0}/c_1$;

$\beta_{r0} = v_{r0}/c_1$ - velocity of a particle in relation to speed of light;

$\gamma_x = \sqrt{1-\beta_y^2-\beta_z^2}$; $\gamma_y = \sqrt{1-\beta_x^2-\beta_z^2}$; $\gamma_z = \sqrt{1-\beta_x^2-\beta_y^2}$;

ε - dielectric permeability of a medium;

ε_t - eccentricity of a trajectory;

μ - magnetic permeability of a medium;

$\mu_1 = \dfrac{q_1q_2(m_1+m_2)}{\varepsilon m_1 m_2}$ or $\mu_1 = -G(m_1+m_2)$ - constant of electromagnetic or gravitational interaction;

ρ - denseness of an electrical charge;

$\sigma = \dfrac{q_2}{4ab}$ - area density of a charge of a slice;

Φ - magnetic flux;

φ_T - angular period of a trajectory;

φ_a - angular period of an ellipse-like or final trajectory, i.e. angular distance(span) from a pericentre up to an apocentre; or half-angle between asymptotes for hyperbolic-like trajectories counted after the clock hand from a negative position of an axes x;

φ_P - full angular period of a final trajectory, during which the particle returns in the same point of space with by constant velocity.

Differential operators:

$\vec{\nabla} = \vec{i}\,\dfrac{\partial}{\partial x} + \vec{j}\,\dfrac{\partial}{\partial y} + \vec{k}\,\dfrac{\partial}{\partial z}$;

$\Delta = \vec{\nabla}\cdot\vec{\nabla} = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ - operator of the Laplace;

$\Box = \Delta - \dfrac{1}{c_i^2}\cdot\dfrac{\partial^2}{\partial t^2}$ - D'Alembertian, where $c_i = c$ or $c_i = c_1$;

$\text{grad}\, u = \vec{\nabla} u$;

$\text{div}\ \vec{A} = \vec{\nabla}\vec{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$;

$\text{rot}\ \vec{A} = \text{curl}\,\vec{A} = [\vec{\nabla}\times\vec{A}]$;

$\text{rot}\ \vec{A} = \vec{i}\left[\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right] + \vec{j}\left[\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right] + \vec{k}\left[\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right]$;

$\text{div grad}\, u = \vec{\nabla}\cdot\left(\vec{\nabla}u\right) = \Delta u$;

$\text{grad div}\vec{A} = \vec{\nabla}\cdot\left(\vec{\nabla}\vec{A}\right) = \Delta\vec{A} + [\vec{\nabla}\times[\vec{\nabla}\times\vec{A}]]$;

$\text{rot rot}\ \vec{A} = \vec{\nabla}\cdot\left(\vec{\nabla}\vec{A}\right) - \vec{\nabla}^2\vec{A}$.

Integrated theorems [24]

$\oint\limits_S \vec{A}(r)\mathbf{d}\vec{S} = \int\limits_V \vec{\nabla}\vec{A}(r)\mathbf{d}V$ - Gauss theorem;

$\int\limits_S [\vec{A}(r)\times\mathbf{d}\vec{S}] = -\int\limits_V [\vec{\nabla}\times\vec{A}(r)]\mathbf{d}V$;

$\int\limits_S u(r)\mathbf{d}\vec{S} = \int\limits_V \vec{\nabla}u(r)\mathbf{d}V$;

$\oint \vec{A}(r)\mathbf{d}\vec{l} = \int\limits_S [\vec{\nabla}\times\vec{A}(r)]\mathbf{d}\vec{S}$ - Stokes theorem.

Used constants

$c = 3 \cdot 10^{10}$ cm/sec - speed of light;

$m_{pr} = 1.672 \cdot 10^{-24}$ g - mass of a proton;

$m_e = m_{po} = 9.1095 \cdot 10^{-28}$ g - mass of an electron and positron;

$e = 4.8 \cdot 10^{-10} \text{cm}^{1.5}\text{g}^{0.5} / \text{sec}$ - charge of an electron;

$R_{po} = 1.4 \cdot 10^{-13}$ cm - radius of a positron;

$R_{pr} = 2.817 \cdot 10^{-13}$ cm - radius of a proton;

$G = 6.67259 \cdot 10^{-11}\ \text{m}^3/(\text{sec}^2 \cdot \text{kg})$ - gravitational constant;

$m_S = 1.97 \cdot 10^{30}$ kg - mass of the Sun;

$m_E = 5.96 \cdot 10^{24}$ kg - mass of the Earth;

$R_S = 6.95 \cdot 10^8$ m - radius of the Sun;

$r_E = 1\,\text{a.e.} = 1.496 \cdot 10^{11}$ m - average distance of the Earth from the Sun;

$R_E = 6.372 \cdot 10^6$ m - average radius of the Earth;

$a_{pl} = 39.52$ AU – major semiaxis of Pluto;

$r_{\alpha C} = 2.31 \cdot 10^{16}$ m - distance up to the nearest star α-Centauri.

Orbit parameters of the Mercury:

$a = 0.387$ AU; $e = 0.206$; $T = 0.204$ *year*; $i = 7^\circ 0.2'$.

AU – astronomical unit.

CHAPTER 1

THE WORLD AROUND AND THE MAIN POSITIONS OF ITS DESCRIPTION

1.1. PHYSICAL THEORY AND DESCRIPTION OF NATURE

The present book is devoted to the theory of interaction. What is a theory? There are theories of the ship, plane, of metalwork business and there is a physical theory, for example the Theory of Relativity, Quantum Theory. In the first three theories the principles and methods of fulfilment of work are described: building of the ship and plane, production of details, assembly of items. These theories differ from practice because they are used in the last one. For example, the airplane or the ship is created, the mechanic takes a file and in according with the theory saws with it a detail.

And what is the physical theory today? It is something other. In the modern physical theory the initial hypothesises are accepted and the picture of the world is created on them. So, in the Theory of Relativity (TR) on the base of principle of a relativity and principle of a limiting velocity it is constructed the world, in which space, time and mass depend on a relative velocity of movement. Adding a hypothesis about the light velocity of gravity and to the principles substituting algebraic mathematics for geometric have led to the General Theory of Relativity (GTR), where the world is curved four-dimensional space-time. Thus, the process of development of the physical theory is the process of creation of the world.

So, there are two varieties of the theories, there are two method of its understanding. One is the description of considered objects: properties of nature, receptions and methods of human activity, etc. And the other is the process of creation of the world. We will understand the Theory of Interaction as the first kind of the theory. There the calculation methods of interactions will be described. Traditionally these problems were considered by mechanics. With emerging of the electrodynamics, TR, the nuclear physics, the theory of elementary particles, the quantum mechanics, and other areas of modern physics, the interactions have become the subject of the research of new sciences and have exceeded the limits of mechanics. Hereinafter they should return to its bosom. At the given stage it is expedient to determine the whole area of interactions as a separate science - the Theory of Interaction, underlying its being in common with the different sections of physics.

Science, in particular Physics, was not developed according to written script. It has simultaneously various sights, various approaches and receptions, which sometimes even are mutually opposite. Therefore, there are different sights of the theory and therefore of science. Supporting one of them, we will not prove the inaccuracy of others. Nevertheless we will try to state some ground of our choice.

1.2. WORLD AROUND AND ITS DESCRIPTION

Everything that surrounding us what we constantly deal with can be divided into two areas. First, the world around: the sky, stars, trees, our house, subjects in it etc, which does not depend on our discourse. The second area is our description of the world around, its reflection in one's mind, its understanding. It is contained in the books, is studied at school, is presented in our consciousness. If the world around we cannot be changed by the reasonings, our understanding of the world constantly varies. For example, earlier man imagined that the Earth is the centre of the universe and the firmament rotates round the Earth. Now we know, that the Earth rotates round the axis, moves round the Sun, the Sun makes movement round the centre of the Galaxy, and the last one makes movement at interaction with other galaxies. However there is a mass of other submissions, which our generations will find by fallacies, and the understanding of the world will be changed by a radical image. Besides even methods of submission of the world vary. So, if in the ancient time the world was represented as clear images: gods, elements, ground, light, ether, etc., now it is represented as mathematical objects: wave functions, singularities, solitons, strings, levels of energy, energy-momentum tensors etc.

As we see, the description of the world can essentially differ from the world. But nevertheless there are some receptions of the description, which give practically constant knowledge of it. We know duration of a year in days and day in hours, we know height of mountains and depth of the seas, we know temperature of water freezing and temperature of steel melting. We are sure in this knowledge. It is based on comparison of objects properties. Let's dwell on it in more details.

1.3. THE CHANGE OF THE OBJECTS THE WORLD AROUND AND TIME AS ITS MEASURE

We begin with selecting the certain properties, characterizing the objects of the world around. For example, all objects change. The man is born a child, grows, turns to the young man, becomes mature, then ages and dies. During the human life other changes happen. Every day Sun moves on a firmament, a day changes a night, phases of Moon, seasons of a year are changed too. Many of changes are repeated and are cyclical, others are unique – emerging a supernew star, collision

of the Levi-Shumeker to the Jupiter. Properties of a change and movement characterise all objects of the world around.

To define a measure of a change of object the man compares this change to some standard change. During the human history the change of different objects was selected as standard: cyclical change of a day and night, change of phases of Moon, change of year seasons, change of generations, change of dynasties, drain of water or sand from capacities, repeating heart beats etc. The magnitude of a change is determined in the results of comparing the object change with the standard change. It expresses in an amount of standard changes or in an amount of their parts and is called time. For example, change of the man, i.e. his life, passes on the average during 70 revolutions of the Earth round the Sun that is identical to an amount of cycles of season changes. There are some results of a comparison of changes: duration, gap of time, instant etc., which are synonyms or update features of changes. The gap of time designates an amount of the cycles of a standard change equivalent to a considered change of the object. And the instant of a considered change designates binding it to the certain cycle of a standard change.

Depending on choice of the measurement standard the same change will be expressed by different magnitude. For example, the age of an oak is equal to 350 years or 5 human lifes. During human activity the system of the measurement standards of changes was created: second, minute, hour, the day, year etc. and it is established the accurate correspondences between them. In the results of comparison of changes of objects with standard changes we received knowledge of the world: the turtle lives 300 years, the sequoia - 5000 years, and the Earth exists 5 billions years. This knowledge will never become fallacious. They can undergo some quantitative changes caused by several reasons. At first, the process of a comparison afterwards can be executed with a greater accuracy. For example, at first the duration of a year was determined in 365 days, but after many years it was updated and now within five significant figures makes 365.25 days. Secondly, the changes of an investigated object or measurement standard can depend on different circumstances. At unfavorable vital conditions the duration of a person's life can not be 70, but 35 years and even less. And the pendular watch, which is transferred in a point with the other acceleration of gravitation, or spring ones - in a location with other temperature, will show other duration of a change. Thirdly, the changes also change. In accordance with wear the movement of mechanical watch changed. With increase of amount of the Earth revolutions round the Sun the duration of a year in days changed. Therefore the results of a comparison of changes should be indicated to circumstances, which they depend on, and to a moment of a comparison. Making extrapolations of it in the future or the past you must conduct additional researches into change of changes of investigated object and standard ones and to correct results of a comparison. For example, the age of the Earth of 5 billion years concerns to present movement of the Earth round the Sun. Probably, from the beginning of his emerging the Earth has made 100 billions of revolutions round the Sun, or probably 1 billion. It is possible, that 2 billions of revolutions ago there was no Earth, but there was a lens-shaped diffuse cloud. As we see, the

careful study by a method of a comparison can open to us great knowledge of the world. And if we take into account circumstances of a comparison, such description of the world will remain valid so long as they will not be changed or there will be no circumstance, which we did not take into account before.

1.4. SIZE OF OBJECTS AND ITS MEASURE

The second property of the world around, which a man most frequently deals with, is magnitude of objects. Being compared one object is found to be less or more than the other. So, a finger pin is less than a stop, the stop is less than a hand (up to a cubit joint), hand is less than a man, the man is less than a tree. Many from the recounted objects were selected as the measurement standards: feet, elbow, step (in Russia). The magnitude of an object is determined in the results of comparing it with the measurement standard and expresses by an amount of the measurement standards or their shares. For example, the magnitude of a man is equalled to 6 ft. Now a large part of mankind uses specially created measurement standard of magnitude or length – the platinum – iridium beam of a x-shape section stored in Sevre (France) at International office of measures. The magnitude of a beam at the temperature of 0^{o}C between two marked primes on it is called meter. Thus, the magnitude of objects both in macro -, and in a microcosms is expressed by amounts or shares of meter.

The magnitude of objects, as well as their change, depends on many circumstances: for example, temperature, pressure etc. Standard meter is made of such material, which in some range of temperature does not change the magnitude. However the change of temperature in a greater range or realization of measurements on the special conditions is necessary for taking into account the definition of objects magnitude.

The magnitude of one man can be more than the other in a direction from the heels up to a head, it is more than the third one in a direction of shoulders and more than the fourth one in a direction of a back – stomach. It is peculiar to other objects, i.e. three kinds of magnitude in three mutually perpendicular directions are inherent to them. However for their measurement the same measurement standard is used. For want of consideration of two bodies it is necessary to define the magnitude of a gap between them. The gap, as well as the magnitude of object, places in three mutually perpendicular directions. The magnitudes of gaps between objects are measured by the same measurement standard of length. Objects and gaps between them will form their container, which is called space.

There are many varieties of measures of magnitude of the objects: a size, length, breadth, height, depth, distance etc. All of them are received comparing the magnitude of an object with the same measurement standard of magnitude. The size of the object is the result of comparison of object magnitude with the magnitude of the measurement standard. Length is the greatest size of an object, breadth is average size, as a rule in a horizontal plane. Depth is size of an object on a vertical downwards. A distance between objects is a size of gap between them. There

are a number of properties of objects: square, volume, and form, which are determined by a combination of sizes of the object.

In the results of a comparison of magnitude of objects we find out a distance between bodies and cities, lengths of the rivers, height of mountains, square of countries, diameter of the Earth, distance up to stars. Many sciences have appeared due to a measurement of object magnitudes: geography, geometry, astronomy etc. Apparently, the mass of knowledge obtained in the results of a comparison of object magnitudes is the largest. The magnitude is a main property of the object. So long as we do not know it reliably, we doubt of existence of an object. For example, the absence of knowledge about magnitudes of elementary particles in a modern Physics calls a doubt in their existence.

1.5. UNHYPOTHETIC DESCRIPTION OF THE WORLD AROUND

Except a changeability and magnitude there are many other properties of the world around: warmth, light, sound etc. For their description the man also has entered the measurement standards, as a result of comparing properties are determined in degrees of temperature, amount of candles of luminosity, decibels of loudness etc. It allows determining properties of objects to determine influence of one property to the other: for example, the influence of the body temperature on its luminosity, length of a string on a tonality of a sound, temperature and pressure on gas volume. As a result the man finds dependencies between properties. Such approach is created all the description areas of behaviour of nature objects, for example, thermodynamics, optics, electrodynamics etc.

Hereinafter they select more complicated properties, which are a combination of simple ones. Other properties, on the contrary, are detailed and appear dependent from the earlier entered magnitudes. So there is a system of the description of the world around, where the magnitudes of properties are determined by a comparison with the measurement standards and the dependencies between properties of objects as dependencies between magnitudes are established.

Thus knowledge about objects of the world around obtained in this way allows to predict their behaviour, allows to be guided among them, to reconstruct their separate stages or even allows to construct such combinations of the phenomena, which did not happen in the world. For example, the man has created objects, which have overcome a terrestrial attraction and went out in space. In this process the results of human operations will correspond to intentions as accurately, as the properties of objects were compared with properties of the measurement standards. For want of it, naturally, the conditions of comparison should be satisfied.

In the represented process of receiving knowledge of a nature the hypothesis and the suppositions about mechanisms of the phenomena are not considered. The picture of the world is not created on them. Here the world around is studied, is

compared and is measured. And here it is necessary to understand the theory as the description of object properties of nature and description methods of human activity and their results.

The usual system of the unhypothetic description is created by mankind during millenniums and depends on many factors. Every man studies it on separate elements and never fully learns the whole system. He, as a rule, does not participate in revealing of new properties of the world and introduction of the new measurement standards. He has an impression that the measures of properties represent some world, which exists irrespective of him, exists objectively and eternally. He imagines, that there is a time, in which all events develop; there is a space, in which all objects of the world around are placed. As the results of such submission the man has questions: what is time? Is it any essence? Can it be discrete? Or is it turns to energy as assumed N.A. Kozyrev, and maybe, to space? The similar questions occur concerning space. Is it any essence? Is it material? Is it curved or rectilinearly and isotropic? Are two parallel direct lines intersected in it or are not intersected? Maybe they are intersected, and maybe are not intersected. Let's accept at first one paradigm, and then the other. These and similar hypothesises and doubts capture people, if they lose submission, where the world around, and where its description.

Entered by the man the measures of properties: space - for magnitude of objects, time - for their variability etc. are the description of the world around. This description could be different. Other properties could be chosen, other measurement standards could be entered. For example, the magnitude of the object can be characterized by capacity or volume, the description of interactions can be characterized by energy and force, the heat at the description of thermal processes can be characterized by entropy and energy. The methods of the description are changed in time. Among the different peoples the changeability of objects and their magnitude was determined differently in different times. Something, which today is considered as space and time, early was represented by the other. The ancient people had fabulous images (gods, titaniums, heroes, asurs, devas; their area of dwelling - hell, paradise and other worlds) alongside with the actual people and geographical objects. The description of other properties changed as well. It is necessary to remember and at researching always need to wonder whether it is the world around or it’s description. For example, what ether, substance, consciousness, spirit, mass, field, energy, force, charge, electron, meson, neutrino, photon, graviton, soliton, planet, star, galaxy, black hole, neutron star, Sannikov Land, Antarctic Continent are. Are these the description of the world around or its objects?

When we become to deal with some strange, we wonder what this is and how it is functioned. However not all questions are pertinent. We can ask about the object of the world around, what parts it consists of, what are its properties are. We can ask questions to the description, how we have defined it, what circumstances were, how it depends on them. The answers will add the unhypothetic description of the world around. It will be objective knowledge of nature, which further will not be discarded by descendants.

People often say that it is impossible to understand the world around without hypothesises and they consider that accepted in the beginning guesses and suppositions are checked up and confirmed, but the unconfirmed ones are thrown. Hypothesises, which will be agreed by the observable phenomena, remain.

The operational analysis of a human brain and process of thinking shows, that it is self-deception. The man has a temptation to be influenced by a bright hypothesis. It is possible to follow a hypothesis at once arises at the man in case, when all circumstances of problems are not clarified. To follow to a at once, and for clearing up of circumstances the years can be required. But the experienced expert will not afford to take a great interest in a hypothesis. He will continue complicated and hard work on study of object, its properties, and influence on them of other objects and circumstances. As the results he will achieve such understanding of features of object, that can foresee and to describe them, not attracting hypothesises.

CHAPTER 2

THE BASIC PRINCIPLES OF THE MECHANICS

2.1. MOVEMENT OF OBJECTS AND ITS MEASURE

The change of the world around happens in different forms. Movement is one of them. The gap between moving objects is changed. Measure of movement is the velocity. An average velocity of moving one object by relation to other is defined by ratio of a change of a distance $\mathbf{d}l$ between them per period $\mathbf{d}t$ to magnitude of this gap, i.e. $\upsilon = \mathbf{d}\,l/\mathbf{d}t$. Because change of gap between two objects is considered that velocity characterizes movement of one object with respect to the second. It is often forgotten and velocity is pertained only to one object. For example, the object moving relatively a surface of the Earth is considered as absolutely moving. They link frame of reference with the Earth, then abstract from the Earth and consider movement objects in this system and their velocities are represented by absolute properties of objects. It frequently results in errors. For example, if to take into account only velocity of a plane relatively the Earth, flight time from Novosibirsk to Moscow will be represented with a large error, if we do not take into account a velocity of a plane regarding air medium: force western wind can delay arrival of a plane for half an hour. Thus, velocity is not the property not of one object, but two. Therefore, for want of interaction of many objects it is necessary to consider their mutual velocities, i.e. to consider a time history of their mutual distances.

The disposition of objects from each other can be in three various directions. Therefore there are three velocities, which at description by their vector magnitude in cartesian frame are determined by three components of a vector of a velocity

$$\vec{v} = \vec{i}\, v_x + \vec{j} v_y + \vec{k} v_z. \qquad (2.1)$$

The velocity $\vec{v}$ can remain constant. In this case the object motions relatively other is equable and rectilinear. In those specific cases of zero velocity the distance between objects is not changed also they are in rest from each other. At the same time relatively other objects they can have any velocities.

The velocity of a body can be changed. For example, motionless object starts moving or during movement the velocity is increased, or its the direction is changed. As the velocity is proportional to change distances between two bodies, the change of a velocity can be caused by change of movement each from them. For performance of a change of a velocity of a body the concept of acceleration $\vec{w}$ was entered which reflects change velocities only of one object. For it some readout system is imagined, which in a moment *t* has identical with a body velocity $\vec{v}$, and hereinafter its velocity is not changed. This system is called inertial in that sense, that it moves on inertia, i.e. nobody influences it changes its velocity. In a further instant $t+\Delta t$ change of a velocity in the attitude to the second body is not considered any more, but change $\Delta\vec{v}$ in relation to this unaccelerated system is considered. The average of acceleration is determined as attitude of a modification of a velocity for a gap Δt to magnitude of a gap and in a vector kind is noted

$$\vec{w} = \frac{\Delta\vec{v}}{\Delta t}. \qquad (2.2)$$

So the certain acceleration of a body is already its absolute performance, instead of relative, bound to other objects.

2.2. INTERACTION AND ITS DESCRIPTION

Why does the body get acceleration? The man has been convinced, that cases accelerated movements of bodies have the reason - other bodies act on them. We will understand action of one object on the other as an ability of the first body to reduce in movement the second body or to change its movement. To change movement of a body means to change its velocity either on magnitude, or on a direction, i.e. to inform it acceleration $\vec{w}$ to him. From here the magnitude of action on a body is determined by magnitude of an acceleration, which it gets or will get, when this action begins. If there is not acceleration, there is no action, or the action of the first body is compensated by a converse on a direction by the other action. For example, the Earth attracts the stone, suspended on a spring, but it does not change the movement, as a spring counteracts this. It creates action, which is

converse in a direction to the action of the Earth, and the stone is in rest. The spring for want of it is stretched on any magnitude Δl.

For expression of action on a body the force was entered. In the indicated example the deformation of the spring Δl visually represents the force of action. By this term we designate a property of a human body to execute any operation, for example, to contract a spring.

The counteraction of the third body can be expressed not only by expansion of a spring, but also by its compression, if it, for example, is located between by attracted bodies. The third body can be not a spring, and its the deformation can be measured by deformation gauges or with the help of piezoeffect. Besides the counteraction of the third body can be expressed by other action. For want of measurement of interaction of two, suspended on filaments, charged balls their deviation from a vertical was counteracted by a gravitational attraction to the Earth.

Historically it has developed so, that action one has begun to determine by magnitude of a deformation Δl. The concept of a force $\vec{F}$ of action, which is determined by magnitude of a deformation Δl, is created by standard action. The scale of a force is constructed so that the unit of a force in any place of a scale corresponds to the same action on certain standard body. Now for a standard body the platinum - iridium cylinder by a diameter and height 39 mm is accepted which is kept in Paris. Being acted by the Earth it stretches a spring on certain length, which expresses magnitude of a force in one kgf (in a technical system MKGFS). So, the action of the Earth on the standard implies that it drops with an acceleration of 9,8 m/sec^2. We circumscribe this action by magnitude forces in F = 1 kgf.

If to join to a spring *n* of the measurement standards, they stretch it on length equivalent *n* kg. And we speak, that the Earth influences them by a force in $F = n$ kgf. Other body, which is under action, can stretch spring too on n_1 kgf, i.e. as n_1 of standard cylinders. But such body, as well as whole other bodies, all drop with an acceleration of 9,8 m/sec^2. Thus, at acting on the different bodies with the same acceleration the force of acting on them will be different. That is, only one force cannot characterise action on a body. Therefore a mass of a body $m = n$ – as amount of standard bodies is inputted when at action with an identical acceleration these bodies stretch the spring on the same magnitude, as body. Then for any action, which is measured by magnitude of force $\vec{F}$, on any body, which is equivalent *m* standards, the acceleration of a body will be

$$\vec{w} = 9.8 \cdot \vec{F}/m. \qquad (2.3)$$

The force has that a direction, as the acceleration, is similar to it(him) and in Space is characterized by three component: $\vec{F} = \vec{i}\, F_x + \vec{j} F_y + \vec{k} F_z$.

Unlike MKGCC in a system of a SI for unit of a force $\vec{F}$ 1 Newton (N) is accepted which is characterized by such action on the standard cylinder, for want of which it is gone with an acceleration 1 m/sec^2. In a system of a SI a parity (ratio) (2.3) in a vector kind will be noted so:

$$\vec{w} = \vec{F}/m. \qquad (2.4)$$

Expression (2.4), known as the second Newton's law, in considered system of units is fair for any actions. And as we see it grows out of our choice of parameters of action and units measurements. Similarly the first and the third Newton's laws are the corollary of our approach. For example, the first law: if other bodies do not act a body, it saves rectilinear and equable movement. It is a corollary of initial determination of action. It is an ability of one body to reduce in movement the second body or to change its movement. From the given determination follows: if other bodies do not act a body it doesn't change its movement and remains in rest or is gone rectilinearly and is equable.

The third Newton's law is, that a force of action of the first body on the second one F_{12} is equal to a force of a counteraction of the second body on the first one F_{21} and is opposite to it in a direction:

$$F_{12} = -F_{21}. \qquad (2.5)$$

This law is the corollary of determination of a force, which expresses a counteraction of the third body to interaction of two bodies. Magnitude of a counteraction, as in example a deformation of a spring, concerns the first body, as well as the second one. That is the same magnitude, which as well as the spring is directed on bodies for want of their mutual attraction and from bodies - for want of their mutual repulsion. In other words, the same force is directed mutually opposite on interacting bodies.

Here we will mark, that despite of expressions, existing in custom, (the force on bodies, directed force etc.) the force is not essence, substation, spirit etc. It is entered by us concept for the description of action.

So, the action on a body is exhibited in it an acceleration. The man expresses and describes the action as a force and mass of a considered body. In a selected system of units the mass unequivocally characterizes connection of an acceleration of a body being under action, with the measured force. Three important conclusions follow from here. At first, in all interactions a mass of a body will be the same. Therefore it is senseless to search for a divergence between a gravitational and inertial mass. These searches are reduced to finding errors of a measurement of same scales, in different interactions. Secondly, from determination of a mass follows, that it cannot be changed from other interaction or movement. That is the mass, contrary to accept in a TR, basically cannot depend on a velocity. Thirdly, the mass is peculiar only to that object, which can get acceleration in outcome of action of the other object, and it is possible to measure this action as a force. As for light, field, energy etc. given process is not realised, it is impossible to attribute a mass by it. Thus, field, energy and the entered articles (photon, graviton etc.) do not have mass.

2.3. DETERMINATION OF FORCE AT DIFFERENT INTERACTIONS

The acceleration of a body is determined by the second law of Newton (2.4). Its mass *m* once measured, is known, and if the expression for a force $\vec{F}$, is known, then as the results of an integration of an acceleration on time a velocity movements of a body is received, and after an integration of a velocity - dependence of a path of movement of a body on time *t*. Thus movement of the bodies is determined completely, which are influenced by the other bodies by a force $\vec{F}$. The force $\vec{F}$ depends on properties of influencing bodies, can depend on a path of movement of a body either distances between bodies $\vec{r}$, velocity $\vec{v}$ and time *t*, i.e. the force can be the function $\vec{F}$ $(\vec{r}, \vec{v}, t)$.

There are many different kinds of actions. For example, the powder gases on a shell in a trunk of a gun by a force act

$$F = SP, \qquad (2.6)$$

where *S* - cross-section of a shell; *P* - overpressure in a trunk of a gun.
In accordance with movement of a shell volume of powder gases grows ($V = Sl$), pressure drops, for example under the isentropic law PV^k = const, and consequently is changed with growth *l* so:

$$P = P_0 (l_0 / l)^k$$

where l_0 - initial volume of powder gases; P_0 - their initial pressure.

After a substitution in (2.6) the following equation for the force

$$F = SP_0 (l_0 / l)^k, \qquad (2.7)$$

which acts on a shell in a trunk of a gun, is received. This force depends on a path according to the exponential law. If instead of powder gases the spring would act on a shell, within the limits of an operation of the law of the Hooke a force would depend on a path bodies under the linear law.

According to from a trunk the shell goes in atmosphere. If it a velocity $\vec{v}$, and velocity of an air medium $\vec{u} = \vec{u}(x,y,z)$, in the elementary case spherical shell by a radius *r* will be acted on it by the force

$$\vec{F} = Cf \frac{\rho |\vec{u} - \vec{v}| (\vec{u} - \vec{v})}{2}, \qquad (2.8)$$

where *C* - aerodynamic factor; ρ - air density; $f = \pi r^2$ - perpendicular to circulated stream the cross-section area of the shell.

Nonlinearly aerodynamic factor *C* depends on a velocity of a shell relatively an air, its viscosity, grain of a shell etc. The density of an air is changed with

height. The velocity of a wind $\vec{u}$ also is changed along a path of a shell. Therefore force $\vec{F}$ by a complicated image depends both on a path and the velocities of a shell and for its determination you will need knowledge of practically all properties of atmosphere. With the certain approximations and simplifications these properties can be given as a field of magnitudes in all atmosphere, where the shell will pass. By a sequential integration of an equation (2.4) the movement of a shell will be determined.

If earlier we spoke about the action on a body of other body, in the last example a shell during its movement is influenced by different parts of atmosphere, i.e. many objects. Nevertheless, by determining a force of their action on a body, we can calculate its movement.

Let our shell supply with a jet engine, which on a command from the Earth in any moment is included and with the help of jet force will introduce it to orbit. This force is a variable in time $\vec{F} = \vec{F}(t)$. Thus we see that the forces can depend on a variable integration $\vec{r}$, $\vec{v}$, t, of the equation (2.4), i.e. from the parameters of the movement of a considered body. Except that force depends on a number of properties, of interacting, objects. One of the primal problems of physics is determination of these forces. The analysis of known forces shows that all of them are determined in the results of a measurement. Measurements, as a rule, depend on many factors. At the beginning it is necessary to select a standard situation and to make a measurement of forces in it. Then the properties are determined, on which the similar situations differ from standard. The measures of these properties are entered and the measurements of their influence on magnitude of a force in a standard situation are made. After fulfilment of such measurements there is a possibility of account of forces in various situations.

If the force of action on a body is established and during movement is known, it is possible to integrate the equation (2.4). As the results the movement of a body will be determined completely. The solution of a problem of interaction of two bodies or actions on a body of many bodies consists of it. It is obvious, the force method consists of three stages. At the beginning we find a force of action, being completely based on the results of measurements. At the second stage we determine the correspondence between a force of action on a body and its acceleration, i.e. measure a mass of a body. At the third stage we consider the movement of a body regards to a known acceleration, in spite of all this the experimental results are not attracted.

2.4. ABOUT DEPENDENCE OF A FORCE ON AN ACCELERATION

Because of theoretical reasoning in an electrodynamics they have entered the force, which depends on acceleration. For example, in many monographs both textbooks, including [26] and [97], the expression for electric field strength of a moving charge q_1 is resulted. Using it, we will note the magnitude of a force of action on a motionless charge q_2:

$$\vec{F} = \frac{q_1 q_2}{\varepsilon\left(r_t - \vec{r}_t\vec{\beta}\right)^{3/2}}\left\{\left(1-\beta^2\right)\left(\vec{r}_t - \vec{\beta}r_t\right) + \vec{r}_t \times \left(\vec{r}_t - \vec{\beta}r_t\right)\times \dot{\vec{\beta}}/c^2\right\}, \qquad (2.9)$$

where $\vec{\beta} = \vec{v}/c$ - dimensionless velocity of a charge q_1; $\vec{v}$ - its velocity relatively a charge q_2; $\dot{\vec{\beta}} = \dot{\vec{v}}/c$ – dimensionless acceleration of a charge q_1; $\vec{r}_t$ – - position vector from a charge q_1 up to a charge q_2 in an instant t ', taken with " by delay ":

$$t' = t - r_t/c. \qquad (2.10)$$

This expression follows from the Lienar-Vihert's retarded potentials and has been known in an electrodynamics since XIX centuries. Apparently, by analogy with (2.9) W. Weber [123] was offered the following expression for forces:

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{\varepsilon r^3}\left(1 - 0{,}5\beta^2 + r\dot{\beta}/c\right). \qquad (2.11)$$

Recently formula of Weber attracts attention of many scientists [80, 124, 125]. Tomas E. Phipps [107, 108] has offered similar expression for a force

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{\varepsilon r^3}\left(\sqrt{1-\beta^2} + \frac{r}{c\sqrt{1-\beta^2}}\dot{\beta}\right), \qquad (2.12)$$

which also depends on an acceleration. The above mentioned scientists intend to advance a new electrodynamics alternate to a Theory of Relativity.

The forces dependent on acceleration were entered also in a mechanics of a liquid. It is supposed, that on a particle by a diameter d, if it is gone with a velocity $\vec{u}$ is acted by the force [27]

$$\vec{F} = -3\pi\rho_l \nu \mathbf{d}\vec{u} - \frac{\pi d_l^3 \rho_l}{12}\frac{\mathbf{d}\vec{u}}{\mathbf{d}t} - \frac{3d^2\rho_l(\pi\nu)^{-0.5}}{2}\int_0^t (t-\tau)^{-0.5}\frac{\mathbf{d}\vec{u}}{\mathbf{d}t}\mathbf{d}\tau, \qquad (2.13)$$

where ρ_l - density of a liquid; ν - its kinematic viscosity.

The first addend in the right part represents a force of Stoks $\vec{F}_S$, which really acts on a particle that is confirmed numerously by experiments for want of small Reynold's numbers. The second addend is named as Basse's force, and third one - force of joined masses. Expression (2.13) is obtained theoretically for want of consideration of local interaction of a spherical particle, which is gone in a liquid with a relative velocity $\vec{v}$. Then the obtained results were indicated to a velocity $\vec{u}$ of a particle relatively undisturbed liquid.

So, the forces dependent on acceleration are used in different areas of mechanics. However there are scientists, who disagree with it. For example, Richard A. Valdron [121] has proved, that the dependence of a force of acceleration contradicts the second Newton's law, Andre K.T. Assis [81] has shown that the force of Weber (2.11) leads to paradoxical results. Nevertheless in a holding controversy [114] he has acted as the supporter of a force, dependent upon accelerations [82].

The dependence of a force upon acceleration has a fundamental significance. Here the question is not limited by, whether there are these forces actually, for example forces of joined masses and Basse (2.13), or these are error forces. The dependence of a force on acceleration in an electrodynamics in the form (2.9) hereinafter results in submissions about a radiation of a moving charge, power loss on a radiation and braking of a charge by this radiation. These conclusions are applied to an electron, moving on orbit around charged nucleus, because of that it was concluded, that, the electron moving with acceleration should lose energy, therefore has inevitably to "fall" on the nucleus. Just for this reason the planetary model of atom has not received further development, all physics of microcosms hereinafter was developed in the direction of creating the mathematical probability models of elementary particles, instead of studying physic nature of microcosms.

Why cannot the force depend on acceleration? To answer this question, at the beginning we will adduce R.A. Valdron's proof, by modifying it. From expressions for a force (2.9), (2.11), (2.12) and (2.13) it is visible, that it is possible to note it as follows:

$$\vec{F} = A\vec{B}(r, \upsilon, w), \tag{2.14}$$

where A - general multiplicand for all component forces, not dependent from accelerations. For expressions (2.9), (2.11) and (2.12) multiplicands $A = q_1 q_2 / \varepsilon$, and for a force in a liquid (2.13) $A = \rho_l$.. Multiplicands A are not only general for all addends of a force, but contains also parameters, from which the multiplicand $\vec{B}$ does not depend. The last condition is indispensable for a further conclusion.

For a small value of acceleration, by decomposing a multiplicand $\vec{B}$ (r, υ, w) in Teilor series on degrees w, we can note the term of series with number i an addend

$$\vec{F} = A\left(\vec{B}_0 + \vec{B}'w + \frac{\vec{B}''}{2}w^2 + \ldots + \frac{\vec{B}^{(\mathrm{i})}}{\mathrm{i}!}w^{\mathrm{i}}\right), \tag{2.15}$$

where $\vec{B}_0$ - significance of a multiplicand B for want of w=0; $\vec{B}', \vec{B}'', \ldots, \vec{B}^{(\mathrm{i})}$ - derivatives from $\vec{B}$ on w.

If the force (2.14) depends on an acceleration vector, then it is necessary to decompose a force on components of a vector in the Teilor number.

For want of above marked conditions the expression (2.15) is general and fair for interactions for want of any change of magnitudes of parameters, from which the multiplicands A, $\vec{B}$ and derivatives depend $\vec{B}', \vec{B}'', \ldots, \vec{B}^{(i)}$. At action on body of force (2.15), according to the second law (2.4), it will get acceleration

$$\vec{w} = \vec{F}/m. \tag{2.16}$$

We will change parameters of an influencing object, which enter only in a multiplicand A so that the force $\vec{F}$ has increased in n times. For it is necessary to increase a charge q_2 of an influencing body in n times for forces (2.9), (2.11) and (2.12) or density of a liquid ρ_l for a force (2.13). So, for want of $A_1 = n\,A$ a force will increase also in n times: $\vec{F}_1 = \vec{F}\,n$. For want of the same mass of a particle and constant distance and velocity the acceleration, according to (2.16), for want of increased force will increase in n time too: $w_1 = n\,w$. The expression (2.15) is general, therefore it should be fair and for the changed parameters of interaction, too

$$\vec{F}_1 = A_1\left(\vec{B}_0 + \vec{B}'w_1 + \frac{\vec{B}''}{2}w_1^2 + \ldots + \frac{\vec{B}^{(\mathrm{i})}}{i!}w_1^{\mathrm{i}}\right). \tag{2.17}$$

After a substitution of significance, $\vec{F}_1$, A_1, w_1 in (2.17) is received

$$\vec{F} = A\left(\vec{B}_0 + \vec{B}'nw + \frac{\vec{B}''}{2}n^2w^2 + \ldots + \frac{\vec{B}^{(i)}}{i!}n^i w^i\right). \tag{2.18}$$

However expression (2.18) will not be agreed expression (2.15). Therefore, or the initial expression for a force (2.14) is incorrect, or Newton's law is incorrect (2.16), from which an acceleration w_1 is determined. Connection between force and acceleration (2.16) is stipulated, as we were convinced, by the determination concepts of a force, mass, acceleration and units of their measurement, and there cannot be the other one. In that case the dependence of a force up on acceleration is the error (see also [113, 114]).

It is uneasy to be convinced, that each the formulas (2.9), (2.11) and (2.13), subjected to the same procedure, conflicts to itself. It means, for want of applica-

tion of forces dependent on an acceleration, absurd results are inevitably received, for example, acceleration in the plane capacitor of a charged particle up to an infinite velocity [82] or unlimited self-acceleration of a charged particle even in the absence of action on it from of the other bodies (see §74 in [26]).

At the beginning of article R.A. Valdron [121] marks, that some authors, including V. Ritss, consider dependence of a force on a velocity and its maximum derivatives. Such submission has development lately, however in application to other magnitudes - strengths of electrical E and magnetic H of fields. Let's remind, that the strengths are forces normalized to a unit of electrical and magnetic charges accordingly. As the electrical field is proportional to a velocity of a change of a magnetic field, and magnetic - velocity of a change of an electrical field, in case of movement of a charge with a variable velocity many authors desire to enter fields of the higher order, which would be determine by maximum derivatives from a velocity of movement of a charge.

So can a force depend on a derivative of the force, on a higher force than acceleration? R.A. Valdron, supposing, that such dependence is possible, the does not proof anything in its support or against.

As we already considered, force of action on a body and its acceleration are different expressions of action. Basically, the description of interactions can be executed and without a force. For example, let us assume we want to study the action of a spring on an aluminium ball in a diameter of 1 sm. We can measure an acceleration w of a ball for want of compression of a spring on magnitude x. By conducting a series of experiments for want of different compression of a spring x, we will receive the dependence of an acceleration of a ball on compression of a spring:

$$w = w_{1a}(x). \qquad (2.19)$$

The equation (2.19) already allows to decide a problem of action of a spring on a ball. The first derivative from transition on time is a velocity ball

$$\frac{\mathbf{d}x}{\mathbf{d}t} = \upsilon\ , \qquad (2.20)$$

and the second one is it an acceleration

$$\frac{\mathbf{d}\upsilon}{\mathbf{d}t} = \frac{\mathbf{d}\upsilon}{\mathbf{d}x}\frac{\mathbf{d}x}{\mathbf{d}t} = \upsilon\frac{\mathbf{d}\upsilon}{\mathbf{d}x} = w_{1a}(x)\ . \qquad (2.21)$$

The solution of a system of the differential equations (2.20) - (2.21) will present a change of a path and velocity of a ball in time, i.e. we will receive the description of its movement.

For an aluminium ball in a diameter of 2 cm the same measurements can be conducted and received dependence $w_{2a}(x)$. For a leaden ball with a diameter 1 cm this dependence $w_{1c}(x)$ also can be obtained. The dependence for accelerations of

other bodies can be similarly measured and with the help of the equations (2.20) - (2.21) their movements are designed.

This forceless method of the description of interactions can be upgraded if to place the correspondence between an acceleration of aluminium balls of different sizes as factor K_a (d). Then on dependence $w_{1a}(x)$ can determine an acceleration of a ball in a diameter of $d = n$ (cm):

$$w_n(x) = K_a(d)\ w_{1a}(x).$$

The same correspondence is determined between accelerations of a leaden ball and aluminium one. This modernization can be distributed to balls from other materials, on bodies of other form etc.

The usual method of the description of interactions with the help of forces is even more general modernization of the considered method. Here acceleration is measured only for one body-standard kg $w = F$, and we call it with a force. The acceleration of all remaining bodies is recalculated with the help of factor of the correspondence m, which shows, how many times for want of the same action the acceleration of a body is less than an acceleration of the measurement standard, i.e. $w = F/m$.

So, the force of action on a body is its acceleration, but expressed in other units. Therefore, naturally, force of action on the body can depend neither of an acceleration of a body, nor of a derivative of an acceleration in time. In this connection all forces, which depend on a derivative of a velocity with time, cannot be measured. They are entered theoretically and are the error. The nearest physics problem is the analysis of influence of such forces on treatment of natural phenomena, it correction and creation of new submission about the world around.

Especially it is necessary to mark the expression for a force (2.9). It is introduced from the equations of electrodynamics, which will be as shown further, by generalising of experiments. But this expression represents a force or acceleration in a period of time, previous considered, in according with the equation (2.10). Thus, (2.9) may not be used for calculation of velocity and and path of movement of a body which it is acted. That is, in essence, the expression (2.9) does not represent a force, which can be used in the second law of mechanics (2.4).

2.5. FORCE AND ENERGY

In mechanics for performance of action on a body the concepts are entered: work, potential and kinetic energy. If a body is influenced by the other bodies by forces F_1, F_2..., F_n (Fig. 2.1) and the body during $\mathbf{d}t$ moves in a path $\mathbf{d}l$, the scalar product of component forces on this transition is called as work:

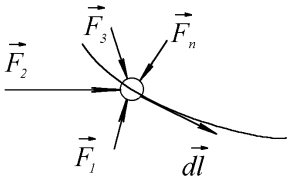


Fig.2.1. Forces of action on a body of several bodies.

$$\mathbf{d}A = \vec{F}_1\,\mathbf{d}\vec{l} + \vec{F}_2\,\mathbf{d}\vec{l} + \ldots = \sum_i \vec{F}_i\,\mathbf{d}\vec{l}\ . \quad (2.22)$$

By analogy to household activity they say, that the influencing the bodies perform the job during $\mathbf{d}t$. For a final space of time t the work of influencing bodies will be

$$A = \int_0^t \sum_i \vec{F}_i\,\mathbf{d}\vec{l} = \sum_i \int_0^t \vec{F}_i\,\frac{\mathbf{d}\vec{l}}{\mathbf{d}t}\mathbf{d}t = \sum_i \int_0^t \vec{F}_i\,\vec{v}\,\mathbf{d}t\,, \quad (2.23)$$

i.e. it is determined by the sum of scalar products acting forces on the velocity of a body.

During the of realization of this work with influencing bodies something happens, for example horse loses a stock of vital forces, and the tractor spends fuel. It is accepted to name such process as power loss of an influencing body, which is spent for fulfilment of work (2.23). The spent energy as though potentially contained in interacting bodies, therefore it is called potential energy U and is equated to the work with a converse:

$$U = -A = -\sum_i \int_0^t \vec{F}_i\,\vec{v}\,\mathbf{d}t\ . \quad (2.24)$$

As the results of action the moving body gets a velocity $\vec{v}$. By movement it can realize any action, for example accelerated shell in the results of impact can destroy a wall of a building. Therefore they say, that the body has a kinetic energy

$$E_c = \frac{mv^2}{2}\,. \quad (2.25)$$

Entered magnitudes of work, potential and kinetic energy allow to consider the process of interaction in other concepts. Nevertheless this method completely follows from a method of forces, as you can see below. In case of action on a body of several bodies we will note the second Newton 's law (2.4) as follows:

$$m\frac{\mathbf{d}\vec{v}}{\mathbf{d}t} = \sum_i \vec{F}_i\ . \quad (2.26)$$

Let us multiple the left and right parts (2.26) on $\vec{v}$ and conduct some sequential transformations

$$m\vec{v}\frac{\mathbf{d}\vec{v}}{\mathbf{d}t} = \sum_i \vec{F}_i\vec{v}; \qquad m\int \vec{v}\,\mathbf{d}\vec{v} = \sum_i \int \vec{F}_i\vec{v}\,\mathbf{d}t; \qquad \frac{mv^2}{2} = \sum_i \int \vec{F}_i\vec{v}\,\mathbf{d}t + C\,, \quad (2.27)$$

where C - independent of time constant of an integration.

With allowance of determinations (2.24) and (2.25) the last expression will be noted in this way:

$$E_f = E_c + U = C. \quad (2.28)$$

The sum of a kinetic energy E_c of a moving body and potential energy U of influencing bodies is named as full mechanical energy E_f of interacting bodies. Full energy of interaction E_f, as(how) is visible from (2.28), is not changed in time and remains a constant during interaction. In physics an energy conservation law is called the law of nature. However we know, that the equation (2.28) is stipulated by the second Newton's law and is the corollary of our approach and our choice of concepts. The fact that the law preservations of mechanical energy (2.28) is not the law of nature, but the corollary of our method of the description of interactions, does not reduce its significance. If it was the law of a nature, as, for example, the law of world gravitations or Coulomb's law, and it was obtained as the result of a measurement of properties bodies, it would be possible to doubt of its universality and exactitude. The fact that the law is stipulated by our approach, testifies that it should be executed with an absolute exactitude at any circumstances.

At a derivation (2.28) we considered a beginning of a movement of a body from quiescence. If the body has an initial velocity v_0, as the result of an integration of expressions (2.27) we receive

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = \sum_i \int_{t_0}^t \vec{F}_i\,\vec{v}\,\mathbf{d}t = \sum_i \int_0^t \vec{F}_i\,\vec{v}\,\mathbf{d}t - \sum_i \int_0^{t_0} \vec{F}_i\,\vec{v}\,\mathbf{d}t = -(U - U_0)\,, \quad (2.29)$$

that is possible to note as:

$$\Delta E_f = \Delta E_c + \Delta U = 0, \quad (2.30)$$

i.e. the increment of full energy of interacting bodies during movement is equal to zero.

At presence of actions on a body it can move at a constant velocity: for example automobile actuated by sprockets, or plane with a jet engine. In these cases the influencing objects are counteracted an air by a resistance to movement. As a velocity is not changed, ΔE_c =0 and, the potential energy is not according to (2.29), is not changed, either that is i.e. ΔU = 0. However energy of fuel ΔE_T is spent. It is spent for heat of an air and surface of a body, i.e. is selected as energy $\Delta E_t = -\Delta E_T$. Supplementing an increment of full mechanical energy according to (2.30) with magnitudes ΔE_t and ΔE_T, we receive

$$\Delta E_f = \Delta E_c + \Delta U + \Delta E_t + \Delta E_T = 0. \quad (2.31)$$

As the increment of each component concerns an initial and final position of the acting bodies, for example $\Delta E_f = E_f - E_{f0}$, the full energy in a final condition will be noted according to (2.31) as

$$E_f = E_c + U + E_t + E_T = E_{c0} + U_0 + E_{t0} + E_{T0} = C. \qquad (2.32)$$

That is the certain energy thus remains constant during interaction.

The connection between mechanical energy and thermal is established as the result of diverse measurements of an amount of heat selected during fulfilment of mechanical work. It is expressed by mechanical equivalent heats η = 4.187 J/cal, or 427 kgf·m /kcal. Energy, selected by fuel, during combustion is measured as a thermal energy. Thus, all kinds of energy as the results experimental standardization are compared to mechanical energy. Therefore energy conservation law becomes applied both for mechanical, and for thermal, chemical and other kinds energies, which are subjected to experimental comparison.

So, the energy conservation law is our approach, instead of the law of nature. Especially it is important to remember during consideration of the new phenomena. The preservation of energy is stipulated by measurements. If the phenomenon leaves frameworks, in which the measurements were conducted, the defiance of the law is possible. Such violation happens during measurement of a thermal energy selected in the reservation after the impact of piercing shells: it exceeds kinetic in 1.2, 1.48 and 4 times when the mass of a shell is 0.0615, 0.085 and 4.05 kg, accordingly (Iavorski V. Energy “from nowhere” // Science and life. - 1998, j 10.-Pp.78 - 79). This violation is stipulated by recalculation of selected heat in mechanical energy with the help of equivalent η = 4.187 J/cal, which was measured during transition of movement in heat with the help of friction. As we can see, during high-speed impacts a mechanical equivalent of a heat is different and additionally depends on a masses: in the indicated cases it drops with magnification mass of a plunger η = 3.5, 2.8 and 1.05 J/cal.

In modern physics the power methods of account interactions predominate: the transfer of energy between interacting bodies, transitions of object to different levels of energy etc are considered. There is a judgement that power methods are more general. In order to prove this statement the movement equations are derived from the law of energy conservation.

However, the initial equations of a mechanics all the same are. The equation (2.28) for energy is obtained in the results of an integration of the equation of movement (2.26). Differentiating (2.27) (namely so was done the derivation of the canonical equations of the Hamilton by minimization of action), we will naturally receive the equations of mechanics. The concepts of mechanics: velocity, acceleration, force and mass are initial ones. The concepts of energy and power equations are based on them. These basis are immediately connected with the measurements properties of the world around, therefore the information of a problem in all vague situations to initial mechanical concepts allows to make clear problems and to find truth.

CHAPTER 3

THE BASIC PRINCIPLES OF THE DESCRIPTION OF CHARGED, AND MAGNETIZED BODIES INTERACTIONS

3.1. INTERACTION OF THE MOTIONLESS CHARGED BODIES

As it is known, originally electrization of bodies is received by friction them one about other. After the other methods of deriving electricity have become known. But in all cases the emergence of electricity in bodies is characterized by that the bodies begin to be attracted or to be repelled from each other. The influence of the charged bodies was studied by a measurement of its force, and as a spring Coulomb used a thin wire, on the angle of which the twisting force of interaction between two charged bodies was determined. By experience it was established, that the forces of influence of the charged bodies against each other depend on a distance between them. It has appeared, that at of same electrization of bodies the force is inversely proportional to the square of a distance.

But as it is possible to make electric the same bodies differently, i.e. the force of influence between two bodies at the same distance can have different magnitude, there was a necessity somehow to describe magnitude of electricity. For it they have selected an identical electricity of two small bodies, which influence each other by force in unit, if the distance between them is equal to unit. For example, at distance of 1 m and the force 1 N the unit of electricity is named a coulomb (K). If at this electricity of one of the bodies other one will have such electricity, that at the same distance the force of their influence will be equal q_2 of units, the magnitude of electricity of the second body, as was accepted, is equal to number q_2.

Selected magnitude of an electrization has been named as a charge of electricity. If at of unit electricity of the second body the electricity of the first one will be of such magnitude, that the force of influence between them at unit distance will be q_1 of units, magnitude of a charge of the first body will be equal q_1. Therefore at electricity of bodies, measured charges q_1 and q_2, the force of influence between them on each unit of a charge of the first body is equal q_2, and for all of q_1 of units of a charge will be equal to a product $q_1\, q_2$. And as the force between the charged bodies is inversely proportional to the square of a distance R between them, the expression for a force at any distances in a vector kind will be noted so:

$$\vec{F} = \frac{q_1 q_2 \vec{R}}{\varepsilon R^3}. \qquad (3.1)$$

This expression is known as law of the Coulomb's law. Factor ε is entered to have a possibility to use magnitudes of charges without dependence from that, in what medium there are interacting bodies. Usually the magnitude of a charge is

measured in any certain medium (in an air or in vacuum) and factor ε takes into account a change of influence between the charged bodies at location them with the same electricity in other medium. In spite of the fact that the magnitude of electricity is founded on a force of action between bodies, the magnitudes of charges of bodies are not changed during changing a medium between them. Thus, the action of the charged bodies against each other, which can have different magnitude in different mediums, is described by the equation (3.1) by means of electrical charges, which do not depend on a medium. This promoted the origin of the estimation, that the electrical charges are some material essence.

From considered it is visible, that the charge is not any essence or substation. This concept entered by the person, for the description of the action of the charged bodies against each other. The charged body can have any charge. If on the same principle the description of gravitational action was constructed, the similar magnitude of a charge for each body would be constant, as any two bodies influence against each other (at the same distance) always uniformly. Units of a charge for gravitational action would remain the same. For example, it is possible to accept, that the body with a mass 123 t has a charge equal 1 C, as it will be attracted to the same body by a force 1 N.

With the passage of time the word "charge" got a new sense. With application of expressions of a type: there are charges, are changed charges, interact charges - understanding a charge as the independent essence was advanced. With discovery of electrons (as particles with an identical electrical charge), the estimation about existence of charges as some substation was ratified. The availability of an electron with a constants property to influence on itself similar one always by the same image, does not prove existence of electrical substation. The introduction of such substation is similar to entering the substations of weight or locomotive traction caused by the Earth acting on a body or the locomotive acting on the coaches. Despite "substation" and "action" are not only the unequivalent concepts, but also generally represent incomparable objects, the substation for gravitational influence as gravitons is entered in a modern physicist by analogy to electricity.

The submission of charges as the carriers of electrical substation promoted emergence of the statements about conservation of charges, though the experiments convince us about emergence and vanishing of an electrization. The opinion about conservation of charges as substation has conduced to the statement about independence of forces of interaction of the charged bodies from their relative movement, which is erroneous, as it will be shown below.

The magnitude of electricity is changed at the charged bodies connected by conductor and the certain changes happen in a conductor, for example heating. The velocity of the change of electricity is entered

$$I = \frac{\mathbf{d}q}{\mathbf{d}t}, \qquad (3.2)$$

which is called electric current. That fact is important at the approach to electrical forces that there is an action of this body on magnet at availability of current in the conducting body. It was established by experiences of Roland, Ahenvald, Roentgen and etc. that not only change of electricity of bodies causes the accelerated movement of the magnetized bodies as well as the movement of bodies with a constant electricity do, i.e. it influences them. It is possible that many have begun to represent current in conductor as movement of charges thereat.

Whether the charges or electrical substation move, whether there are quantums of energy and those similar hypothetical models of this phenomenon - we will not consider and review. There is a precise and certain problem before us - to have a possibility to determine interaction of the charged and magnetized bodies in case of a change of their electrization and magnetization or at their movement. Such possibility can be reached only by measurement of actions of the standard bodies against each other which is in the certain condition and by the comparison of magnitude of the condition of considered bodies with the appropriate condition of the standard bodies instead introduction of prospective mechanisms of these actions. So the standard action of bodies with such electrization is measured for the charged bodies when they are attracted with the force of one unit at a distance of one unit. The magnitude of electricity as a charge is measured for any body by comparing its force of influence with the force of standard influence. It is necessary to say that the magnitude of a charge can be measured by a comparison of other properties of the charged bodies for example by a comparison of parted substance at electrolysis etc. However all comparisons are based on units of electricity defined on interaction of bodies.

3.2. PROBLEM OF THE DESCRIPTION OF INTERACTIONS OF THE MOVED CHARGED BODIES

The force of interaction of two motionless charged bodies is expressed by the equation (3.1). It is enough to know force and mass of each bodies m_1 and m_2 for the evolution of their accelerations w_1 and w_2 and consequent movements. But when the movements of bodies begin, will the magnitude of a force be described by the equation (3.1) or will it depend on a velocity of their relative movement?

Before answering this question let’s consider how it would be possible to define dependence of force on velocity of relative movement of bodies. If the force of their influence against each other can be measured at resting bodies by a spring located between them then the spring can be joined only to the second body in case of movement of one body. The moving body will realize its movement and the motionless body which is under its influence will deform a spring, the second end of which should now be joined either to installation or to the Earth. The magnitude of deformation of a spring represents the magnitude of force from the moving body on the motionless one. And as factor of the correspondence m is known (unit of acceleration of the motionless body corresponds to m units of force) then acceleration of movement will be also defined by this measurement. If the spring is unconnected then the body will begin to move with this acceleration relatively of that body which was connected by the second end of the spring i.e. in relation

to the Earth. Therefore acceleration determined in this way represents a change of movement of a body relatively of the Earth.

Measuring the force on a motionless body at various velocities of a moved body in this way it is possible to receive expression for the force and, consequently, for the acceleration of movement of a motionless body depending on the velocity. But the moved body tests influence of a motionless one, too. Is it possible to define its acceleration?

Deformations of a spring pertain both to one and the other body in case of motionless interacting bodies. It is enough to know force of their influence for defining the accelerations if masses of bodies are known.

Can we tell that the spring's deformation, which is made by a motionless body, will also represent force on moved one and, consequently, express its acceleration? It would be possible to answer this question by a direct measurement of the force on a moved body.

It is necessary to connect it with more massive body, which does not change interaction with motionless body and move with necessary constant velocity relatively of motionless one, by a spring therefore. As we don't know such measurement we will be searching for the answer to the question being based on the indirect facts. Each from two motionless bodies originally begins to move accelerately in the outcome of interaction. The forces causing movement of the first body $F_{12}= m_1w_1$ and movement of the second one $F_{21} = m_2w_2$, are equal. Therefore accelerations of the bodies will be inversely proportional to masses $w_1/w_2 = m_2/m_1$. The point, which distances from objects are inversely proportional to masses ($R_1/R_2 = m_2/m_1$), is known as centre-of-mass. It should move without acceleration in this case. This statement is confirmed. From what is observed in all cases of movement of such bodies in nature it follows that their centre-of-mass goes evenly and linearly or rests in case of absence of external influence on bodies of a system. Due to the stated fact we come to the conclusion about equality of forces with which two objects interact one another in case of their relative movement.

We remind that the statement about equality of forces is a corollary of describing the method of interaction in nature by forces but it is not the choice of nature. The method excludes existence of different in magnitude forces of action on each of two bodies.

So knowing masses of bodies we can calculate acceleration of moved and originally motionless bodies by measuring force in magnitude of deformation of a spring by a motionless body. And magnitude of force can depend on a velocity of a moved body, but we can define acceleration of bodies and, consequently, clarify the whole motion pattern by considered method nevertheless.

3.3. CHANGE OF INTERACTION FORCE IN CASE OF MOVEMENT OF CHARGED BODY AND MAGNET

If the body with the charge q_1 move with velocity υ in regards of the second one with the charge q_2 that these bodies will interact, putting each other accelerations. We could define dependence of force on velocity by measuring the force F_{21} of action of a moved body on a motionless one. As the similar measurements have not been conducted we consider results of other experience whereby of which it is possible to calculate the action force of a moved charged body on a motionless one [112, 113, 115, 116].

Let's begin with experience in which the action of moved charged bodies on a magnet was studied (magnet means the magnetized body or electromagnet). If the motionless charged body does not influence on the magnet then there is an interaction between them in case of movement relatively of the magnet. This interaction was studied by experiments about the influence of the rotated charged capacitor on magnetic arrow, which were conducted by G. Roland, A. Ahenvald and etc. It was compared with action of the capacitor where a current passed so that the direction of the current was in according with movement of the charged plates of the capacitor. In the issue it was established that the action on the magnet by the rotated charged capacitor is equal to the action of the capacitor with such current under which the change of a electric charge happens per unit of time, equal to a change of the charge under rotation of the capacitor. This outcome proves that the action on a magnet of a body with changed electricity depends on velocity of a change of this electricity, i.e. depends on current $I = \mathbf{d}q/\mathbf{d}t$. Therefore the force of action on a magnet depends on current I irrespective of that whether the change of electrical force is expressed by current owing to current of conductivity or owing to movement of a charged body. So, the change of an electrical force of action from a moved charged body expressed by current I, results in emergence of a force of action of this body on a magnet.

We will consider the second experienced fact, at which the force of action of conductor with current on a magnet was measured. Different scientists, including Oersted, Ampere etc, accomplished the experiments on action of conductor with current on magnetic arrow or magnet at different configurations of bodies. Laplace has founded, that the action of conductor with current on a magnet decreases in inverse proportion to square of a distance between them. Due to these experiments the expression for an increment of strength $\mathbf{d}\vec{H}$, created by a part of conductor with length $\mathbf{d}l$, current I and distance R from it up to magnet as is obtained

$$\mathbf{d}\vec{H} = \frac{I}{R^3 c}\left[\mathbf{d}\vec{l} \times \vec{R}\right], \tag{3.3}$$

which is known as Biot-Savart-Laplace's law. Here $\vec{H}$ represents the magnitude of a force on a unit magnetic pole.

As the influence of a moved charged body can be described by current I, the expression (3.3) describes also its action on a magnet. The magnitude of action is determined by what magnitude of current I will represent movement of a charged body.

But the magnet, in its turn, at movement influences a charged body. The describing this action equation is founded by M. Faradey because of experiments, in which the movement of a magnet was made relatively of conductor. On extremities of such conductor there were charges, which were determined as a potential difference. It means emergence of a force of the action on a charged body, if it is located on a place of conductor. A residual of potentials on extremities of conductor in the experiments was measured, instead of the force on a charged body, as it is required at our consideration. However the last can be calculated on a potential difference.

The magnitude of an appearing charge on extremities of conductor depends on magnitude describing action of a magnet on other magnet, and of a velocity of movement. The electricity on extremities of conductor occurs as well at motionless magnet, if the force of its action to other magnet is changed. For example, the electromagnet, in which winding the current varies, also creates a charge on extremities of conductor.

As in the first case of a moved constant magnet, so as in the second case motionless one with changed magnitude of a magnetization, the magnitude of an appearing electrization in conductor could be founded depending on a velocity of change of strength $\vec{H}$ in a conductor place. The directed electricity was measured on magnitude of current in the closed conducting circuit, and the strength $\vec{H}$ was considered through a magnetic flux, which is taking place through a circuit, as follows:

$$\Phi = \mu \int \vec{H} \mathbf{d}\vec{s} \,, \tag{3.4}$$

where μ - the magnetic permeability describing a medium, where there is a conductor.

In the total the dependence of electricity on a velocity of a change of a flux Φ magnetic strength $\vec{H}$ through this circuit was installed. The given equation has a title Faraday's law of an induction has a sight

$$u = -\frac{1}{c}\frac{\mathbf{d}\Phi}{\mathbf{d}t} \,. \tag{3.5}$$

Here u in a system of units CGC represents an electromotive force in the closed conducting circuit, c - speed of light.

So, there is an action u on a charged body, which in the equation (3.5) is stipulated by a velocity of a change of action on a magnet in that place, where there is a charged body.

We have considered three experimental facts: 1) the action of moved charged body on a magnet is equivalent to current; 2) forces of action of current on a magnet are determined by the law (3.3); 3) the action of a moved magnet on a charge are carried out according to expression (3.5). Now let's come back to consideration of action of a moving body with a charge q_1. At its movement there is an action on a magnet, which depends on a velocity of a change of action on a charged body in a place of a determination of a magnet. This position is expressed by the equation (3.3). But at movement of a charged body the action on a magnet will be changed and, as the expression (3.5) testifies, in that place there will be an action on a charged body. The given action is proportional to a velocity of a change of magnetic action, which, in its turn, according to the equations (3.3), is proportional to a velocity of a change of electrical action. Therefore jointly equation (3.3) and (3.5) express that fact, that from a moved body with a charge q_1 there is an additional action on a motionless body with a charge q_2 dependent from a change of main action, which forms by a body q_1 in case, when it rests relatively of a body q_2.

From the above mentioned experiments follows that the action of a moved charged body on motionless ones does not happen as their interaction at rest. In all three experiences the electrical and magnetic actions occur at availability of relative movement of two interacting bodies. For example, in a magnet coil there will be current only at relative movement of a magnet. At joint movement of a magnet and the magnet coil with any velocity and in relation to any bodies there will not be current in a magnet coil, if there is no movement of a magnet relatively of it. Therefore, the interaction of two charged bodies depends on their relative velocity.

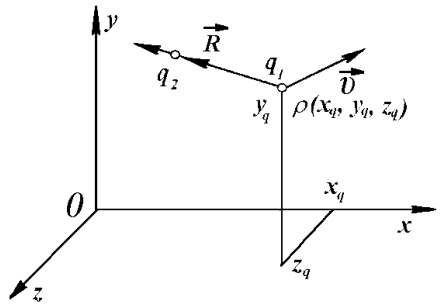


Fig.3.1. Action of a moved point body with a charge q_1 on a motionless point body with a charge q_2.

At definition of a charge the unit of charge was accepted such electricity, identical at two motionless bodies, at which they at distance equal to unit deformed a spring per unit of a force. But at availability of relative movement of these bodies the magnitude of a force depends on a velocity. So bodies on that distance will influence each other by force, distinct from unit. As this magnitude of a force determines charges of bodies, the other ones will become their magnitude. Thus, at availability of movement of charged bodies the magnitude of a charge is changed. In an electrodynamics it is accepted to consider the magnitude of a body charge constant. In this connection in Kauphman's and similar experi-

ments on a change of the attitude of a charge to a mass of electrons and other particles they came to a conclusion about a change of a mass of these particles at their movement. But as the action depends on movements a given conclusion is erroneous. Kauphman's experiments confirm a change of a charge of a body at its movement, however in physics it is accepted to consider magnitude of a charge as a constant. Therefore hereinafter we will suppose it constant, but take into account, that the magnitude of a charge is determined on force action of motionless bodies.

3.4. DERIVATION OF MAXWELL'S EQUATIONS FROM THE EXPERIMENTAL BASES

For defining the action between relatively moved bodies we will enter a coordinate system *x*, *y*, *z* (see fig.3.1), in which coordinates relating to a moved body with the charge q_1, we designate x_q, y_q, z_q accordingly. Let $\rho\,(x_q, y_q, z_q)$ is a density of electricity, which with the help of charge of a body is determined as

$$q_1 = \int_V \rho \mathbf{d}V\,, \tag{3.6}$$

where *V* - volume of a body, and the voxel is equal

$$\mathbf{d}V = \mathbf{d}x_q \mathbf{d}y_q \mathbf{d}z_q \tag{3.7}$$

The volume element $\mathbf{d}V$ has a charge $\mathbf{d}q_1 = \rho\mathbf{d}V$ and influences a charge q_2 by a force according to Coulomb's law (3.1). Integrating it on the whole volume, we receive

$$\vec{F} = \frac{q_2}{\varepsilon}\int_V \frac{\rho\vec{R}}{R^3}\mathbf{d}V\,, \tag{3.8}$$

where $\vec{R} = \vec{r}(x,y,z) - \vec{r}(x_q,y_q,z_q)$ - a position vector from a charge q_1 to a charge q_2. After an integration of (3.8) the force $\vec{F}$ will be function only of coordinates of a charge q_2, i.e. $\vec{F} = \vec{F}(x,y,z)$. In any point of coordinates it is possible to locate a body q_2 and to define a force. In mathematical sense the force $\vec{F}$ can be considered as the function of coordinates and applied to it all mathematical operations.

Included in integrand expression (3.8) multiplicands $\vec{R}/R^3$ can be noted as

$$\vec{R}/R^3 = -\text{grad}(1/R)\,, \tag{3.9}$$

where in operation of a gradient the derivation is carried out on coordinates *x*, *y*, *z*. With allowance for (3.9) force (3.8) will be:

$$\vec{F} = -\frac{q_2}{\varepsilon}\int_V \text{grad}\left(\frac{\rho}{r}\right)\mathbf{d}V\,. \tag{3.10}$$

Let's find a divergence from the right and left parts (3.10) on variables *x*, *y*, *z*:

$$\text{div}\,\vec{F} = -\frac{q_2}{\varepsilon}\int_V \text{div}\left[\text{grad}\left(\frac{\rho}{r}\right)\right]\mathbf{d}V = -\frac{q_2}{\varepsilon}\int_V \frac{\Delta\rho}{r}\mathbf{d}V\,. \tag{3.11}$$

Let's take advantage of the Poisson's theorem for scalar function *U*:

$$U = -\frac{1}{4\pi}\int_V \frac{\Delta U}{r}\mathbf{d}V\,. \tag{3.12}$$

With allowance for (3.12) the expression (3.11) will accept with a sight:

$$\text{div}\,\vec{F} = \frac{4\pi q_2}{\varepsilon}\rho\,. \tag{3.13}$$

From the expression (3.13) it is visible, that the derivatives depend on a force on coordinates *x*, *y*, *z* from a density of a charge in this point. In essence, the equation (3.13) does not introduce anything new and reflects a property of a charge q_1 and its density ρ as of force performance of action on a body with a charge q_2. It is other form of an entry of the Coulomb's law, namely differential, when the charge of an influencing body is expressed as a density of electricity ρ and is distributed in space circumscribed by coordinates *x*, *y*, *z*. If Coulomb's law describes interaction of two point bodies, the law of interaction in a differential kind (3.13) describes the action of charged bodies of any configuration, which the density of charge is set by its density $\rho\,(x_q, y_q, z_q)$.

Some differential equations are of interest for motionless charges. For their derivation we will take the operation rot from the right and left parts (3.10). As for any scalar function rot(grad *U*) = 0,

$$\text{rot}\,\vec{F} = 0\,. \tag{3.14}$$

Such distribution of forces created by motionless charges, is named irrotational. Now we will take the operation grad from the right and left parts (3.13):

$$\text{grad}(\text{div}\,\vec{F}) = \Delta\vec{F} + \text{rot}(\text{rot}\,\vec{F}) = \frac{4\pi q_2}{\varepsilon}\text{grad}\,\rho\,.$$

With allowance for (3.14) these equations accept a sight

$$\Delta\vec{F} = \frac{4\pi q_2}{\varepsilon}\operatorname{grad}\rho\,. \qquad (3.15)$$

It is named Laplace's equation and as well as Poisson's equation (3.13), is the differential form of an entry of Coulomb's law.

We will consider the action of a moved body with a charge q_1 on motionless second of q_2. At movement of a charged element $\mathbf{d}V$ of the first body happens the time history of magnitude of a charge in points, motionless relatively of the second body. That is relatively of it the current of electricity happens which is determined as the velocity of a change: $I = \dfrac{\mathbf{d}q_1}{\mathbf{d}t}$. Expressing magnitude of a charge through a density according to (3.6) and differentiating, we receive

$$I = \frac{\mathbf{d}}{\mathbf{d}t}\int \rho\mathbf{d}V = \int \frac{\partial\rho}{\partial t}\mathbf{d}V + \int \operatorname{div}(\rho\vec{v})\mathbf{d}V\,.$$

Here integration is carried out on coordinates x_q, y_q, z_q of an influencing charge q_1, instead of on coordinates x, y, z, in which there can be a charge q_2. The partial derivative on time can be expressed through a force $\vec{F}$ according to the law (3.13):

$$\frac{\partial\rho}{\partial t} = \frac{\varepsilon}{4\pi q_2}\operatorname{div}\left(\frac{\partial\vec{F}}{\partial t}\right). \qquad (3.16)$$

After a substitution of a derivative expression for current of electricity I, created by a moved charge q_1 is received:

$$I = \int \operatorname{div}\left(\frac{\varepsilon}{4\pi q_2}\frac{\partial\vec{F}}{\partial t} + \rho\vec{v}\right)\mathbf{d}V\,. \qquad (3.17)$$

As it was already mentioned, the moved charged body affects a magnet, which according Biot-Savart-Laplace's law (3.3) is proportional to current (3.17). Let's transform expression (3.3) to distributed in space x, y, z magnitudes. With this purpose let's integrate it for an infinite direct conductor with current I (see Fig. 3.2). Here $\mathbf{d}\vec{l}$ - element of conductor

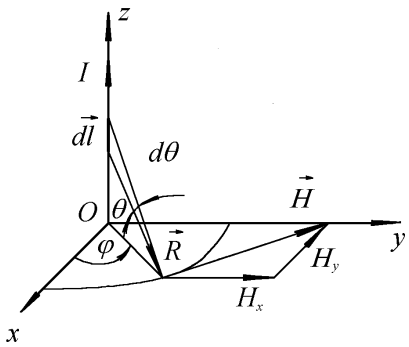


Fig. 3.2. Magnetic strength of an infinite conductor with current.

conterminous on a direction current, and $\vec{R}$ - its distance up to a magnet. Then, with allowance of geometric equations

$$[\mathbf{d}\vec{l}\times\vec{R}] = \vec{\tau}\,\mathbf{d}l\,R\cos\theta\,,$$

$$\mathbf{d}l\cos\theta = R\mathbf{d}\theta \quad\text{and}\quad R = R_P/\cos\theta\,,$$

where $\vec{\tau}$ - basis vector of a tangent to a circle in a plane xy, and R_P - the least distance from a point x, y up to conductor with current I, the magnetic strength (3.3) will be noted:

$$\vec{H} = \frac{I\vec{\tau}}{c}\int\frac{\mathbf{d}l\cos\theta}{R^2} = \frac{I\vec{\tau}}{R_P}\int_{-\pi/2}^{\pi/2}\cos\theta\,\mathbf{d}\theta = \frac{2I\vec{\tau}}{cR_P}\,. \qquad (3.18)$$

The vector $\vec{H}$ is directed (see Fig. 3.2) on a circle, where current passes in the centre perpendicularly to its plane, and in the cartesian frame it can be noted:

$$\vec{H} = \frac{2I}{cR_P}\left(-\vec{i}\sin\varphi + \vec{j}\cos\varphi\right).$$

Magnetic strength $\vec{H}$ - magnitude of a force of action on a magnetic pole with a magnetic charge $M = 1$. Sources of an electrical force are the charges. The mathematical connection between a density of charges and divergence of a force is expressed by the equation (3.13). Let's define a divergence of magnetic strength:

$$\operatorname{div}\vec{H} = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = \frac{2I}{c}\left[-\sin\varphi\frac{\partial(1/R_P)}{\partial x} + \cos\varphi\frac{\partial(1/R_P)}{\partial y}\right].$$

As, $R_P = \sqrt{x^2+y^2}$ and $\sin\varphi = x/R_P$, $\cos\varphi = y/R_P$, after a substitution of derivatives is received

$$\operatorname{div}\vec{H} = 0\,. \qquad (3.19)$$

The equation (3.19) testifies to such character of a magnetic force, which cannot be expressed as separate magnetic charges. The sources of a magnetic force exist as two opposite magnetic poles - northern and southern.

The expression (3.18) can be noted as

$$2\pi R_P\vec{H} = \frac{4\pi I}{c}\vec{\tau}\,,$$

i.e. the circulation of magnetic strength on a ring circuit is determined only by the magnitude of current I, enveloped by an circuit. It can be copied so:

$$\oint_l \vec{H}\mathbf{d}\vec{l} = \frac{4\pi I}{c}\,. \qquad (3.20)$$

Using the Stokes theorem for an integral on a closed loop, we shall write (3.20) as follows:

$$\int \text{rot}\,\vec{H}\,\mathbf{d}\vec{S} = \frac{4\pi I}{c}, \qquad (3.21)$$

and in the equation (3.17) for a current I with the help of Ostrogradsky- Gauss theorem the integral on volume we shall express through an integral on a surface:

$$I = \int\left(\frac{\varepsilon}{4\pi q_2}\frac{\partial\vec{F}}{\partial t} + \rho\vec{v}\right)\mathbf{d}\vec{S}\ .$$

After substitution of the current I in (3.21) we obtain the following expression:

$$\text{rot}\,\vec{H} = \frac{\varepsilon}{cq_2}\frac{\partial\vec{F}}{\partial t} + \frac{4\pi}{c}\rho\vec{v}\ . \qquad (3.22)$$

This equation determines a force of action $\vec{H}$ on a unit magnetic pole stipulated by the moved charged body q_1. It is known as the second Maxwell's law [65, 96] and, in essence, is other form of an entry of the experimental Biot-Savart-Laplace's law (3.3).

The action $\vec{H}$ on a magnet arises in a place of a determination of a body with a charge q_2 (see.fig.3.1). As the body q_1 goes, the magnetic action $\vec{H}$ is changed. In according with equations (3.4) and (3.5) in a place of a determination q_2 there will be an action on a charged body. In (3.4) they consider the flux of magnetic strength $\vec{H}$ through a surface S resting on a circuit l, in which according to (3.5) the electromotive force u is induced. It can be expressed through circulation of electrical strength $E = \vec{F}/q_2$ as follows:

$$u = \oint\frac{\vec{F}}{q_2}\mathbf{d}\vec{l} = \int\text{rot}\frac{\vec{F}}{q_2}\mathbf{d}\vec{S}\ . \qquad (3.23)$$

After a substitution in (3.5) u and of the Faradey's law of induction accepts a kind

$$\int\text{rot}\frac{\vec{F}}{q_2}\mathbf{d}\vec{S} = -\frac{\mu}{c}\int\frac{\partial\vec{H}}{\partial t}\mathbf{d}\vec{S}\ .$$

As this expression should be fair at any integrands, a ratio follows from here

$$\text{rot}\frac{\vec{F}}{q_2} = -\frac{\mu}{c}\frac{\partial\vec{H}}{\partial t}, \qquad (3.24)$$

which is known as the first Maxwell's equation.

3.5. DIFFERENTIAL EQUATIONS FOR FORCES

As we can see, the first and second Maxwell's equations are other form of an entry of two experimental laws: Faradey's electrical induction and Biot-Savart-Laplace's magnetic action of current accordingly. A charged skew body moving with a velocity $\vec{v}$, the density of which electricity ρ, in according with the equations (3.24) and (3.22) creates these actions. If we exclude magnitude $\vec{H}$ from the equations, we receive one equation for magnitude of action of a charged skew body moved with a velocity $\vec{v}$, on a motionless one. With this purpose we will take the operation rot from (3.24)

$$\text{rot}\left[\text{rot}\frac{\vec{F}}{q_2}\right] = -\frac{\mu}{c}\frac{\partial}{\partial t}\left(\text{rot}\,\vec{H}\right).$$

Conducting sequential transformations of the left part of this equation, we receive

$$\text{grad}\,\text{div}\frac{\vec{F}}{q_2} - \frac{\Delta\vec{F}}{q_2} = \frac{4\pi}{\varepsilon}\text{grad}\,\rho - \frac{\Delta\vec{F}}{q_2} = -\frac{\mu}{c}\frac{\partial}{\partial t}\left(\text{rot}\,\vec{H}\right). \qquad (3.25)$$

Here Δ - Laplace's operator, and in the first addend is $\text{div}\frac{\vec{F}}{q_2}$ expressed through a density of a charge ρ in according with (3.13).

Now let's differentiate in time the second Maxwell's equation (3.22)

$$\frac{\partial}{\partial t}\text{rot}\,\vec{H} = \frac{\varepsilon}{cq_2}\frac{\partial^2\vec{F}}{\partial t^2} + \frac{4\pi}{c}\frac{\partial\left(\rho\vec{v}\right)}{\partial t}$$

also we will substitute in a right member (3.25). After transformation we have

$$\Delta\vec{F} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{F}}{\partial t^2} = \frac{4\pi\mu q_2}{c^2}\frac{\partial\left(\rho\vec{v}\right)}{\partial t} + \frac{4\pi q_2}{\varepsilon}\text{grad}\,\rho\ . \qquad (3.26)$$

Let's enter a label

$$c_1 = \frac{c}{\sqrt{\mu\varepsilon}} \qquad (3.27)$$

also we will note this equation so:

$$\Delta\vec{F} - \frac{1}{c_1^2}\frac{\partial^2\vec{F}}{\partial t^2} = \frac{4\pi q_2}{\varepsilon}\left[\frac{1}{c_1^2}\frac{\partial\left(\rho\vec{v}\right)}{\partial t} + \text{grad}\,\rho\right]. \qquad (3.28)$$

It is named as the d'Alembert's equation. This equation without a right member

$$\Delta\vec{F} - \frac{1}{c_1^2}\frac{\partial^2\vec{F}}{\partial t^2} = 0 \qquad (3.29)$$

is named wave. It supposes [24, 52] private solutions of a type

$$\vec{F}(r,t) = \vec{F}_k(r)\mathrm{e}^{\mp i\omega t},\ \omega = kc_1, \qquad (3.30)$$

where k - any constant, defined by boundary conditions. A special case (3.30) is the solution

$$\vec{F}(x,y,z,t) = \vec{F}_0\cos\omega\left[t \mp (k_1x + k_2y + k_3z)/c + t_0\right], \qquad (3.31)$$

where, $k_1^2 + k_2^2 + k_3^2 = 1$, t_0 - constant of an integration.

The solution (3.31) represents plane waves, where k_1, k_2, k_3 - direction cosines of a plane, on which the constant significance of a force $\vec{F}_0$ is saved. This plane moves in space with a velocity c_1, which is called a velocity of distribution of electromagnetic interaction.

If the charged objects are motionless and are constant (ρ = const), their action on a charge q_2 is constant ($\partial\vec{F}/\partial t = 0$), the equation (3.28) turns to the equation of Laplace (3.15). For moved or non-stationary charges the force of their action will be other, it is determined by d'Alembert's equation (3.28).

As we already considered, the moved charged body influences a magnet. The equations (3.22) and (3.24) allow to define such action depending on a density of its charge and velocity $\vec{v}$ of a body relatively of a magnet ρ. With this purpose we will take the operation rot from the right and left parts of the second of the Maxwell's equation (3.22):

$$\mathrm{rot}\left(\mathrm{rot}\,\vec{H}\right) = -\Delta\vec{H} + \mathrm{grad}\left(\mathrm{div}\,\vec{H}\right) + \frac{\varepsilon}{cq_2}\frac{\partial}{\partial t}\left(\mathrm{rot}\,\vec{F}\right) + \frac{4\pi}{c}\mathrm{rot}\left(\rho\vec{v}\right) \qquad (3.32)$$

and differentiate in time the first Maxwell's equation (3.24)

$$\frac{1}{q_2}\frac{\partial}{\partial t}\left(\mathrm{rot}\,\vec{F}\right) = -\frac{\mu}{c}\frac{\partial^2\vec{H}}{\partial t^2}. \qquad (3.33)$$

Excluding $\vec{F}$ from expressions (3.32) and (3.33), with allowance for (3.27) for the magnetic action of moved charged bodies d'Alembert equation is received

$$\Delta\vec{H} - \frac{1}{c_1^2}\frac{\partial^2\vec{H}}{\partial t^2} = -\frac{4\pi}{c}\mathrm{rot}\left(\rho\vec{v}\right). \qquad (3.34)$$

CHAPTER 4

THE INTERACTION OF TWO MOVING CHARGED POINT BODIES

4.1. D'ALEMBERT'S EQUATION *FOR A POINT CHARGE*

If the size of bodies is considerably less than distances between them it is possible to consider such bodies as point ones. Let's define a force with which the point charge q_1 moving with constant velocity $\vec{v}$ actions on the other point one q_2 (see fig.4.1). The relative movement is considered: the charge q_1 goes relatively the charge q_2. The charge q_2 can be in any point x, y, z in the cartesian frame. The charge q_1 is at the origin of the frame in the moment $t = 0$.

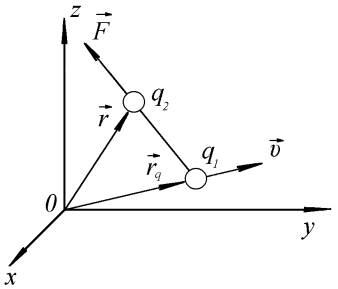


Fig.4.1. To the derivation of d'Alembert's equation for a point charged body.

The charge density q_1 can be written as a point object by δ-function [17], which has the following kind in depending on the coordinate x:

$$\delta(x - x') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\, i(x - x')k\mathbf{d}k = \begin{cases}0, x \neq x',\\ \infty, x = x'\end{cases} \qquad (4.1)$$

and

$$\int_{-\infty}^{\infty}\delta(x - x')\mathbf{d}x = 1. \qquad (4.2)$$

As it is visible δ-function has only one nonzero significance and it is equal to infinity in the point x', where there is the particle. It is equal to zero in all other points. However integral from the δ-function is a finite quantity and is equal to unit on the whole range of changes x.

The charge density q_1 frame will be written in space by the δ-function so:

$$\rho = q_1 \delta(x - \upsilon_x t)\,\delta(y - \upsilon_y t)\delta(z - \upsilon_z t), \qquad (4.3)$$

where $\vec{r} = \vec{i}\,x + \vec{j}y + \vec{k}z$ - the position vector of a space point;

$\vec{r}_q = \vec{i}\,\upsilon_x t + \vec{j}\upsilon_y t + \vec{k}\upsilon_z t$ - is the position vector of the charge q_1.

Then it is received after the substitution of the expressions for the δ-functions

$$\rho = \frac{q_1}{8\pi^3}\int_{-\infty}^{\infty} \exp i\left[k_1(x - \upsilon_x t) + k_2\left(y - \upsilon_y t\right) + k_3(z - \upsilon_z t)\right]\mathbf{d}k\,, \qquad (4.4)$$

where the integral $\int_{-\infty}^{\infty}\mathbf{d}k$ is triple $\iiint \mathbf{d}k_1\mathbf{d}k_2\mathbf{d}k_3$. Taking into consideration (4.2) it is easy to be convinced that the integral is on the whole space $\int \rho\mathbf{d}x\mathbf{d}y\mathbf{d}z = q_1$, where ρ is determined by the expression (4.4). Thus the δ-function has allowed distributing the point charge with simultaneous localization of it in a determination place of the charge q_1 on the whole space. Now it can take advantage of the d'Alemberts equation (3.28) in which the non-stationary electricity ρ density actions on the motionless charge q_2 by force $\vec{F}$. Further we will consider the unit force

$$\vec{E} = \vec{F}/q_2\,, \qquad (4.5)$$

acting per unit of the charge q_2. Following the tradition we will call it electrical intensity. Let's rewrite the d'Alembert's equation (3.28) in projections on axes of a coordinate system subject to (4.5):

$$\Box E_x = \frac{4\pi}{\varepsilon}\left[\frac{\partial\rho}{\partial x} + \frac{1}{c_1^2}\frac{\partial\rho}{\partial t}\upsilon_x\right];$$

$$\Box E_y = \frac{4\pi}{\varepsilon}\left[\frac{\partial\rho}{\partial y} + \frac{1}{c_1^2}\frac{\partial\rho}{\partial t}\upsilon_y\right]; \qquad (4.6)$$

$$\Box E_z = \frac{4\pi}{\varepsilon}\left[\frac{\partial\rho}{\partial z} + \frac{1}{c_1^2}\frac{\partial\rho}{\partial t}\upsilon_z\right],$$

where $\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}$ is D'Alembertian.

After the substitution of the charge density (4.4) we have for the projection E_x:

$$\Box E_x = \frac{q_1 i}{2\pi^2\varepsilon}\int_{-\infty}^{\infty}\left[\left(1 - \beta_x^2\right)k_1 - \beta_x\beta_y k_2 - \beta_x\beta_z k_3\right]\exp(i\,r_v)\mathbf{d}k\,, \qquad (4.7)$$

where

$$r_v = k_1(x - \upsilon_x t) + k_2\left(y - \upsilon_y t\right) + k_3(z - \upsilon_z t), \qquad (4.8)$$

$$\beta_x = \frac{\upsilon_x}{c_1}, \quad \beta_y = \frac{\upsilon_y}{c_1}, \quad \beta_z = \frac{\upsilon_z}{c_1}.$$

The x-projection of action force of the moved charge q_1 per unit of the motionless charge q_2 determines by the d'Alembert equation. The force component on the axes *y* and *z* will be similarly written.

4.2. SOLUTION OF THE D'ALEMBERT'S EQUATION

D'Alembert's opetrator does not depend on the variable integration k_1, k_2, k_3 in the right side. Therefore the equation (4.7) can be symbolically written so:

$$E_x = \frac{q_1 i}{2\pi^2\varepsilon}\int_{-\infty}^{\infty}\left[\left(1 - \beta_x^2\right)k_1 - \beta_x\beta_y k_2 - \beta_x\beta_z k_3\right]\Box^{-1}\exp\,(i\,r_v)\mathbf{d}k\,, \qquad (4.9)$$

where $r_v = k_1(x - \upsilon_x t) + k_2\left(y - \upsilon_y t\right) + k_3(z - \upsilon_z t)$.

The intensity E_x will be represented as an integral if it is defined

$$\Box^{-1}\exp\,(i\,r_v) = G\,. \qquad (4.10)$$

The function *G* is called the Green's function. It can be discovered from the equation now

$$\Box G = \exp\,(i\,r_v)\,, \qquad (4.11)$$

which is a second-order partial linear differential equation with a right side. Its solution consists of the partial solution from the right side and a general solution ∈G = 0. The equation solution ∈G = 0 gives the solution from a zero charge density i.e. it represents an electrical action from another source. There is only one charge q_1 under the conditions of the problem in space. The general solution of

equation without the right side is rejected: the charge q_1 action interests only. The partial solution is searched as

$$G = C \cdot \exp(i\ r_v). \tag{4.12}$$

Its substitution in the equation (4.11) determines the coefficient

$$C = -\frac{1}{k_1^2 + k_2^2 + k_3^2 - (\beta_x k_1 + \beta_y k_2 + \beta_z k_3)^2}.$$

After the substitution of the coefficient C in (4.12) Green's function will be the following:

$$G = -\frac{\exp(i\ r_v)}{k_1^2 + k_2^2 + k_3^2 - (\beta_x k_1 + \beta_y k_2 + \beta_z k_3)^2}. \tag{4.13}$$

Substituting expression (4.10) in (4.9) by the Green function we receive electrical intensity as the integral

$$E_x = -\frac{q_1 i}{2\pi^2 \varepsilon} \int_{-\infty}^{\infty} \frac{(1-\beta_x^2)k_1 - \beta_x\beta_y k_2 - \beta_x\beta_z k_3}{k_1^2 + k_2^2 + k_3^2 - (\beta_x k_1 + \beta_y k_2 + \beta_z k_3)^2} \exp(i\ r_v)\mathbf{d}k. \tag{4.14}$$

Let's transform (4.14) by selecting the integral over the variable k_1:

$$E_x = -\frac{q_1 i}{2\pi^2 \varepsilon} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp i\left[k_2(y - \upsilon_y t) + k_3(z - \upsilon_z t)\right] I_1 dk_2 dk_3, \tag{4.15}$$

where

$$I_1 = \int_{-\infty}^{\infty} \frac{\left[(1-\beta_x^2)k_1 - (\beta_x\beta_y k_2 + \beta_x\beta_z k_3)\right] \exp i(x - \upsilon_x t)\, k_1}{k_1^2 - \beta_x^2 k_1^2 - 2\beta_x k_1(\beta_y k_2 + \beta_z k_3) + a^2} \mathbf{d}k_1, \tag{4.16}$$

$$a^2 = k_2^2 + k_3^2 - (\beta_y k_2 + \beta_z k_3)^2. \tag{4.17}$$

The solution of integrals (4.15) and (4.16) is considered in case of velocity of charge movement not exceeding velocity c_1, i.e.

$$\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2 \le 1. \tag{4.18}$$

The sign of expression is researched for a^2. In case of limiting significances of a velocity β the quantity a^2 is enclosed in limits from $a^2 = k_2^2 + k_3^2$ when

$\beta_y = \beta_z = 0$ $a^2 = [k_2(1-\beta_y^2)^{1/2} - k_3\beta_y]^2 > 0$ to when $\beta_y^2 + \beta_z^2 = 1$, i.e. it is positive.

We will rewrite the expression (4.17) as

$$a^2 = k_2^2(1-\beta_y^2) + k_3^2(1-\beta_z^2) - 2\beta_y\beta_z k_2 k_3.$$

It can be negative only when the third summand is positive for example β_y, β_z, k_2 and k_3 have identical signs or in pairs identical signs. We will find the increment $\mathbf{d}a^2$ in case of change β from (4.17):

$$\mathbf{d}a^2 = -2(\beta_y k_2 + \beta_z k_3)(k_2\mathbf{d}\beta_y + k_3\mathbf{d}\beta_z).$$

It is easy to be convinced that the increment $\mathbf{d}a^2 < 0$ in case of these signs, i.e. a^2 is monotonically changed. And as the quantity a^2 is positive in extreme cases by β, it is positive in the whole range.

It will enter the conventions: $e_1 = \sqrt{1-\beta_x^2}$; $b = \beta_x(\beta_y k_2 + \beta_z k_3)$; $x_q = x - \upsilon_x t$, and ζ – is a complex number. Let's consider the integral on a closed loop in the complex plane

$$\oint \frac{(e_1^2\zeta - b)\exp ix_q\zeta}{e_1^2\zeta^2 - 2b\zeta + a^2}\mathbf{d}\zeta. \tag{4.19}$$

It is visible by these conventions that the integrand expression (4.19) coincides with integrand expression (4.16) when $\zeta = k_1$. The integrand denominator has zero in (4.19) when

$$\zeta_{1,2} = \frac{b \pm \sqrt{b^2 - a^2 e_1^2}}{e_1^2}, \tag{4.20}$$

where

$$b^2 - a^2 e_1^2 = -\left\{(k_2^2 + k_3^2)(1-\beta_x^2) - (\beta_y k_2 + \beta_z k_3)^2\right\}. \tag{4.21}$$

It is possible to show by the similar method to in case of the sign a^2 proof that $b^2 - a^2 e_1^2 < 0$. Then after the substitution of the significances b, a^2, e_1 in (4.21) zeros of the denominator will be

$$\zeta_{1,2} = \frac{\beta_x(\beta_y k_2 + \beta_z k_3) \pm i\sqrt{(k_2^2 + k_3^2)(1-\beta_x^2) - (\beta_y k_2 + \beta_z k_3)^2}}{1-\beta_x^2}. \tag{4.22}$$

It is known [24] that the integral on a closed loop like of (4.19) is determined by the sum of derivations C_{-1}

$$\oint f(\zeta)\exp(ix_q\zeta)\mathbf{d}\zeta = 2\pi i\zeta C_{-1}\,.$$

We will consider the complex integral (4.19) along a circuit of a semi-ring with a radius R in upper half-plane of the complex variable $\zeta = \xi + i\eta$. The pole ζ^+, i.e. the special point in case of positive sign before i, is in this half-plane by (4.22). Let's divide the integral (4.19) in two components: by the top-semicircle C_R and the horizontal diameter:

$$\oint = \int\limits_{C_R} + \int\limits_{-R}^{R} = 2\pi i C_{-1}\left(\zeta^+\right). \qquad (4.23)$$

Its significance is written in the right side by the derivation ζ^+.

The integral along a semicircle with infinite radius $\lim\limits_{\mathrm{R}\to\infty}\int\limits_{C_R}\frac{e_1^2\zeta - b}{e_1^2\zeta^2 - 2b\zeta + a^2}\exp ix_q\zeta\mathbf{d}\zeta$ is equal to zero by Jordan's lemma, if $x_q > 0$ и $\lim\limits_{\zeta\to\infty}\frac{e_1^2\zeta - b}{e_1^2\zeta^2 - 2b\zeta + a^2}$ and is a final quantity, it willtake place in this case. If $x_q = x - \upsilon_x t < 0$ then the Jordan's lemma will be performed in the lower half-plane and then the integral (4.19) is divided in two components by the lower semi-circle and the horizontal diameter:

$$\oint = 2\pi i C_{-1}\left(\zeta^-\right) = \int\limits_{-C_R} - \int\limits_{-R}^{R}\,. \qquad (4.24)$$

Thus the integrals are equal to zero both along upper semicircle in (4.23) and along lower one in (4.24) and the integrals along the horizontal diameter change to the integral (4.16) in the limit:

$$\lim_{\mathrm{R}\to\infty}\int\limits_{-R}^{r}\frac{e_1^2\zeta - b}{e_1^2\zeta^2 - 2b\zeta + a^2}\exp ix_q\zeta\cdot\mathbf{d}\zeta = I_1\,.$$

From this solutions of an integral (4.16) will be provided $x - \upsilon_x t > 0$
$I_1 = 2\pi i C_{-1}(\zeta^+)$ and provided $x - \upsilon_x t < 0$ $\quad I_1 = -2\pi i C_{-1}(\zeta^-)$ by (4.23) and (4.24).

Let's find the derivation of the pole in the upper half-plane:

$$C_{-1}\left(\zeta^+\right) = \lim_{\zeta\to\zeta^+} f(\zeta)\exp\left(i\,x_q\zeta\right)\left(\zeta - \zeta^+\right).$$

After the substitution $f(\xi)$ we receive (4.19) according to

$$C_{-1}\left(\zeta^+\right) = \lim_{\xi\to\xi^+}\frac{\left(e_1^2\zeta - b\right)\exp\left(i\,x_q\zeta\right)\left(\zeta - \zeta^+\right)}{(1-\beta_x)^2\left(\zeta - \zeta^+\right)\left(\zeta - \zeta^-\right)} = \frac{\left(e_1^2\zeta^+ - b\right)\exp\left(i\,x_q\zeta^+\right)}{2i\sqrt{\left(k_2^2 + k_3^2\right)\left(1-\beta_x^2\right) - \left(\beta_y k_2 + \beta_z k_3\right)^2}}\,.$$

Substituting e_1 and b here we can rewrite the derivation as

$$C_{-1}\left(\zeta^+\right) = (1/2)\exp\left[\frac{x_q\beta_x\left(\beta_y k_2 + \beta_z k_3\right)}{1-\beta_x^2} - \right.$$
$$\left. - \frac{ix_q\sqrt{\left(k_2^2 + k_3^2\right)\left(1-\beta_x^2\right) - \left(\beta_y k_2 + \beta_z k_3\right)^2}}{1-\beta_x^2}\right]. \qquad (4.25)$$

The derivation of the pole is similarly determined in the lower half-plane:

$$C_{-1}\left(\zeta^-\right) = (1/2)\exp\left[\frac{x_q\beta_x\left(\beta_y k_2 + \beta_z k_3\right)}{1-\beta_x^2} + \right.$$
$$\left. + \frac{ix_q\sqrt{\left(k_2^2 + k_3^2\right)\left(1-\beta_x^2\right) - \left(\beta_y k_2 + \beta_z k_3\right)^2}}{1-\beta_x^2}\right]. \qquad (4.26)$$

After the derivations substitution in the significances of the integral I_1 we find:
provided $x > \upsilon_x t$

$$I_1 = \pi i\exp i(x - \upsilon_x t)\frac{\beta_x\beta_y k_2 + \beta_x\beta_z k_3}{1-\beta_x^2}\times$$
$$\times\exp\left[-(x - \upsilon_x t)\times\frac{\sqrt{\left(k_2^2 + k_3^2\right)\left(1-\beta_x^2\right) - \left(\beta_y k_2 + \beta_z k_3\right)^2}}{1-\beta_x^2}\right],$$

provided $x < \upsilon_x t$

$$I_1 = -\pi i\exp i(x - \upsilon_x t)\frac{\beta_x\beta_y k_2 + \beta_x\beta_z k_3}{1-\beta_x^2}\times$$
$$\times\exp(x - \upsilon_x t)\times\frac{\sqrt{\left(k_2^2 + k_3^2\right)\left(1-\beta_x^2\right) - \left(\beta_y k_2 + \beta_z k_3\right)^2}}{1-\beta_x^2}\,.$$

The general integral expression for both cases can be written as

$$I_1 = \pi i \frac{x-\upsilon_x t}{|x-\upsilon_x t|} \exp\left[i(x-\upsilon_x t)\frac{\beta_x(\beta_y k_2 + \beta_z k_3)}{1-\beta_x^2} - |x-\upsilon_x t| \times \right.$$

$$\left. \times \frac{\sqrt{(k_2^2+k_3^2)(1-\beta_x^2)-(\beta_y k_2+\beta_z k_3)^2}}{1-\beta_x^2} \right].$$

After the substitution I_1 in (4.15) electrical intensity will be written so:

$$E_x = \frac{q(x-\upsilon_x t)}{2\pi\varepsilon|x-\upsilon_x t|} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[i\left(y-\upsilon_y t + \frac{x-\upsilon_x t}{1-\beta_x^2}\beta_x\beta_y \right)k_2 + \right.$$

$$\left. + \left(z-\upsilon_z t + \frac{x-\upsilon_x t}{1-\beta_x^2}\beta_x\beta_z \right)k_3 \right] - \frac{|x-\upsilon_x t|}{1-\beta_x^2} \times \qquad (4.27)$$

$$\times \sqrt{1-\beta_x^2-\beta_y^2} \sqrt{\frac{(k_2^2+k_3^2)(1-\beta_x^2)-(\beta_y k_2+\beta_z k_3)^2}{1-\beta_x^2-\beta_y^2}} \Bigg] \mathbf{d}k_2 \mathbf{d}k_3 .$$

Let's enter new conventions:

$$L_2 = y-\upsilon_y t + \frac{x-\upsilon_x t}{1-\beta_x^2}\beta_x\beta_y , \qquad (4.28)$$

$$L_3 = z-\upsilon_z t + \frac{x-\upsilon_x t}{1-\beta_x^2}\beta_x\beta_z , \qquad (4.29)$$

$$A_g = \frac{\beta_y^2\beta_z^2}{1-\beta_x^2-\beta_y^2} , \qquad (4.30)$$

$$A_i = \sqrt{\frac{1-\beta_x^2-\beta_z^2-\beta_y^2\beta_z^2(1-\beta_x^2-\beta_y^2)^{-1}}{1-\beta_x^2-\beta_y^2}} , \qquad (4.31)$$

$$A_{\pm} = A_g \pm A_i i , \qquad (4.32)$$

$$u = \frac{|x-\upsilon_x t|}{1-\beta_x^2}\sqrt{1-\beta_x^2-\beta_y^2} > 0 \qquad (4.33)$$

and also replace variable

$$s^2 = (k_2 - k_3 A_+)(k_2 - k_3 A_-) > 0 , \qquad (4.34)$$

$$n = k_2 L_2 + k_3 L_3 . \qquad (4.35)$$

As at change k_2 and k_3 in one of the half-planes, for example $0<k_2<\infty$, $-\infty<k_3<\infty$, new variables s, n pass all their values, for example $0<s<\infty$, $-\infty<n<\infty$, at integration on n and s the value of integral it is necessary to double.

The substitution of new conventions (4.28) - (4.35) in (4.27) conduces to the expression for electrical intensity

$$E_x = \frac{q_1(x-\upsilon_x t)}{2\pi\varepsilon|x-\upsilon_x t|} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\{ i(L_2 k_2 + L_3 k_3) - us \} \mathbf{d}k_2 \mathbf{d}k_3 . \qquad (4.36)$$

Let's define an element of area in the new variable n and s. As

$$\mathbf{d}s\mathbf{d}n = \left(\frac{\partial s}{\partial k_2}\frac{\partial n}{\partial k_3} - \frac{\partial s}{\partial k_3}\frac{\partial n}{\partial k_2} \right) \mathbf{d}k_2 \mathbf{d}k_3 ,$$

then considering (4.34) - (4.35) we receive

$$\mathbf{d}s\mathbf{d}n = \frac{\mathbf{d}k_2\mathbf{d}k_3}{s}\left[k_2(L_3 + L_2 A_g) - k_3(L_3 A_g + L_2 A^2) \right], \qquad (4.37)$$

where

$$A^2 = \frac{\beta^2{}_y\beta_z^2}{(1-\beta_x^2-\beta_y^2)^2} + \frac{1-\beta_x^2-\beta_z^2-\frac{\beta^2{}_y\beta^2{}_z}{1-\beta_x^2-\beta_y^2}}{1-\beta_x^2-\beta_y^2} . \qquad (4.38)$$

It will express k_2 and k_3 considering (4.34) - (4.35) by s and n:

$$k_2 = \frac{s}{L_2} - k_3\frac{L_3}{L_2} , \qquad (4.39)$$

$$s^2 = k_2^2 - 2k_2k_3A_g + k_3^2A^2 ,$$

$$k_3^2\left(\frac{L_3^2}{L_2^2} + 2A_g\frac{L_3}{L_2} + A^2 \right) - 2k_3 n\left(\frac{L_3}{L_2^2} + \frac{A_g}{L_2} \right) + \frac{n^2}{L_2^2} - s^2 = 0 .$$

The solution of the obtained quadratic equation gives

$$k_3 = \frac{n\left(L_3 + L_2 A_g\right) \pm L_2\sqrt{s^2 B^2 - n^2 A_i^2}}{B^2}, \qquad (4.40)$$

where

$$B^2 = L_3^2 + 2A_g L_3 L_2 + A^2 L_2^2 . \qquad (4.41)$$

And k_2 is determined in case of the substitution k_3 in (4.39):

$$k_2 = \frac{n\left(L_3 A_g + L_2 A^2\right) \mp L_3\sqrt{s^2 B^2 - n^2 A_i^2}}{B^2} . \qquad (4.42)$$

The substitution k_2 and k_3 in (4.37) gives the resultant expression for an element of area in the new variable:

$$\mathbf{d}k_2 \mathbf{d}k_3 = \mp \frac{s}{\sqrt{s^2 B^2 - n^2 A_i^2}} \mathbf{d}s\mathbf{d}n . \qquad (4.43)$$

The double signs testify to a duality of the values of k_2 and k_3 in case of same values of n and s, that was already noted. It is visible by the expressions (4.40) and (4.42) that k_2 and k_3 will be valid if $s^2 B^2 - n^2 A_i^2 \geq 0$, then $|n| \leq sBA_i^{-1}$. After the substitution of the area element (4.43) in (4.36) and the multiplication into 2 the electrical intensity will be expressed

$$E_x = \frac{q_1\left(x - \upsilon_x t\right)}{\pi\varepsilon\left|x - \upsilon_x t\right|} \int_0^\infty \mathbf{d}s \int_{-\infty}^{\infty} \frac{s\exp\left(in - us\right)}{\sqrt{s^2 B^2 - n^2 A_i^2}} \mathbf{d}n . \qquad (4.44)$$

Let's execute the transition to polar coordinates r, in the plane ns:

$$n = r \cdot \sin\alpha , \qquad s = r \cdot \cos\alpha , \qquad (4.45)$$

for which an area element and a change range will be the following:

$$\mathbf{d}n\mathbf{d}s = r\mathbf{d}r\mathbf{d}\alpha, \qquad 0 < r < \infty, \qquad -\alpha_0 \leq \alpha < \alpha_0 ,$$

where

$$\alpha_0 = \operatorname{arctg}\frac{B}{A_i} . \qquad (4.46)$$

Then (4.44) becomes in polar coordinates

$$E_x = \frac{q_1\left(x - \upsilon_x t\right)}{\pi\varepsilon\left|x - \upsilon_x t\right|} \int_{-\alpha_0}^{\alpha_0} \cos\alpha \int_{-\infty}^{\infty} \frac{r\exp\left(i\sin\alpha - u\cos\alpha\right)r}{\sqrt{B^2\cos^2\alpha - A_i^2\sin^2\alpha}} \mathbf{d}\alpha\mathbf{d}r ,$$

after integrating it around r it is received

$$E_x = \frac{q_1\left(x - \upsilon_x t\right)}{\pi\varepsilon\left|x - \upsilon_x t\right|} \int_{-\alpha_0}^{\alpha_0} \frac{\cos\alpha}{\sqrt{B^2\cos^2\alpha - A_i^2\sin^2\alpha}} \frac{\mathbf{d}\alpha}{\left(i\sin\alpha - u\cos\alpha\right)^2} . \qquad (4.47)$$

Let's select the real part from (4.47) and transform:

$$E_x = \frac{2q_1\left(x - \upsilon_x t\right)}{\pi\varepsilon\left|x - \upsilon_x t\right|} \int_0^{B/A_i} \frac{\left(u^2 - \operatorname{tg}^2\alpha\right)\mathbf{d}\operatorname{tg}\alpha}{\left(u^2 - \operatorname{tg}^2\alpha\right)^2\sqrt{B^2 - A_i^2\operatorname{tg}^2\alpha}} .$$

Let's make the substitution (a and b are new parameters):

$$\operatorname{tg}\alpha = \frac{B}{A_i}\sin\gamma, \qquad a = u^2 , \qquad b = \frac{B^2}{A_i^2} > 0 , \qquad (4.48)$$

after substitution of which it is received

$$E_x = \frac{2q_1\left(x - \upsilon_x t\right)}{\pi\varepsilon\left|x - \upsilon_x t\right|A_i} \int_0^{\pi/2} \frac{a - b\sin^2\gamma}{\left(a + b\sin^2\gamma\right)^2} \mathbf{d}\gamma . \qquad (4.49)$$

The included integral in (4.49) can be expressed by the known integrals [12]:

$$I_2 = \int_0^{\pi/2} \frac{a - b\sin^2\gamma}{\left(a + b\sin^2\gamma\right)^2} \mathbf{d}\gamma = 2a\int_0^{\pi/2} \frac{\mathbf{d}\gamma}{\left(a + b\sin^2\gamma\right)^2} - \int_0^{\pi/2} \frac{\mathbf{d}\gamma}{a + b\sin^2\gamma} . \qquad (4.50)$$

Let's write the significance of the second integral:

$$\int_0^{\pi/2} \frac{\mathbf{d}\gamma}{\left(a + b\sin^2\gamma\right)^2} \mathbf{d}\gamma = \left.\frac{\operatorname{arctg}\left(\sqrt{\frac{a+b}{a}}\operatorname{tg}\gamma\right)}{\sqrt{a^2 + ab}}\right|_0^{\pi/2} = \frac{\pi}{2\sqrt{a^2 + ab}} .$$

Let's find the first integral:

$$\int_0^{\pi/2}\frac{\mathbf{d}\gamma}{\left(a+b\sin^2\gamma\right)^2}=\frac{1}{2a(a+b)}\left[(2a+b)\int_0^{\pi/2}\frac{\mathbf{d}\gamma}{a+b\sin^2\gamma}+\right.$$

$$\left.+\frac{b\sin\gamma\cos\gamma}{a+b\sin^2\gamma}\Big|_0^{\pi/2}\right]=\frac{2a+b}{2a(a+b)}\frac{\pi}{2\sqrt{a^2+ab}}.$$

Substituting the integrals significances in (4.50) we discover

$$I_2=\frac{\pi}{2}\frac{\sqrt{a}}{(a+b)^{3/2}}. \qquad (4.51)$$

After the substitution of the integral (4.51) in (4.49) electrical intensity will be

$$E_x=\frac{q_1(x-\upsilon_x t)}{\varepsilon|x-\upsilon_x t|}\frac{\sqrt{a}}{A_i(a+b)^{3/2}}. \qquad (4.52)$$

It will define the parameters a and b by (4.48):

$$\sqrt{a}=u=\frac{|x-\upsilon_x t|}{1-\beta_x^2}\sqrt{1-\beta_x^2-\beta_y^2},$$

$$b=\frac{B^2}{A_i^2}=\frac{\left(L_3^2+2A_gL_3L_2+A^2L_2^2\right)\left(1-\beta_x^2-\beta_y^2\right)^2}{(1-\beta_x^2)\,(1-\beta^2)},$$

$$a+b=\frac{1-\beta_x^2-\beta_y^2}{(1-\beta_x^2-\beta_z^2)\,(1-\beta_x^2-\beta_y^2)-\beta_y^2\beta_z^2}\times$$

$$\times\left\{\left(1-\beta_x^2-\beta_z^2\right)(x-\upsilon_x t)^2+\left(1-\beta_x^2-\beta_z^2\right)(y-\upsilon_y t)^2+\right.$$

$$+\left(1-\beta_x^2-\beta_y^2\right)(z-\upsilon_z t)^2+2\beta_x\beta_y(x-\upsilon_x t)(y-\upsilon_y t)+$$

$$\left.+2\beta_x\beta_z(x-\upsilon_x t)(z-\upsilon_z t)+2\beta_y\beta_z(y-\upsilon_y t)(z-\upsilon_z t)\right\}.$$

And this tested backward, is equalled

$$a+b=\frac{1-\beta_x^2-\beta_y^2}{\left(1-\beta_x^2\right)\left(1-\beta^2\right)}\left\{(\vec{r}-\vec{\upsilon}t)^2-\left[\vec{\beta}\times(\vec{r}-\vec{\upsilon}t)\right]^2\right\}.$$

After the substitution A_i, a and b in (4.52) it is received

$$E_x=\frac{q_1(x-\upsilon_x t)\dfrac{|x-\upsilon_x t|}{1-\beta_x^2}\sqrt{1-\beta_x^2-\beta_y^2}}{\varepsilon|x-\upsilon_x t|\left[\dfrac{\left(1-\beta_x^2\right)\left(1-\beta^2\right)}{\left(1-\beta_x^2-\beta_y^2\right)^2}\right]^{1/2}}\times$$

$$\times\frac{1}{\left[\dfrac{1-\beta_x^2-\beta_y^2}{\left(1-\beta_x^2\right)\left(1-\beta^2\right)}\right]^{3/2}\left\{(\vec{r}-\vec{\upsilon}t)^2-\left[\vec{\beta}\times(\vec{r}-\vec{\upsilon}t)\right]^2\right\}^{3/2}}.$$

After the simplification this expression becomes

$$E_x=\frac{q_1(x-\upsilon_x t)\left(1-\beta^2\right)}{\varepsilon\left\{(\vec{r}-\vec{\upsilon}t)^2-\left[\vec{\beta}\times(\vec{r}-\vec{\upsilon}t)\right]^2\right\}^{3/2}}. \qquad (4.53)$$

The solutions of the equations (4.6) for component intensity E_y and E_z will be similar. Therefore action force of the evenly and linearly moved charge q_1 with velocity per unit motionless charge q_2 will be vector written so:

$$\vec{E}=\frac{q_1(\vec{r}-\vec{\upsilon}t)\left(1-\beta^2\right)}{\varepsilon\left\{(\vec{r}-\vec{\upsilon}t)^2-\left[\vec{\beta}\times(\vec{r}-\vec{\upsilon}t)\right]^2\right\}^{3/2}}. \qquad (4.54)$$

4.3. INTERACTION FORCE OF TWO BODIES

If $\vec{R}=\vec{r}-\vec{\upsilon}t=\vec{r}-\vec{r}_q$ – is a vector of the distance from the charge q_1 to the charge q_2 then the charge q_1, moved relatively it, acts on the charge q_2 in correspondence with (4.54) by the force

$$\vec{F} = \frac{q_2 q_1 \left(1-\beta^2\right)\vec{R}}{\varepsilon\left\{ R^2 - \left[\vec{\beta}\times\vec{R}\right]^2 \right\}^{3/2}} \qquad (4.55)$$

As it is visible, the force of interaction between charged bodies depends only on their relative parameters: distance between them, their relative velocity and angular position between distance and velocity. The interaction of two bodies does not depend on coordinate systems, frame of references, ether and field. This outcome testifies that there are no mediums such as ether or field in which the movement happens. If they were interaction would depend on velocity relatively of these hypothetical essences. The expression (4.55) also testifies that it is simpler possible to consider interactions of two bodies by their relative distance and velocity than in different frames of references as it is accepted in TR.

The expression (4.54) is known as electric field intensity created by a charge moved with constant velocity in modern electrodynamics [40] and TR [26]. It is rewritten as

$$\vec{E} = \frac{q_1\left(1-\beta^2\right)\vec{R}}{R^3\left(1-\beta^2\sin^2\varphi\right)^{3/2}},$$

where φ - is angle between $\vec{R}$ and $\vec{v}$.

This expression is deduced by the transformations of electrical strength of a point charge from a motionless frame in a moved one. According to O.D. Jefimenko [98] first it was received by Oliver Haviside on the basis of the Maxwell's equations using invented by him operational calculus [94] in 1888. O.D. Jefimenko also deduced the equation (4.54) on the basis of the lag theory, which is stipulated by final velocity of electromagnetic action propagation. T.G. Barnes with his colleagues deduced (4.54) on the basis of induction by a moved charge of electrical field, which creates induction in its turn, i.e. it applies a new correction of field [84]. Thus it is obtained infinite series of intensity E after summation of which they derived the expression (4.54). Being based on the experimental laws (3.3) and (3.5) C.W. Lucas (Jr.) and J.W. Lucas [102] deduced the equation (4.54) as a result of repeated integration of transitions of an electric field in magnetic, and magnetic one in electric, which are described by these laws.

The authors are under impression of the ideology in the above mentioned works that the charged body creates electrical field with strength $\vec{E}$, in case of movement of which it is additionally produced magnetic field with intensity $\vec{H}$. According to this ideology, except electrical force $\vec{F}_E = q_2\vec{E}$, magnetic force must act on a body by Lorentz's law

$$\vec{F}_H = \frac{\mu q}{c}\left[\vec{v}\times\vec{H}\right]. \qquad (4.56)$$

None of them interprets the expression (4.54) as action force quantity of the moved charged body q_1 per unit of a motionless charged body. Moreover nobody cites directly the expression for interaction force of two moved charges. It is connected with the fact that the expressions for forces conflict with the main laws of mechanics within the field concepts. For example, O.D. Jefimenko shows that the third law of mechanics [97] is infringed or there are other paradoxes [98].

The expression (4.55) deduced by us in 1968 immediately follows from the experimental laws (3.3) and (3.5). We also have introduced the first (3.24) and second (3.22) Maxwell's laws from these laws. We have sequentially shown that the same concept is used both in case of the experimental researches, which conduced to derivation of the laws (3.3) and (3.5), and in obtained expression (4.55) it is the force of action on the body.

We obtain the force for constant velocity $\vec{v}$ of movement of the charged body q_1 in accordance to (4.55). If the velocity $\vec{v}$ depends on time then instead of the d'Alembert's equations (4.6) solved by us the other will be, one of which will be written in a projection on the axes *x* by (3.36) so

$$\Box E_x = \frac{4\pi}{\varepsilon}\left(\frac{\partial\rho}{\partial x} + \frac{1}{c_1^2}\frac{\partial\rho}{\partial t}v_x\right) + \frac{\rho}{c_1^2}\frac{\partial v_x}{\partial t}, \qquad (4.57)$$

i.e. component of the derivative $\partial v_x/\partial t$ emerges. The solution of the equation (4.57) must give equation $\vec{E}$ on not only distance and velocity, but also derivative of velocity. Classics of electrodynamics have apparently applied by these reasonings with reference to the equations for scalar φ and vector $\vec{A}$ potentials and imagined that the fields created by moved charges must depend on their accelerations. Electromagnetic forces of action on a particle dependent on its acceleration emerged from here.

We consider that the availability of the derivative $\partial v_x/\partial t$ in (4.57) is stipulated by the cost of differential bondings, which arise in case of research of a moved charged body action on magnet and of a moved magnet on a charged body. The time derivative of velocity in (4.57) is a partial derivative in the motionless point of space, relatively which the body is moved. And acceleration of a body arises in case of its transition in space. Therefore conclusion about dependence of force on acceleration from the expression (4.57) does not follow. We would have the full right to tell that the forces depend on acceleration if it was revealed for movement of a charged body with velocity $\vec{v}$ that for acceleration $\vec{w}$ the force of action on magnet differs from the force, when acceleration is absent, or if the dependence of action on a charged body from acceleration was revealed for magnet movement. If such dependence on acceleration was established then the experimental laws (3.3) and (3.5) would include the addends with *w*. However we do not know such results of experiments, therefore we will consider by the fore-quoted general proofs that the electromagnetic forces do not depend on acceleration. Therefore the expression (4.55) determines the force of action for any movement velocity of a charged body.

4.4. LAW OF RELATIVE MOVEMENT OF TWO OBJECTS

Let us consider the interaction of two electrified point objects with charges q_1 and q_2, with masses m_1 and m_2 (see Fig. 4.2). If $\vec{r}_1$ and $\vec{r}_2$ - their position vectors, the velocities of charges will be $\vec{u}_1 = \frac{\mathbf{d}\vec{r}_1}{\mathbf{d}t}$ and $\vec{u}_2 = \frac{\mathbf{d}\vec{r}_2}{\mathbf{d}t}$.

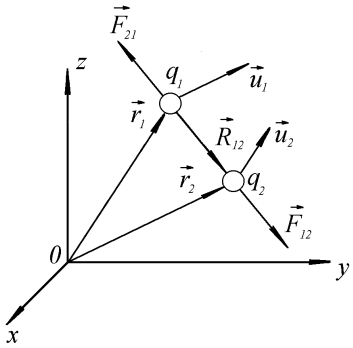


Fig.4.2. Forces of interaction of two point charged bodies.

The position vector from a charge q_1 up to a charge q_2 will be $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$, and it a velocity relatively a charge q_2

$$\vec{v} = \vec{u}_1 - \vec{u}_2 = \frac{\mathbf{d}\vec{r}_1}{\mathbf{d}t} - \frac{\mathbf{d}\vec{r}_2}{\mathbf{d}t}.$$

In correspondence with (4.55) the force of action of the first charge on the second one

$$\vec{F}_{12} = \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}_{12}\left(1-\beta^2\right)}{\left\{ R_{12}^2 - \left[\vec{\beta}\times\vec{R}_{12}\right]^2 \right\}^{3/2}}. \quad (4.58)$$

Knowing a force, it is possible to rewrite the equation of movement of the second charge:

$$m_2 \frac{\mathbf{d}^2\vec{r}_2}{\mathbf{d}t^2} = \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}_{12}\left(1-\beta^2\right)}{\left\{ R_{12}^2 - \left[\vec{\beta}\times\vec{R}_{12}\right]^2 \right\}^{3/2}}. \quad (4.59)$$

As the second charge goes relatively the first with a velocity ($-\vec{v}$) and position vector from it to the first charge $\vec{R}_{12} = \vec{r}_1 - \vec{r}_2$, in according with (4.55) the force of its action on the first charge will be the following

$$\vec{F}_{21} = \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}_{21}\left(1-\beta^2\right)}{\left\{ R_{21}^2 - \left[\vec{\beta}\times\vec{R}_{21}\right]^2 \right\}^{3/2}}.$$

Then the equation of movement of the first charge will be written

$$m_1 \frac{\mathbf{d}^2\vec{r}_1}{\mathbf{d}t^2} = \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}_{21}\left(1-\beta^2\right)}{\left\{ R_{21}^2 - \left[\vec{\beta}\times\vec{R}_{21}\right]^2 \right\}^{3/2}}. \quad (4.60)$$

And as $\vec{R}_{12} = -\vec{R}_{21} = \vec{r}_2 - \vec{r}_1 = \vec{R}$, the subtraction of the equation (4.60), multiplied on m_2, from the equation (4.59), multiplied on m_1, gives

$$m_1 m_2 \frac{\mathbf{d}^2\vec{R}}{\mathbf{d}t^2} = \left(m_1 + m_2\right) \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}\left(1-\beta^2\right)}{\left\{ R^2 - \left[\vec{\beta}\times\vec{R}\right]^2 \right\}^{3/2}}.$$

After transformation the law of relative movement of two charged point objects is received

$$\frac{\mathbf{d}^2\vec{R}}{\mathbf{d}t^2} = \mu_1 \frac{\vec{R}\left(1-\beta^2\right)}{\left\{ R^2 - \left[\vec{\beta}\times\vec{R}\right]^2 \right\}^{3/2}}, \quad (4.61)$$

where

$$\mu = \frac{q_1 q_2 \left(m_1 + m_2\right)}{\varepsilon m_1 m_2}. \quad (4.62)$$

Equation (4.61) includes only relative quantities: the distance $\vec{R}$ and velocity $\vec{v}$ of interactioning objects, i.e. their movement does not depend either on frame of references or the observers, fields etc. In case of small velocities of movement $\beta \to 0$ the equation (4.61) passes in the law of motion of charges interacting to Coulomb's law:

$$\frac{\mathbf{d}^2\vec{R}}{\mathbf{d}t^2} = \mu_1 \frac{\vec{R}}{R^3}. \quad (4.63)$$

The equation (4.63) can be treated treat as equation of movement in case of infinite velocity of distribution of interaction, as in case $c_1 \to \infty$ of dimensionless velocity $\beta \to 0$. That means, the equation (4.61) describes the movement of objects, which interaction is distributed with a final velocity. If the gravitational action is distributed with a velocity c_1, it is necessary to expect, that the movement of two interaction masses m_1 and m_2 will be determined by the equation (4.61). In this case by Newton's law of gravitation a constant of interaction

$$\mu_1 = -G\left(m_1 + m_2\right), \quad (4.64)$$

where G - gravitational constant.

4.5. INTEGRATION OF THE LAW OF MOTION

Let us consider relative movement of particles in a plane of vectors $\vec{\upsilon}$ and $\vec{R}$ (see fig.4.3). Let's express acceleration $\vec{w} = \dfrac{\mathbf{d}^2\vec{R}}{\mathbf{d}t^2}$ in the left part (4.61) by acceleration in projections: on a direction of a position vector and on a direction, perpendicular to it(him). Component of acceleration on a perpendicular direction $\vec{w}_t = \dfrac{\vec{\tau}}{R}\dfrac{\mathbf{d}\left(\omega R^2\right)}{\mathbf{d}t}$ is called transversal.

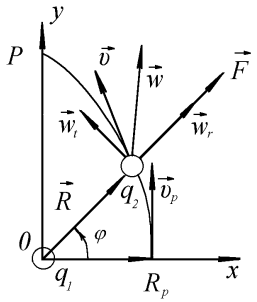


Fig. 4.3. Plane movement of two interacting bodies.

Here $\vec{\tau}$ - unit vector, and ω - angular velocity. The radial component of acceleration will be:

$$\vec{w}_r = \frac{\vec{R}}{R}\left(\frac{\mathbf{d}^2R}{\mathbf{d}t^2} - R\omega^2\right).$$

After the substitution of these accelerations in (4.61) we have

$$\frac{\vec{\tau}}{R}\frac{\mathbf{d}\left(\omega R^2\right)}{\mathbf{d}t} + \frac{\vec{R}}{R}\left(\frac{\mathbf{d}^2R}{\mathbf{d}t^2} - R\omega^2\right) = \mu_1 \frac{\vec{R}\left(1-\beta^2\right)}{\left\{R^2 - \left[\vec{\beta}\times\vec{R}\right]^2\right\}^{3/2}}. \qquad (4.65)$$

Equating expressions in the right and left parts of this equality in case of appropriate vectors $\vec{\tau}$ also $\vec{R}$ we receive two equations, from which the first one will be transformed so:

$$\frac{1}{R}\frac{\mathbf{d}\left(\omega R^2\right)}{\mathbf{d}t} = 0, \quad \frac{\mathbf{d}\left(\omega R^2\right)}{\mathbf{d}t} = 0, \quad \omega R^2 = \text{const}.$$

That is during movement a kinematic angular momentum

$$\omega R^2 = h \qquad (4.66)$$

remains constant. The movement happens in that plane, in which the vectors $\vec{R}$ and $\vec{\upsilon}$ lie. Such property of movement also is inherent in interactions according to the Coulomb and Newton's laws.

The coefficients in case of vector $\vec{R}$ in (4.65) give the second equation of movement:

$$\frac{\mathbf{d}^2R}{\mathbf{d}t^2} - R\omega^2 = \mu_1 \frac{R\left(1-\beta^2\right)}{\left\{R^2 - \left[\vec{\beta}\times\vec{R}\right]^2\right\}^{3/2}}. \qquad (4.67)$$

As the components of a velocity of a particle will be: radial - $\upsilon_r = \dot{R}$ and transversal $\upsilon_t = \omega R$,

$$1-\beta^2 = 1-\frac{\upsilon^2}{c_1^2} = 1-\frac{1}{c_1^2}\left(\dot{R}^2 + \omega^2R^2\right),$$

$$R^2 - \left[\vec{\beta}\times\vec{R}\right]^2 = R^2 - \frac{1}{c_1^2}\omega^2R^4 = R^2\left(1-\frac{\omega^2R^2}{c_1^2}\right),$$

and after the substitution in (4.67) we receive

$$\frac{\mathbf{d}^2R}{\mathbf{d}t^2} - R\omega^2 = \mu_1 \frac{1-\dfrac{1}{c_1^2}\left(\dot{R}^2 + \omega^2R^2\right)}{R^2\left(1-\dfrac{\omega^2R^2}{c_1^2}\right)^{3/2}}. \qquad (4.68)$$

This is nonlinear differential second-kind equation. Let's consider its solution. With this purpose we will proceed to new variable

$$y = 1/R \qquad \text{and} \qquad \varphi = \int \omega\mathbf{d}t, \qquad (4.69)$$

where φ- angular coordinate (see Fig. 4.3). Let's transform derivatives taking into account (4.66):

$$\frac{\mathbf{d}R}{\mathbf{d}t} = -\frac{1}{y^2}\frac{\mathbf{d}y}{\mathbf{d}\varphi}\frac{\mathbf{d}\varphi}{\mathbf{d}t} = -\frac{\omega}{y^2}\frac{\mathbf{d}y}{\mathbf{d}\varphi} = -h\frac{\mathbf{d}y}{\mathbf{d}\varphi},$$

$$\frac{\mathbf{d}^2R}{\mathbf{d}t^2} = -h\frac{\mathbf{d}^2y}{\mathbf{d}\varphi^2}\omega = -h^2y^2\frac{\mathbf{d}^2y}{\mathbf{d}\varphi^2}.$$

After the substitution of a new variable both derivatives in (4.68) and its transformation, we will rewrite:

$$\frac{\mathbf{d}^2 y}{\mathbf{d}\varphi^2} - \frac{\mu_1}{c_1^2}\frac{\left(\frac{\mathbf{d}y}{\mathbf{d}\varphi}\right)^2}{\left(1-\frac{h^2}{c_1^2}y^2\right)^{3/2}} = -y - \frac{\mu_1}{h^2}\frac{1}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}. \qquad (4.70)$$

As equation (4.70) concerns to a class of the nonlinear equations, which independent argument φ obviously does not enter, it is possible to make a replacement

$$p = \frac{\mathbf{d}y}{\mathbf{d}\varphi}. \qquad (4.71)$$

Then $\frac{\mathbf{d}^2 y}{\mathbf{d}\varphi^2} = \frac{\mathbf{d}p}{\mathbf{d}y}\frac{\mathbf{d}y}{\mathbf{d}\varphi} = p\frac{\mathbf{d}p}{\mathbf{d}y}$ also equation (4.70) accepts a kind

$$p\frac{\mathbf{d}p}{\mathbf{d}y} - \frac{\mu_1}{c_1^2}\frac{p^2}{\left(1-\frac{h^2}{c_1^2}y^2\right)^{3/2}} = -y - \frac{\mu_1}{h^2}\frac{1}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}. \qquad (4.72)$$

Thus, we have reduced (4.68) to the nonlinear differential equation of the first order. Let's proceed to new variables:

$$z = p^2, \qquad f(y) = -2\frac{\mu_1/c_1^2}{\left(1-\frac{h^2}{c_1^2}y^2\right)^{3/2}}, \qquad (4.73)$$

$$g(y) = -2\left(y + \frac{\mu_1}{h^2}\frac{1}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}\right), \qquad (4.74)$$

which reduce (4.72) to the linear differential equation of the first order:

$$\frac{\mathbf{d}z}{\mathbf{d}y} + f(y)z = g(y). \qquad (4.75)$$

Taking into account boundary conditions in a point $y = y_0$, $z = z_0$ and designating $F = \int_{y_0}^{y} f(y)\mathbf{d}y$, the solution of the equation (4.75) will be written as

$$z = e^{-F}\left(z_0 + \int_{y_0}^{y} g(y)e^{F}\mathbf{d}y\right). \qquad (4.76)$$

Now we find function F, by substituting in integrand expression the significance for $f(y)$ by (4.73):

$$F = -2\int_{y_0}^{y}\frac{\mu_1/c_1^2}{\left(1-\frac{h^2}{c_1^2}y^2\right)^{3/2}}\mathbf{d}y = \frac{2\mu_1}{c_1^2}\left(\frac{y_0}{\sqrt{1-\frac{h^2}{c_1^2}y_0^2}} - \frac{y}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}\right).$$

Further let us take an integral in (4.76):

$$\int_{y_0}^{y} g(y)e^{F}\mathbf{d}y = -\exp\frac{2\mu_1 y_0}{c_1^2\sqrt{1-\frac{h^2}{c_1^2}y_0^2}}\int_{y_0}^{y} 2\left(y + \frac{\mu_1}{h^2\sqrt{1-\frac{h^2}{c_1^2}y_0^2}}\right)\times$$

$$\times\exp\left(-\frac{2\mu_1 y}{c_1^2\sqrt{1-\frac{h^2}{c_1^2}y^2}}\right)\mathbf{d}y = \exp\frac{2\mu_1 y_0}{c_1^2\sqrt{1-\frac{h^2}{c_1^2}y_0^2}}\frac{c_1^2}{h^2}\left(1-\frac{h^2}{c_1^2}y^2\right)\times$$

$$\times\exp\left(-\frac{2\mu_1 y}{c_1^2\sqrt{1-\frac{h^2}{c_1^2}y^2}}\right)\Bigg|_{y_0}^{y} = y_0 - \frac{c_1^2}{h^2} + \left(\frac{c_1^2}{h^2} - y^2\right)\times$$

$$\times \exp\frac{2\mu_1}{c_1^2}\left(\frac{y_0}{\sqrt{1-\frac{h^2}{c_1^2}y_0^2}}-\frac{y}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}\right).$$

After the substitution of these expressions in (4.76) solutions accept a kind

$$z=-y^2+\frac{c_1^2}{h^2}+\left(y_0^2+z_0-\frac{c_1^2}{h^2}\right)\exp\frac{2\mu_1}{c_1^2}\left(\frac{y}{\sqrt{1-\frac{h^2}{c_1^2}y^2}}-\frac{y_0}{\sqrt{1-\frac{h^2}{c_1^2}y_0^2}}\right). \quad (4.77)$$

By (4.73) and (4.71) we will transform variable *z*:

$$z=p^2=\left(\frac{\mathbf{d}y}{\mathbf{d}\varphi}\right)^2=\frac{1}{R^4}\left(\frac{\mathbf{d}R}{\mathbf{d}\varphi}\right)^2,$$

$$z_0=\frac{1}{R^4}\left(\frac{\mathbf{d}R}{\mathbf{d}t}\right)^2\left(\frac{\mathbf{d}\varphi}{\mathbf{d}t}\right)^{-2}\Bigg|_{R=R_0}=\frac{v_r^2}{R^4\omega^2}\Bigg|_{R=R_0}=\frac{v_{r0}^2}{h^2},\quad y_0=\frac{1}{R_0}.$$

Here it used the second boundary condition $v_r(R_0)=v_{r0}$. The first boundary condition was used for a transversal velocity $v_t(R_0)=\omega R_0=h/R_0$. Let's mark that the entry conditions are required in case of binding solutions to time in case of integration $R=\int v_r\mathbf{d}t$ и $\varphi=\int\omega\mathbf{d}t$.

After the substitution of significances *y*, *z* and y_0, z_0 in (4.77) the law of motion (4.61) is received:

$$\frac{1}{R^2}\frac{\mathbf{d}R}{\mathbf{d}\varphi}=\sqrt{\frac{c_1^2}{h^2}-\frac{1}{R^2}+\left(\frac{1}{R_0^2}+\frac{v_{r0}^2}{h^2}-\frac{c_1^2}{h^2}\right)\exp\frac{2\mu_1}{c_1^2}\left(\frac{1/R}{\sqrt{1-\frac{h^2}{c_1^2R^2}}}-\frac{1/R_0}{\sqrt{1-\frac{h^2}{c_1^2R_0^2}}}\right)}. \quad (4.78)$$

As

$$\frac{\mathbf{d}R}{\mathbf{d}\varphi}=\frac{\mathbf{d}R/\mathbf{d}t}{\mathbf{d}\varphi/\mathbf{d}t}=\frac{v_r}{\omega}=\frac{v_r}{h}R^2, \quad (4.79)$$

then, excluding $\frac{\mathbf{d}R}{\mathbf{d}\varphi}$ from (4.78) and (4.79), we receive a radial velocity of movement of interacting particles

$$v_r=c_1\sqrt{1-\beta_t^2-\left(1-\beta_0^2\right)\exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2-\frac{h^2}{c_1^2}}}-\frac{1}{\sqrt{R_0^2-\frac{h^2}{c_1^2}}}\right)}. \quad (4.80)$$

The dimensionless velocities here are entered

$$\beta_{r0}=\frac{v_{r0}}{c_1},\qquad \beta_{t0}^2=\frac{h^2}{c_1^2R_0^2}=\frac{\omega^2R_0^2}{c_1^2}=\frac{v_{t0}^2}{c_1^2},\qquad \beta_0^2=\beta_{t0}^2+\beta_{r0}^2. \quad (4.81)$$

In case of known, according to (4.80), radial velocity the equation of a trajectory according to (4.79) will be written so:

$$\varphi=h\int\frac{\mathbf{d}R}{R^2v_r}. \quad (4.82)$$

The equations (4.80) and (4.82) determine movements of objects, which interaction is distributed with a final velocity c_1. The interesting corollaries follow from these equations.

4.6. TRANSITION TO CLASSICAL *AND RELATIVISTIC MECHANICS*

Let us show that the equations (4.80) and (4.82) pass in the equations of a classical mechanics if to consider distribution of interactions instantaneous. Taking into account (4.81) we will find a limit of quadrate of a radial velocity (4.78) in case of $c_1\to\infty$:

$$\lim_{c_1^2\to\infty}v_r^2=\lim_{c_1^2\to\infty}\frac{1-\frac{h^2}{c_1^2R^2}-\left(1-\frac{v_{r0}^2}{c_1^2}-\frac{h^2}{c_1^2R_0^2}\right)\exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2-\frac{h^2}{c_1^2}}}-\frac{1}{\sqrt{R_0^2-\frac{h^2}{c_1^2}}}\right)}{1/c_1^2}.$$

The limit represents indeterminacy of a type $\frac{0}{0}$. After application L'Hospital's rule and simplification it we have

$$\lim_{c_1^2\to\infty} v_r^2 = v_{r0}^2 - \frac{h^2}{R^2} + \frac{h^2}{R_0^2} - 2\mu_1\left(\frac{1}{R} - \frac{1}{R_0}\right) = v_{r0}^2 + \left(\frac{\mu_1}{h} + \frac{h}{R_0}\right)^2 - \left(\frac{\mu_1}{h} + \frac{h}{R}\right)^2.$$

From here we discover a radial velocity

$$v_r = \sqrt{v_{r0}^2 + \left(\frac{\mu_1}{h} + \frac{h}{R_0}\right)^2 - \left(\frac{\mu_1}{h} + \frac{h}{R}\right)^2}\,, \tag{4.83}$$

after substitution which in (4.82) is received the equation of a trajectory in a classical mechanics in case of movement in a central field

$$\varphi = \int \frac{h\mathbf{d}R}{R^2\sqrt{v_{r0}^2 + (\mu_1/h + h/R_0)^2 - (\mu_1/h + h/R)^2}}\,. \tag{4.84}$$

Let's show that the equations (4.80) and (4.82) in case of velocities of movement of objects which are not close to speed of light, are reduced to the equation of movement in a centrally symmetric field in theory relativity [26]:

$$\varphi = \int \frac{M\mathbf{d}R}{R^2\sqrt{W_0^2/c^2 - \left(m^2c^2 + M^2/R^2\right)\left(1 - R_g/R\right)}}\,. \tag{4.85}$$

With this purpose we will simplify expression for a radial velocity (4.80), decomposing multiplicands in a number and neglecting addends of the small order:

$$\left(1 - \frac{h^2}{c_1^2}\frac{1}{R^2}\right)^{-\frac{1}{2}} = 1 + \frac{h^2}{2c_1^2}\frac{1}{R^2} + \ldots;$$

$$\exp\frac{2\mu_1}{c_1^2 R}\left(1 - \frac{h^2}{c_1^2}\frac{1}{R^2}\right)^{-\frac{1}{2}} = 1 + 2\frac{\mu_1}{c_1^2}\frac{1}{R} + 2\frac{\mu_1^2}{c_1^4}\frac{1}{R^2} + \frac{\mu_1 h^2}{c_1^4 R^3} + \ldots$$

The exponential multiplicand (4.80) with R_0 will be similarly written. Taking into account only terms c_1^2 in a denominator the quadrate of a radial velocity will be expressed

$$v_r^2 = c_1^2\left[\left(1 - \frac{h^2}{c_1^2 R^2}\right) - \left(1 - \beta_0^2\right)\left(1 + 2\frac{\mu_1}{c_1^2 R}\right)\left(1 - 2\frac{\mu_1}{c_1^2 R_0}\right)\right]. \tag{4.86}$$

Let's enter quantity

$$R_g = -2\frac{\mu_1}{c_1^2}, \tag{4.87}$$

which is call a gravitational radius and bear from a right side (4.86) coefficients:

$$v_r^2 = c_1^2\left(1 - \frac{h^2}{c_1^2 R^2}\right)\left(1 - \beta_0^2\right)\left[\left(1 - \beta_0^2\right)^{-1} - \left(1 - \frac{h^2}{c_1^2 R^2}\right)^{-1}\left(1 - \frac{R_g}{R}\right)\left(1 + \frac{R_g}{R_0}\right)\right].$$

But as in case of velocities not close to c_1 simplifications will be fair

$$\left(1 - \beta_0^2\right)^{-1} = 1 + \beta_0^2 + \ldots,$$

$$\left(1 - \frac{h^2}{c_1^2 R^2}\right)^{-1} = 1 + \frac{h^2}{c_1^2 R^2} + \ldots,$$

then after the substitution them and regarding terms of the order c_1^2 we receive

$$v_r^2 = c_1^2 + v_0^2 - \left(c_1^2 + \frac{h^2}{R^2}\right)\left(1 - \frac{R_g}{R}\right)\left(1 + \frac{R_g}{R_0}\right). \tag{4.88}$$

After the substitution (4.88) in (4.82) we receive approximate up to degrees c_1^2 the equation of a trajectory

$$\varphi = \int \frac{h\mathbf{d}R}{R^2\sqrt{c_1^2 + v_0^2 - \left(c_1^2 + \frac{h^2}{R^2}\right)\left(1 - \frac{R_g}{R}\right)\left(1 + \frac{R_g}{R_0}\right)}}\,. \tag{4.89}$$

Let's designate a mass of the moved object by m, for example mass of a charge q_2 in Fig.4.3. Then $M = mh$ - is its angular momentum. According to a relativity theory, quadrate of energy of object moved with a velocity v_0, can be rewritten

$$\frac{W_0^2}{c_1^2} = \frac{1}{c_1^2}\left(\frac{mc_1^2}{\sqrt{1 - \beta_0{}^2}}\right)^2 = \frac{m^2c_1^2}{1 - \beta_0{}^2} \approx m^2c_1\left(1 + \beta_0^2\right) = m^2\left(c_1^2 + v_0^2\right).$$

If a numerator and denominator in a right side (4.89) will be multiplied on m and we use expressions for M and W_0, we receive the equation of a trajectory of movement of a point body with a mass m under action of other point body

$$\varphi = \int \frac{M\mathbf{d}R}{R^2\sqrt{W_0^2/c^2 - \left(m^2c^2 + M^2/R^2\right)\left(1 - R_g/R\right)\left(1 + R_g/R_0\right)}}. \qquad (4.90)$$

As the gravitational radius $R_g << R_0$, then, neglecting R_g/R_0, from (4.90) the relativistic equation (4.85) is received.

The equation (4.89) is fair for any velocity of distribution of interaction c_1 in a medium, and not just for a limiting velocity of distribution of light c in vacuum, as it has a place in relativity theory. It is fair for electromagnetic and gravitational interactions. By choice μ_1, regarding (4.62) or (4.64), in a parameter R_g in (4.87) the kind of interaction is determined. The relativistic expression (4.85) is only possible for using it, when the mass of object m is less significant than the mass of an influencing body. The equation (4.89) is fair in case of any masses of interacting bodies.

In case of derivation (4.89) in expansion the terms of the order c_1^2 were taken into account. Therefore given equation, as and relativistic (4.85), is inexact and can give incorrect results. Further we will receive more point expression for an integral of a trajectory and decide it.

From a classical mechanics it is known, that attracting objects can reach in case of radial rapprochement of any large velocity, which limits only by μ_1 - a constant of their interaction. But if the interaction of objects propagates with a final velocity c_1, the objects reaching such velocity will not be able to act each other. You should expect, that such objects can not speed up one another up to a velocity, greater velocity of distribution of their action. With this purpose let us consider interaction radial of moved particles (h = 0), initial the movement in infinity ($\upsilon_{r0} = 0$). The radial closing speed, regarding (4.80), in case of these conditions will be written

$$\upsilon_r = c_1\sqrt{1 - \exp\left(\frac{2\mu_1}{c_1^2}\frac{1}{R}\right)}. \qquad (4.91)$$

As in case of attraction $\mu_1 < 0$, only in case of rapprochement up to R = 0, as it is visible from (4.91), the velocity will reach a velocity of distribution of interaction.

If in any point R_0 the velocity of object is equaled velocities of distribution of interaction (β_0 = 1), it follows from (4.80) $\upsilon_r = c_1\sqrt{1-\beta_t^2} = \sqrt{c_1^2 - \upsilon_t^2}$, i.e. full velocity $\upsilon = \upsilon_r^2 + \upsilon_t^2 = c_1^2$. Therefore, such object will have a velocity c_1 and in all remaining points of a trajectory, i.e. will move with a constant velocity equal to a velocity of distribution of interaction.

In summary we will consider isoforce lines (4.55). As we can see, the force is directed on a radius, and module of a force of action moved with a velocity υ of a charge it can be written as

$$F = \frac{q_1q_2}{\varepsilon}\frac{1-\beta^2}{R^2\left(1-\beta^2\sin^2\varphi\right)^{3/2}}, \qquad (4.92)$$

where $\beta = \upsilon/c_1$ - dimensionless velocity of relative movement of charges; φ - angle between a velocity $\vec{\upsilon}$ and position vector $\vec{R}$ from a charge q_1 up to a charge q_2.

From (4.92) we receive the equation of a line of constant significance

$$R = \sqrt{A\frac{1-\beta^2}{R^2\left(1-\beta^2\sin^2\varphi\right)^{3/2}}}, \qquad (4.93)$$

where

$$A = \frac{q_1q_2}{\varepsilon} = \text{const}. \qquad (4.94)$$

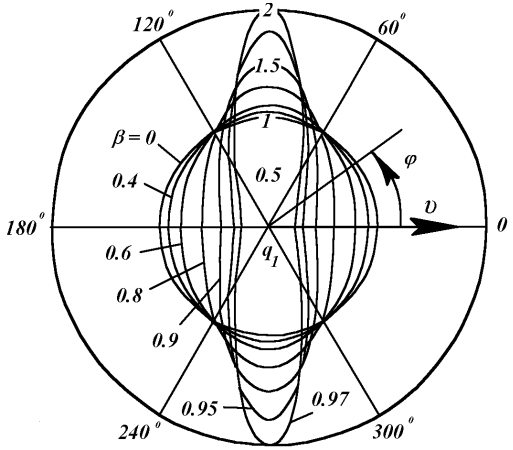


Fig. 4.4. Isoforce lines of action moved with a velocity υ of a charged body q_1 on a motionless charged body q_2 were from it on different angular distances φ in case of different quantities of a velocity $\beta = \upsilon/c_1$.

The equation (4.93) in case of A = 1 is represented in a Fig. 4.4 in polar coordinates. The lines of an equal force (4.93) represent cuts by a central plane of surfaces of an equal force, which are generated by rotation of isoforce lines round a vector of a velocity υ. In case of zero velocity of a charge q_1 the isosurface is an orb with a radius R = 1. With magnification of a velocity of an isoforce lines are contractioned along a line of movement and are stretched in a plane, perpendicular movement. In case of β = 1 isosurface of a force turns to perpendicular velocities a unlimited plane. Let's mark, that the isoforce lines (4.54) are simultaneously isolines of intensity E (4.54) of the moved charged body.

CHAPTER 5

THE TRAJECTORIES OF TWO BODIES MOVEMENT

5.1 CLASSICAL TRAJECTORIES

At the beginning we will consider trajectories of interacting particles by Coulomb or Newton's laws that are described by an equation of trajectory (4.84). In some point trajectory called a pericentre, the interacting objects approach at a minimum distance R_p (see Fig. 4.3): their radial velocity is equal to zero, and transversal $\upsilon_t = \upsilon_p = h/R_p$ receives the greatest value. Let's transform equations for radial velocity (4.83) and trajectory (4.82) to a dimensionless kind:

$$\bar{\upsilon}_r = \sqrt{(\alpha_1 + 1)^2 - \left[\alpha_1 + \frac{1}{\bar{R}}\right]^2}\,, \tag{5.1}$$

$$\varphi = \int \frac{\mathbf{d}\bar{R}}{\bar{R}^2 \bar{\upsilon}_r}\,, \tag{5.2}$$

where $\bar{R} = R/R_p$ - relative radius; $\bar{\upsilon}_r = \upsilon_r/\upsilon_p$ - relative radial velocity; $\beta_p = \upsilon_p/c_1$; $\alpha_1 = \mu_1/\left(R_p \upsilon_p^2\right)$ - parameter of trajectory.

The expression (5.2) in case of radial velocity (5.1) replacement $y = 1/\bar{R}$ is easily integrated, and in case of boundary condition $\bar{R} = 1$, in case of φ = 0, we obtain an equation of fundamental classical trajectories

$$\bar{R} = \frac{1}{(\alpha_1 + 1)\cos\varphi - \alpha_1}\,. \tag{5.3}$$

By of transformations $\bar{x} = \bar{R}\cos\varphi$ and $\bar{y} = \bar{R}\sin\varphi$, where

$$\bar{x} = x/R_p\,, \qquad \bar{y} = y/R_p \tag{5.4}$$

are adduced coordinates, , the equation of trajectory (5.3) in Cartesian coordinate system receives a kind:

$$\alpha_1^2 \bar{y}^2 - (2\alpha_1 + 1)\bar{x}^2 + 2(\alpha_1 + 1)\bar{x} = 1\,. \tag{5.5}$$

The equations (5.3) or (5.5) depend only on one parameter α_1 , i.e. the kind of trajectory is determined by its size. Let's consider trajectories of attraction in case of $\alpha_1 < 0$. In case of $\alpha_1 < -1$ of (5.3) it follows, that $\bar{R} = R/R_p < 1$, but it contradicts adopted, that a radius of a pericentre R_p - is the least distance between interacting objects. Values therefore are possible $\alpha_1 \geq -1$. In case of α_1 = -1, in accordance with (5.3), $\bar{R} = 1$, i.e. the movement happens in case of constant distance between particles, namely in a circle.

In case of α_1 = -0.5, in according to (5.5),

$$\bar{x} = 1 - 0.25\bar{y}^2\,, \tag{5.6}$$

i.e. trajectory is the parabola. In range of a change -1 < α_1 < -0.5 trajectories represent ellipses. In case of $\varphi = \pi$ the distance between particles, according to (5.3), will be greatest:

$$\bar{R}_a = -1/(2\alpha_1 + 1)\,. \tag{5.7}$$

Size R_a - radius of an apocentre. As $\bar{h} = h/\left(R_p \upsilon_p\right) = 1$, in apocentres the particle has the least transversal velocity, equal

$$\bar{\upsilon}_a = 1/\bar{R}_a = -(2\alpha_1 + 1)\,, \tag{5.8}$$

and it the radial velocity is equal to zero. Let's note, that for a parabola in case of α_1 = -0.5, according to (5.7), radius of an apocentre $\bar{R}_a \to \infty$, and velocity to infinity pursuant to (5.8) $\upsilon_\infty = 0$. In case of change α_1 from -1 up to -0.5 trajectories become more and more prolate ellipses and in a limit are transformed into parabolic trajectory. In range -0.5 < α_1 < 0 trajectories are hyperbolas. The half-angle between asymptotes of hyperbolas is determined from (5.3) in case of $\bar{R} \to \infty$:

$$\varphi_a = \pi - \arccos\left[\alpha_1/(\alpha_1 + 1)\right]. \tag{5.9}$$

The velocity of movement of a particle on infinity becomes exclusively radial and equal, according to (5.1),

$$\bar{\upsilon}_\infty = \sqrt{2\alpha_1 + 1}\,. \tag{5.10}$$

With approach α_1 to zero the half-angle between asymptotes φ_a comes nearer to π/2. In case of α_1 = 0 hyperbolas are degenerated in direct pursuant to (5.3), equal,

$$\bar{R} = 1/\cos\varphi\,. \tag{5.11}$$

As it is seen from (5.10), the particle goes in it in infinity with $\bar{\upsilon}_\infty = 1$ similar to the same one in pericentres. Using (5.1) it is easy to show, that the full velocity of a particle in all points of trajectory will be $\bar{\upsilon} = \sqrt{\upsilon_r^2 + \upsilon_t^2} = 1$, i.e. the particle goes with constant velocity. Parameter of trajectory α_1 = 0 in two cases: for want of interaction μ_1 = 0 and in case of infinite velocity $\bar{\upsilon}_p \to \infty$. In case of it trajectory

will be a direct line (5.11). Further we will see, that in case of final velocity of interaction c_1 of a particle will move on this trajectory, if their velocity $\upsilon = c_1$.

The trajectories (5.3) or (5.5) describe sections of a cone by a plane and are known for a long time. However they are resulted [24] depending on two parameters:

$$\overline{R} = \frac{\overline{P}}{1+\varepsilon_t \cos\varphi}, \tag{5.12}$$

where $P = \overline{P} R_p$ - focal parameter (in a Fig. 4.3 - coordinate of trajectory *y*, in case of x = 0); ε_t - eccentricity.

From comparison (5.3) and (5.12) it is visible, that

$$\overline{P} = -1/\alpha_1 , \tag{5.13}$$

$$\varepsilon_t = -\left(1+1/\alpha_1\right). \tag{5.14}$$

Let's express also through parameter of trajectory α_1 large and small semi-axises of an ellipse:

$$\overline{a} = \frac{\alpha_1}{2\alpha_1+1}, \qquad \overline{b} = \sqrt{-\frac{1}{2\alpha_1+1}}. \tag{5.15}$$

5.2. TIME OF MOVEMENT ON TRAJECTORY

Now we will consider time of movement in case of interactions Coulomb or Newton's laws. In case of known radial velocity the time of movement is determined as

$$t = \int \mathbf{d}R/\upsilon_r \ . \tag{5.16}$$

Let's consider an integral in an adduced variable. As not in all cases of movements there are data in pericentres, we will refer variables to values in any point R_0, in which the particle has radial υ_{r0} and transversal $\upsilon_{t0} = h/R_0$ making velocities and where begins a reference of time, i.e. $t_0 = 0$. Then with allowance for of radial velocity (4.83) in a dimensionless kind the time of movement will be

$$\bar{t} = \int_1^{\overline{R}} \frac{\mathbf{d}\overline{R}}{\sqrt{\overline{\upsilon}_{r0}^{02} + \left(\alpha_1^0+1\right)^2 - \left(\alpha_1^0 + \frac{1}{\overline{R}}\right)^2}}, \tag{5.17}$$

where

$$\overline{\upsilon}_{r0}^0 = \upsilon_{r0}/\upsilon_{t0} , \qquad \alpha_1^0 = \mu_1/\left(R_0\upsilon_{t0}^2\right), \qquad \overline{R} = R/R_0 , \qquad \bar{t} = t\,\upsilon_{t0}/R_0 .$$

Pursuant to definition, connection between parameters of trajectory, is the following,

$$\alpha_1 = \alpha_1^0 \, R_p / R_0 . \tag{5.18}$$

Let's transform expression (5.17), and an integral reduce to known [12]:

$$\bar{t} = \int_1^{\overline{R}} \frac{\overline{R}\,\mathbf{d}\overline{R}}{\sqrt{A\overline{R}^2 - 2\alpha_1^0\overline{R} - 1}} = \left.\frac{\sqrt{A\overline{R}^2 - 2\alpha_1^0\overline{R} - 1}}{A}\right|_1^{\overline{R}} + \frac{\alpha_1^0}{A}\int_1^{\overline{R}} \frac{\mathbf{d}\overline{R}}{\sqrt{A\overline{R}^2 - 2\alpha_1^0\overline{R} - 1}}, \tag{5.19}$$

where

$$A = \left(\overline{\upsilon}_{r0}^0\right)^2 + 2\alpha_1^0 + 1 . \tag{5.20}$$

Let's consider case $A > 0$. It corresponds to small values $|\alpha_1|$, i.e. hyperbolic trajectories. The solution of an integral in (5.19) depends on a discriminant of a radicand

$$D = -4A + 4\left(\alpha_1^0\right)^2 = -4\left[\overline{\upsilon}_{r0}^{02} + \left(\alpha_1^0+1\right)^2\right] < 0 .$$

In case of this value *D* the integral in (5.19) will be recorded so:

$$I = \frac{1}{\sqrt{A}} \ln\left(2\sqrt{A\overline{R}^2 - 2\alpha_1^0\overline{R} - 1} + 2A\overline{R} - 2\alpha_1^0\right). \tag{5.21}$$

After a substitution (5.21) in (5.19) and transformation time of movement on hyperbolic trajectory is obtained

$$\bar{t} = \frac{\overline{R}\overline{\upsilon}_r^0 - \overline{\upsilon}_{r0}^0}{A} + \frac{\alpha_1^0}{A^{3/2}} \ln \frac{\overline{R}\overline{\upsilon}_r^0\sqrt{A} + A\overline{R} - \alpha_1^0}{\overline{\upsilon}_{r0}^0\sqrt{A} + A - \alpha_1^0}, \tag{5.22}$$

where

$$\overline{\upsilon}_r^0 = \sqrt{\left(\overline{\upsilon}_{r0}^0\right)^2 + \left(\alpha_1^0+1\right)^2 - \left(\alpha_1^0 + 1/\overline{R}\right)^2} . \tag{5.23}$$

The case $A < 0$ corresponds to large values $|\alpha_1|$, i.e. ellipses, and the integral in (5.19) will be

$$I = -\frac{1}{\sqrt{-A}} \arcsin \frac{A\overline{R} - \alpha_1^0}{\sqrt{\left(\overline{\upsilon}_{r0}^0\right)^2 + \left(\alpha_1^0+1\right)^2}} . \tag{5.24}$$

After a substitution (5.24) in (5.19) and transformation time of movement on elliptical orbit is discovered

$$\bar{t} = \frac{\bar{R}\bar{v}_r^0 - \bar{v}_{r0}^0}{A} + \frac{\alpha_1^0}{(-A)^{3/2}}\left\{\arcsin\frac{A\bar{R} - \alpha_1^0}{\sqrt{\left(\bar{v}_{r0}^0\right)^2 + \left(\alpha_1^0 + 1\right)^2}} - \arcsin\frac{A - \alpha_1^0}{\sqrt{\left(\bar{v}_{r0}^0\right)^2 + \left(\alpha_1^0 + 1\right)^2}}\right\} \tag{5.25}$$

Let's consider case $A = 0$. From here, according to (5.20) $\alpha_1^0 = -0.5 - 0.5\left(\upsilon_{r0}\right)^2$. In case of normalisation to R_p, where $\upsilon_{r0} = 0$, $\alpha_1^0 = \alpha_1 = -0.5$, i.e. it is case of parabolic trajectory. Then the first integral in (5.20) will be

$$\bar{t} = \int_1^{\bar{R}} \frac{\bar{R}\,\mathbf{d}\bar{R}}{\sqrt{-2\alpha_1^0\bar{R} - 1}} = \frac{1}{\sqrt{-2\alpha_1^0}} \int_1^{\bar{R}} \left(\sqrt{\bar{R} + 1/\left(2\alpha_1^0\right)} + \frac{1}{2\alpha_1^0\sqrt{\bar{R} + 1/\left(2\alpha_1^0\right)}} \right) \mathbf{d}\bar{R}\,.$$

After an integration and reduction time of movement on parabolic orbit is obtained

$$\bar{t} = \frac{\left(-2\alpha_1^0\bar{R} - 1\right)^{3/2} - \left(-2\alpha_1^0 - 1\right)^{3/2}}{6\left(\alpha_1^0\right)^2} + \frac{\sqrt{-2\alpha_1^0\bar{R} - 1} - \sqrt{-2\alpha_1^0 - 1}}{\sqrt{2}\left(-\alpha_1^0\right)^{3/2}}\,. \tag{5.26}$$

We have considered time of movement on trajectory in case of availability of transversal velocity. In case purely of radial movement, in case of $h = 0$, the radial velocity, according to (4.83), will be

$$\upsilon_r = \sqrt{\upsilon_{r0}^2 - 2\mu_1\left(1/R - 1/R_0\right)}\,. \tag{5.27}$$

Then pursuant to (5.16) time will be recorded

$$t = \int_{R_0}^{R} \frac{\mathbf{d}R}{\sqrt{\upsilon_{r0}^2 - 2\mu_1\left(1/R - 1/R_0\right)}} = \frac{1}{2\mu_1}\int_{R_0}^{R} \frac{\mathbf{d}x}{(a + bx)^2\sqrt{x}}\,, \tag{5.28}$$

here

$$x = \upsilon_{r0}^2 - 2\mu_1\left(1/R - 1/R_0\right), \qquad a = \frac{1}{R_0} + \frac{\upsilon_{r0}^2}{2\mu_1}, \qquad b = -\frac{1}{2\mu_1}\,. \tag{5.29}$$

The integral in (5.28) is uncovered by tabular integrals [24]:

$$\int \frac{\mathbf{d}x}{(a + bx)^2\sqrt{x}} = \frac{\sqrt{x}}{a(a + bx)} + \frac{1}{a\sqrt{ab}}\operatorname{arctg}\sqrt{\frac{bx}{a}}\,,$$

after which substitution in (5.28) and transformation with use (5.29) time of movement in case of radial interaction of two objects is obtained

$$\bar{t}^{\,m} = \frac{\bar{R}\bar{v}_r^{\,m} - \bar{v}_{r0}^{\,m}}{1 - \left(\bar{v}_{r0}^{\,m}\right)^2} - \frac{1}{\left(1 - \left(\bar{v}_{r0}^{\,m}\right)^2\right)^{3/2}}\left\{\operatorname{arctg}\frac{\bar{v}_{\mathrm{r}}^{\,m}}{\sqrt{1 - \left(\bar{v}_{r0}^{\,m}\right)^2}} - \operatorname{arctg}\frac{\bar{v}_{\mathrm{r0}}^{\,m}}{\sqrt{1 - \left(\bar{v}_{r0}^{\,m}\right)^2}}\right\}, \tag{5.30}$$

where

$$\bar{t}^{\,m} = t\sqrt{\frac{-2\mu_1}{R_0^3}}; \qquad \bar{v}_r^{\,m} = \upsilon_r\sqrt{\frac{R_0}{-2\mu_1}}; \qquad \bar{v}_{r0}^{\,m} = \upsilon_{r0}\sqrt{\frac{R_0}{-2\mu_1}}. \tag{5.31}$$

So, the time of movement on trajectory in case of interaction of two bodies is subdivided into four cases of movement, which three of which take place in case of availability of transversal component velocity: 1) hyperbolic (5.22) in case of $A > 0$; 2) elliptical (5.25) in case of $A < 0$; 3) parabolic (5.26) in case of $A = 0$. The trajectories in case of it are determined (5.3) in case of $-0.5 < \alpha_1 < 0$, $-1 \le \alpha_1 < -0.5$, $\alpha_1 = -0.5$ accordingly. In the fourth case of radial movement time and law of motion are set by the equation (5.30). The radial velocity $\bar{v}_r^{\,m}$, adduced according to (5.31) is included here, which, regarding (5.27), has a kind

$$\bar{v}_r^{\,m} = \sqrt{\frac{1}{\bar{R}} - 1 + \left(\upsilon_{r0}^{\,m}\right)^2}\,. \tag{5.32}$$

The equations for time (5.22), (5.25) and (5.26) in case of availability of transversal velocity become simpler in case of reference to parameters in pericentres. For this it is necessary to replace α_1^0 on α_1 in indicated expressions and to equate $\bar{v}_{r0} = 0$. For example, the time of movement on parabolic trajectory, according to (5.26), will be

$$\bar{t} = \frac{2}{3}\left(\bar{R} - 1\right)^{3/2} + 2\sqrt{\bar{R} - 1} \tag{5.33}$$

and pursuant to (5.25) on elliptical orbit

$$\bar{t} = \frac{\bar{R}\bar{v}_r}{2\alpha_1 + 1} - \frac{\alpha_1\left(\dfrac{\pi}{2} + \arcsin\dfrac{(2\alpha_1 + 1)\bar{R} - \alpha_1}{-\alpha_1 - 1}\right)}{\left(-2\alpha_1 - 1\right)^{3/2}}\,. \tag{5.34}$$

From the last expression in case of $\bar{R} = \bar{R}_a$, where $\bar{R}_a$ is determined (5.7), we will discover time of movement from a pericentre in an apocentre

$$\bar{t}_a = -\frac{\alpha_1 \pi}{\left(-2\alpha_1 - 1\right)^{3/2}}. \qquad (5.35)$$

As cycle time on elliptical orbit $\bar{T} = 2\bar{t}_a$,

$$\bar{T} = -\frac{2\alpha_1 \pi}{\left(-2\alpha_1 - 1\right)^{3/2}}. \qquad (5.36)$$

Thence follows that for circular orbit (α_1 = -1) adduced pursuant to (5.17) period

$$\bar{T} = 2\pi. \qquad (5.37)$$

After a substitution in (5.36) semimajor axes of elliptical orbit *a* pursuant to (5.15) is obtained

$$\bar{T}^2 = -\frac{4\pi^2}{\alpha_1}\bar{a}^3. \qquad (5.38)$$

The last expression for circular orbits (α_1 = -1) expresses the third Kepler's law: "The squares of times of the rotations of planets round the Sun pertain as cubes of their mean distances from it ". The expression (5.38) in a comparison with Kepler's Law is more exact and gives a ratio between *T* and *a* for elliptical orbits ($\alpha_1 \neq -1$), allowing to take into account weights of both interacting bodies by means of parameter μ_1 in α_1.

5.3. TRAJECTORY AT FINAL VELOCITY INTERACTION PROPAGATION

After a reference of velocity (4.80) to parameters in pericentres (5.2) the adduced radial velocity will be

$$\bar{v}_r = \frac{1}{\beta_p}\sqrt{1 - \frac{\beta_p^2}{\bar{R}^2} - \left(1-\beta_p^2\right)\exp\left[2\alpha_1\beta_p^2\left[\frac{1}{\sqrt{\bar{R}^2-\beta_p^2}} - \frac{1}{\sqrt{1-\beta_p^2}}\right]\right]}. \qquad (5.39)$$

The expression (5.39) together with (5.2) represents in a dimensionless kind of trajectory of movement in case of interaction of two bodies with final velocity of its propagation. They describe also considered above classical trajectories, as in case of expression (4.80) passes small velocity β_p to (4.83). The equation (5.39) in difference from (5.1) is two-parameter, as depends not only on parameter α_1, but also on relative velocity β_p.

The integration of equations of trajectory (5.2) and (5.39), and also time (5.16) and (5.39) was executed numerically [60] on the personal computer by the help of a packet MATHCAD. With the purpose of increasing velocity and accuracy of an integration the range $\bar{R}$ was divided into segments, where the integration implemented, and the results then were summarized. By asymptotic solutions adduced in item 5.7, and test examples it is proved, that the error of an integration does not exceed 0.001. In case of calculations the parameters α_1 (-0.1; -0.2; -0.3; -0.4; -0.5; -0.6; -0.7; -0.8; -0.9) and β_p (0.1; 0.3; 0.7; 0.9) are varied. Other values of parameters, which were determined by characteristic of trajectories, were also set. The programs for processing of results were written on a Fortran.

Consequently the program on the MATHCAD language, which actuated all stages of evaluations, was developed. It has allowed considerably to simplify calculation of trajectories and to increase accuracy of results. The given program is adduced in Appendix 1.

Only the most characteristic kinds of trajectories will be shown below, but the time of movement on them will not be considered. All calculated trajectories (about 90) can be found in the monograph [59] in tabular form of sizes $\bar{R}, \bar{v}_r, \varphi, \bar{x}, \bar{y}, \Delta\varphi, \bar{t}$, where $\bar{x}$ and $\bar{y}$ - are Cartesian's coordinates. There the values $\bar{R}, \varphi$ and $\bar{t}$ for limit points of classical trajectories that are determined in same parameter α_1 are given. In Appendix 2 the initial parameters and values in final computational points of trajectories, which are systematised under the forms of trajectories, are adduced.

5.4. THE HYPERBOLIC TRAJECTORIES

In a Fig. 5.1 in Cartesian coordinate system ($\bar{x} = x/R_p$; $\bar{y} = y/R_p$) the half-branches of hyperbolic-like trajectories are shown. The attracting centre is at the origin, and the particle moves from a pericentre ($\bar{x} = 1$; $\bar{y} = 0$) to infinity or visa versa. The inversion of movement is supposed by process of integration for all trajectories, expect elimination 7.

When β_p = 0.1 trajectories practically coincide with the classical one. With increase of velocity in the pericentres the half-angle between asymptotes φ_a decreases and in case of limiting $\beta_p = \beta_{pc} = 0.954$ becomes negative. The trajectory 7 differs from remaining not only by the angle $\varphi_a < 0$. Let us consider some details of the integration. The point $\bar{R} = 1$ is a singular point of an integral (5.2), since it transforms (5.39) into zero. Therefore numerical was integration implemented from $\bar{R} = 1.001$ up to $\bar{R} = 1000$, and the angle increment φ in the area $1 < \bar{R} < 1.001$ was determined from the asymptotic solution

$$\varphi = \frac{1}{\sqrt{1 + \alpha_1 / \sqrt{1-\beta_p^2}}}\frac{\sqrt{\bar{R}^2 - 1}}{\bar{R}}, \qquad (5.40)$$

which derivation is given in item 5.7. As one can see, the denominator in the first multiplier vanishes to zero when

$$\beta_p = \beta_{pc} = \sqrt{1-\alpha_1^2}\,, \qquad (5.41)$$

i.e. in this case when β_{pc} = 0.954 and $\overline{R}=1$ the angle φ tends to infinity. This result was verified by numerical integration, namely by sequential setting on the integration variable starting at $\overline{R}=1.0001$, $\overline{R}=1.00001$ etc. Thus, the particle of the trajectory with limiting velocity β_{pc} at the pericentres, while moving from infinity, reaches a circle with radius $\overline{R}=1$ and rests on it infinitely long. It is occurs that the particle moving from infinity is captured by attracting centre into a circular orbit.

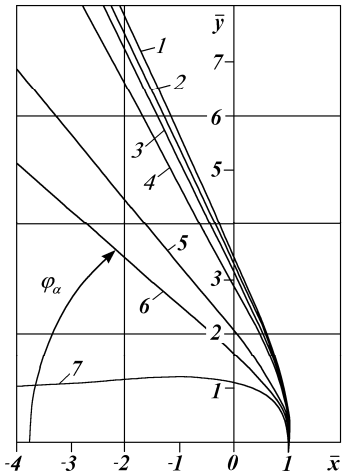


Fig. 5.1. Trajectories at α_1 = - 0.3 and sub-light speed at the pericentres ($\beta_p \le \beta_{pc}$) with half-angles between asymptotes φ_a and approaching velocities in infinity $\beta_{r\infty}$ (sign! testifies to the beginning of an integration with $\overline{R}$ = 1.001).

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*
β_p	0.1	0.3	0.5	0.7	0.9	0.93	0.954
α	-0.006	-0.054	-0.15	-0.294	-0.486	-0.519	-0.546
$\beta_{r\infty}$	0.063	0.208	0.329	0.480	0.649	0.668	0.667
φ_a°	64.8	64.4	63.8	61.9	50.6	40.3	-5.58!

We consider the given process in more details. Rushing φ to infinity, and consequently, a circular orbit are possible at $\overline{v}_r = 0$ in expression (5.2). From expression (5.39) with allowance for this condition we get

$$\alpha_1 = 0.5\frac{\ln\left[\left(1-\dfrac{\beta_p^2}{\overline{R}^2}\right)\Big/\left(1-\beta_p^2\right)\right]}{\beta_p^2\Big/\sqrt{\overline{R}^2-\beta_p^2}-\beta_p^2\Big/\sqrt{1-\beta_p^2}}\,. \qquad (5.42)$$

The radius of a circular orbit is simultaneously a radius of the pericentre, i.e. $\overline{R}=1$. Having found the limit of the right-hand side (5.42) where $\overline{R}\to 1$ for circular orbit $\alpha_{1c} = \alpha_1$ and $\beta_{pc} = \beta_p$ we get the following relation:

$$\alpha_{1c} = -\sqrt{1-\beta_{pc}^2}\,. \qquad (5.43)$$

With small velocities ($\beta_p \to 0$) it follows from (5.43) that α_1 = -1. This really corresponds to a circular orbit. Since the expressions (5.41) and (5.43) are identical, this fact convince us again that trajectory 7, having attained the limiting velocity β_{pc} at the pericentre, passes to a circular orbit as shown in Fig. 5.1.

Fig. 5.1 also shows the radial velocities of particles on infinity $\beta_{r\infty}$ and half-angles φ_a between asymptotes. For trajectory 7 the plotting of a polar angle begins at $\overline{R}=1.001$. With increase of the particle velocity in the infinity its velocity also increases at the pericentres. The violation of this rule for trajectories *6* and *7* is explained by the fact that the trajectory parameter α_1 depends on velocity at the pericentres v_p. Therefore it is expedient to consider the interaction parameter, which is independent of velocity,

$$\alpha = \frac{2\mu_1}{R_p c_1^2} = -\frac{R_g}{R_p}\,, \qquad (5.44)$$

which is connected with α_1 by a equation

$$\alpha = 2\alpha_1\beta_p^2\,. \qquad (5.45)$$

As follows from Fig. 5.1, the interaction parameter α for all hyperbolic-like trajectories is less than 1, i.e. according to (5.44) the pericentre radiuses is larger than the gravitational one.

The third property of trajectory *7* in Fig. 5.1 is its finitude. In the area $\beta_{pc} < \beta_p < 1$ the radicand in (5.39) is negative, i.e. no trajectories exist. With the purpose of numerical research of the other possible trajectories the parameters of equations (4.80) and (4.82) were, as well as in (5.17), referred to parameters v_{t0}, R_0 at an arbitrary of trajectory. In this case, Equation (5.2) remains without change, and instead of (5.39) we get

$$\overline{v}_r^{\,0} = \frac{1}{\beta_{t0}}\sqrt{1-\frac{\beta_{t0}^2}{\overline{R}^2}-\left(1-\beta_{r0}^2-\beta_{t0}^2\right)\exp\left[2\alpha_1^0\beta_{t0}^2\left[\frac{1}{\sqrt{\overline{R}^2-\beta_{t0}^2}}-\frac{1}{\sqrt{1-\beta_{t0}^2}}\right]\right]}\,, \qquad (5.46)$$

where

$$\overline{R} = R/R_0\,,\quad \beta_{t0} = v_{t0}/c_1\,,\quad \beta_{r0} = v_{r0}/c_1\,,\quad \overline{v}_r^{\,0} = v_r/v_{t0}\,,\quad \alpha_1^0 = \mu_1/\left(R_0 v_{t0}^2\right).$$

The equations (5.2), (5.46), and also (5.16) and (5.46) were integrated on two segments: $\overline{R} > 1$ and $\overline{R} < 1$. The values β_{t0} = 0.96 were set to relative transversal velocity; 0.97; 0.98; 0.987, which are more than β_{pc} and the radial velocity β_{r0} (see Fig. 5.2) was varied. In all calculations we found, that decreasing $\overline{R}$ up to some value $\overline{R} \to \beta_{t0}$ the radial velocity tends to zero, i.e. this point is the pericentre $\overline{R} = \overline{R}_p$. And the value $R_p / R_0 = \beta_{t0}$, indicates according to the angular momentum conservation law $\overline{h} = \overline{R}\beta_{t0} = 1$, that the tangential velocity at the given point tends to the speed of light, but it does not reach it: $\beta_p = \upsilon_p / c_1 = 1_{-0}$. These trajectories with almost light speed at the pericentres are restandardized to R_p and shown in Fig. 5.2. Since the trajectory parameter $\alpha_1 = \mu_1 / (h\upsilon_p)$ when $\upsilon_p = c_1$ is connected $\alpha_1^0 = \mu_1 / (h\upsilon_{t0})$ by the equation

$$\alpha_1 = \alpha_1^0 \beta_{t0}, \qquad (5.47)$$

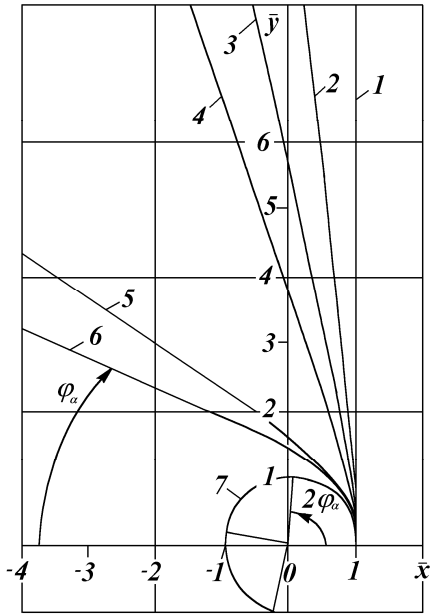


Fig. 5.2. Trajectories at α_1^0 = - 0.3 and light speed at the pericentres ($\beta_p = 1_{-0}$). A sign * - conformity of the data.

N°	*5*	*4*	*2*	*1*	*5*	*3*	*1*	*6*	*1*	*7*	*1*
β_{t0}	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.98	0.98	0.987	0.987
α	-0.576	-0.576	-0.576	-0.576	-0.582	-0.582	-0.582	-0.588	-0.588	-0.592	-0.592
β_{r0}	0.1	0.2	0.25	0.28	0.1	0.2	0.243	0.1	0.199	0.1	0.161
$\beta_{r\infty} / \overline{R}_a^*$	0.712	0.850	0.949	1.0	0.707	0.897	1	0.681	1	1.04*	1
φ_a°	35	73	84	90	35	78	90	23	90	41.2	90

it follows that in this case $\alpha_1 \approx$ -0.3 and trajectory of a Fig. 5.2 is possible to consider as prolongation of trajectories of Fig. 5.1 with the particle velocity increasing to infinity. However in contrast to the trajectories of a Fig. 5.1, with an increase $\beta_{r\infty}$ (see trajectories 6, 5, 4, 3, 2 in Fig. 5.2) the angle between asymptotes φ_a increases and for the particle moving with the light speed ($\beta_{r\infty} = 1$), the angle is equal to $\pi/2$ (see trajectories *1*), i.e. the particle with speed moves along vertical line. It the trajectory is described by expression (5.11). The movement of a particle

infinite velocity happens interacting according to Coulomb's law takes place on such trajectory.

Numerical solutions have shown, that the trajectories with light speed at the pericentres are received at tangential a particle velocity

$$\beta_{t0} > \beta_{pc}. \qquad (5.48)$$

With further increase in tangential velocity (see trajectory *7* in Fig. 5.2) the orbit becomes final and the angle of its apocentre from the pericentre is φ_a = 41.2°. In this case period of return to a pericentre will be implemented over angle 82.4°, and for one revolution will be more than four such periods. A final orbit we shall mean a trajectory on which the particle does not leave in infinity. For such orbit the angle φ_a means an angular distance up to the apocentre, i.e. the angular halfcycle of orbit. As the radius of the apocentre in the considered example $\overline{R}_a = 1.04$ does not differ from the pericentre radius, the movement will take place along the circular orbit with four small hops per turn. During hops the velocity particle discreases, and at the pericentres it tends to light speed. As in this case φ_a is not a multiple of π/n, where n is an integer, the position of pericentres in space will vary, they will rotate with an a angle per turn

$$\Delta\varphi_p = 2\varphi_a(n+1) - 2\pi, \qquad (5.49)$$

where n = INTEGER (π/φ_a) is an integer; INTEGER is the whole part of number.

Apparently, for the final orbits similar to 7 in Fig. 5.2, the radicand in (5.46) in at large $\overline{R}$ should be negative. Let's discover the limiting parameters α_{1p}^0 and $\beta_{0p} = \sqrt{\beta_{r0}^2 + \beta_{tp}^2}$ from the condition $\bar{v}_r^0 = 0$ when $R \to \infty$. After transformation (5.46) we get

$$\alpha_{1p}^0 = 0.5\frac{\sqrt{1-\beta_{tp}^2}\,\ln\left(1-\beta_{0p}^2\right)}{\beta_{tp}^2}. \qquad (5.50)$$

With $R_0 = R_p$ and small velocities when $\beta_{0p} = \beta_{tp} \to 0$, it follows that $\alpha_{1p}^0 = -0.5$, i.e. the equation (5.50) determines the parameter of parabolic trajectories. If the tangential velocity of the particle is larger than the limiting velocity β_{pc} and larger than β_{tp}, then the trajectory will be final and have light speed at the pericentres.

In accordance with positions the parameters there were shown $\alpha_1^0 = -0{,}498$, $\beta_{t0} = 0.93$, where final trajectories are brightly expressed (Fig. 5.3). With an increase in the radial velocity for trajectories *1, 2, 3* sizes of a hop

$\Delta\overline{R} = \overline{R}_a - 1$ and the angle up to the apocentre increase. With further increase β_{r0} the value R_a grows continuously, the angle φ_a reaches a maximum for trajectory *4*, and then decreases. In this case the trajectories *4* and *5* have an apocentre separating from a pericentre more, than one revolution. With a further increase of radial velocity the trajectory (see line *6*) is broken. When β_0 approaches to unit the trajectories are flattened and, coming nearer to vertical at light speed they turn into straight lines (similarly to trajectories *3, 2, 1* in a Fig. 5.2). It is necessary to note, that the hyperbolic-like trajectories in Fig. 5.1 and 5.2 can in separate regions coincide. However, the particles different movements interaction correspond parameters to the trajectories and the velocities along them are different.

So, in the region -0.5 < α_1 < 0 we can note the hyperbolic-like trajectories at particles velocities at the pericentres $\beta_p < \beta_{pc}$, the trajectories of acquising the particles from infinity into circular orbit when $\beta_p = \beta_{pc}$ and the trajectories of the particles moving at the speed of light in the pericentres when $\beta_{t0} > \beta_{pc}$. In the last case, where $\beta_{t0} > \beta_{tp}$, we can see the final trajectories, which period can differ from 2π essentially.

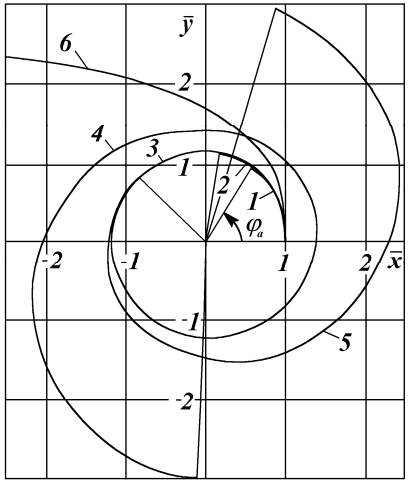


Fig. 5.3. Final trajectories (*1÷5*) with light speed at the pericentres ($\beta_p = 1_{-0}$).

α_1° = - 0.498; β_{t0}= 0.93; α = - 0.926

N°	*1*	*2*	*3*	*4*	*5*	*6*
β_{r0}	0.100	0.120	0.128	0.129	0.130	0.200
$\overline{R}_a/\beta_{r\infty}$*	1.103	1.133	1.176	2.981	3.035	0.095*
φ_a°	59.8	82.1	135.6	626.8	432.9	-18.24

5.5. THE PARABOLA-LIKE AND ELLIPSE-LIKE TRAJECTORIES

In a Fig. 5.4 the sub-lightsped trajectories when of α_1 = -0.5 we show, that in the classical case ($\beta_p \to 0$) gives a parabola. Even with $\beta_p = 0.1$, trajectory *1* is a highly-stretched ellipse. With an increase of velocity the distance to the apocentre decreases, and the angular distance φ_a increases, and for the limiting trajectory *6* it exceeds 2π. For this trajectory, as well as for a limited trajectory *7* in Fig. 5.1, the angle is counted from $\overline{R} = 1.001$. In the area $1.001 > \overline{R} \geq 1$ $\varphi \to \infty$, i.e. the acquisition of a particle from the final area of space into the circular orbit happens.

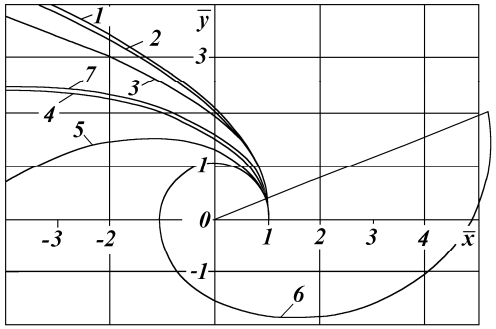
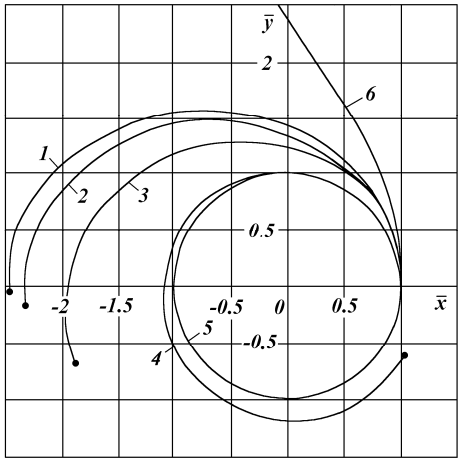


Fig. 5.4. Trajectories at α_1 = - 0.5 and with sub-lightsped at the pericentres ($\beta_p \leq \beta_{pc}$).

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*
β_p	0.1	0.3	0.5	0.7	0.8	0.866	1_{-0}
α	-0.01	-0.09	-0.25	-0.49	-0.64	-0.75	-0.9
$\overline{R}_a/\beta_{r\infty}$*	23641	2574	250.7	37.28	13.16	5.456	0.195*
φ_a°	1 180.2	182.2	186.8	200.7	224.3	383.6!	-7.243



In Fig. 5.4 the trajectory *7* with $\beta_{t0} = 0.9$ and $\beta_{r0} = 0.2$ which has the light speed at the pericentres is also presented here. This hyperbolic trajectory with a negative angle between asymptotes is similar to trajectories *6* in a Fig. 5.3.

So, in case of the constant parameter of trajectory α_1 with an increase in the particle velocity, the parabolic trajectory transforms into an ellipse-like, in which the pericentre turns by an angle $\Delta\varphi$ per one turn according to (5.49).

Fig. 5.5. Trajectories at α_1 = - 0.7 and with sub-lightspeed (*1÷5*) at the pericentres ($\beta_p \leq \beta_{pc}$).

N°	*1*	*2*	*3*	*4*	*5*	*6*
β_p	0.1	0.3	0.5	0.7	0.714	1_{-0}
α	-0.014	-0.126	-0.350	-0.686	-0.714	-1.12
$\overline{R}_a/\beta_{r\infty}$	2.482	2.334	1.991	1.220	1.031	0.331*
φ_a°	180.4	184.5	197.5	328.1	1340!	40.26

The minimum distance to the apocentre $\overline{R}_a = 5.456$ is peculiar for a limited trajectory. With even higher speeds, the trajectories become hyperbolic, with light speed at the pericentres.

Ellipse-like trajectories, as follows from Fig. 5.5, when increasing their speed decrease the eccentricity and increase the pericentre revolution. The limited trajectory *5* has less expressed apocentres, therefore practically it does not differ from the circular orbit. Here we show the trajectory *6* with light speed at the pericentres obtained at $\beta_{t0} = 0.8$ and $\beta_{r0} = 0.4$. The interaction parameter $|\alpha|$ in this case exceeds the unit, i.e. at the pericentres the particle goes inside the gravitational radius sphere.

To determine the possible values of α we take advantage of connection (5.45) with parameter α_1. Then for limiting trajectories (5.43) we obtain the dependence of the interaction parameter $\alpha_c = \alpha$ with velocity at the pericentre β_{pc} as

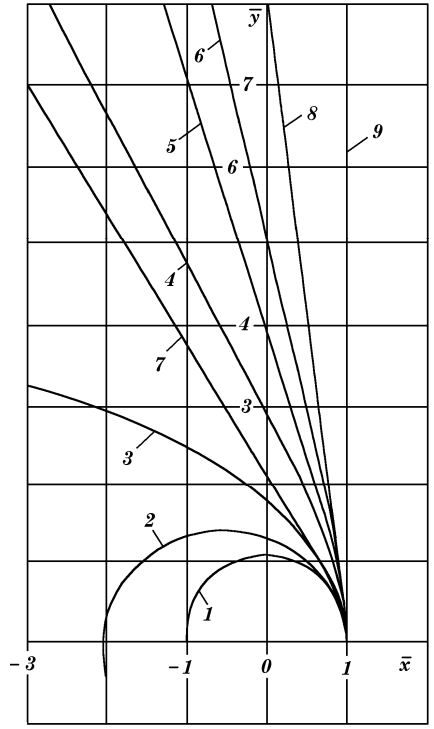
$$\alpha_c = -2\beta_{pc}^2\sqrt{1-\beta_{pc}^2}\ . \qquad (5.51)$$

It is easy to show, that this expression has an extremum at $\beta_{pc} = \sqrt{2/3}$ and the highest value of the interaction parameter will be $\alpha_c = -4/\sqrt{27}$. When $|\alpha| > |\alpha_c|$ the trajectories already have light speed at the pericentres, and they are either hyperbolic, or terminal. Thus, taking into account (5.44), it is possible to make conclusions. At first, for an attracting centre with a radius of a smaller gravitational radius R_g (the so-called " black hole ") the particles can penetrate inside the gravitational radius circle and do not fall on the attracting centre. Secondly, the particles at the pericentres reach the light speed, and owing to decreasing action on them go to infinity (or to the apocentre - for terminal trajectories). The elimination is made only by a particle, which vector velocity is directed strictly on a radius. According to (4.80), with $h = 0$ and $R_0 \to \infty$ we obtain

$$\beta_r = \sqrt{1-(1-\beta_{r0}^2)\exp(-R_g/R)} \qquad (5.52)$$

In this case particle will fall on the attracting centre, but its velocity, as follows from (5.52), will be less than light speed. In the classical case (4.83) for the particle, moving radially from infinity, the velocity is

$$\beta_r = \sqrt{\beta_{r0}^2 + R_g/R}\ . \qquad (5.53)$$



If the particle rested ($\beta_{r0} = 0$) at the infinity, on reaching the radius $R = R_g$ its velocity will be the equal to light speed. As this process is identical to both electromagnetic and gravitational interactions, the value R_g is best referred to as the light radius.

So, the obtained results testify that for interactions propagating with velocity c_1, the attracting centre at radius $R \le R_g$, i.e. "the black hole", involves substance more weakly, than the classical attracting centre, which action is propagated instantly.

The interactions of trajectories with constant parameter α_1 were analysed in above mentioned cases. In case of variations β_p parameters of interacting objects vary. Let's consider speed influence at constant characteristics of interacting objects. In Fig. 5.6 the trajectories with constant parameter α are shown.

Fig. 5.6. Trajectories with constant interaction parameter α = - 0.3.

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
β_p	0.408	0.463	0.548	0.707	0.866	0.913	0.988	1_{-0}	1
α_1	-0.9	-0.7	-0.5	– 0.3	-0.2	-0.18	-0.154	-0.151	-0.15
$\overline{R}_a/\beta_{r\infty *}$	1.036	2.074	157	0.488*	0.739*	0.809*	0.913*	0.968*	1.0*
φ_a°	189.9	193.5	189.1	62.56	73.78	77.56	59.40!	82.72	90

With increase of velocity at the pericentres eccentricities of the ellipse-like trajectories (*1 - 3*) are increased, and then they are broken and pass into hyperbola-like trajectories (*4 - 7*), which angle between the asymptotes grows. With even higher velocity in infinity $\beta_{r\infty}$ (the trajectory *8*) speed is reached at the pericentres, the angle φ_a increases and tends to $\pi/2$ φ_a for light trajectory *9*. It should be mentioned, that for $\beta_p = \beta_{pc}$ (trajectory *7*) the angle between asymptotes is less, as the integration is executed up to $\overline{R} = 1.001$.

The variations of trajectories for different α_1 and β_p are shown in Fig. 5.7. The curve *1* from (5.43) limits from below, and on the right the area of existence of trajectories with sub-lightsped in pericentres. This curve gives parameters of trajectories which are transient into circular orbit. The curve *2*, presenting relation (5.50), detaches the hyperbola-like trajectories from ellipse-like ones. The parameters of parabolic-like trajectories are located on it. Curves *1* and *2* are intersected at the point α_1 = -0.450764 and β_p = 0.89264.

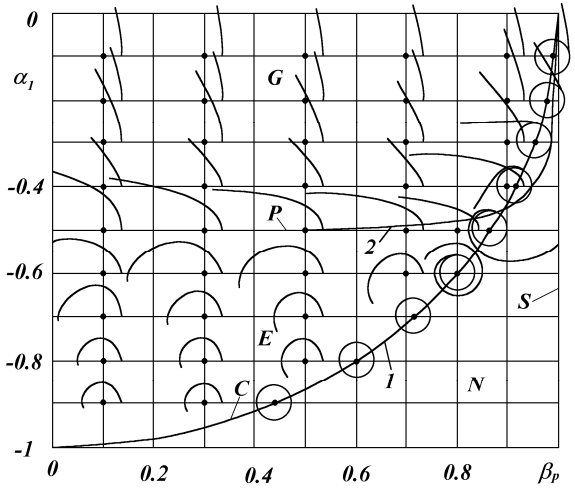


Fig. 5.7. The panorama of trajectories of two-body interactions depending on parameters α_1 and β_p.
1 - Formula (5.43); *2* - formula (5.50) at $\beta_p = \beta_{tp} = \beta_{0p}$.
Kinds of trajectories: *G* - hyperbolic; *P* - parabolic-like; *E* - ellipse-like; *C* - boundary trajectories, transient in a circle; *S* - with light speed at the pericentres; *N* - absence of trajectories.

5.6. THE TRAJECTORIES OF REPULSION AND FULL PERIOD OF TRAJECTORIES

Fig. 5.8 presents the hyperbolic trajectories when the interacting bodied are repulsed. The repelling centre is in the origin. The repelling centre is in origin. The calculations were performed at three values of α_1 and variation of β_p. With an increase in β_p the half-angle between asymptotes φ_a increases and tends to $\pi/2$ for the light speed trajectory. The interaction parameter α is positive for the repulsion trajectories also can be more unity. The particle velocity increases when it moves away from the centre. Trajectories halfcycle were considered earlier. In Fig. 5.9 the trajectories during full period are presented. The cyclical trajectories *1* and *4* are open-ended. The trajectory with hops *1* has three periods per one turn at 360°, and the trajectory *4* has three and one-half turns per period.

Used here concept "period" is applicable to the distance between the interacting particles, as it increases from minimum $\overline{R}_p = 1$ up to maximum $\overline{R}_a$ and then decreases up to $\overline{R}_p$ when the angle changes on $\varphi_P = 2\varphi_a$, where φ_P is a function period $\overline{R}(\varphi)$. Unlike classical case, here the particle does not return to an initial

point of space through the angle φ_P, i.e. the vector functions $\bar{R}(\varphi)$ and $\bar{v}(\varphi)$ are not periodic. Therefore in general case the motion along terminal trajectories is not periodic. However if φ_a is multiple to number π, the motion will be periodic. From Fig. 5.9 one can see, that for trajectories *1* and *4* the angular distance up to the apocentre is possible to present as:

$$\varphi_a = \pi n + \pi/k, \tag{5.54}$$

where n = 0,1,2..., k = 1, 2, 3..., i.e. the particle will come in the same point of space with full period

$$\varphi_P = 2k\varphi_a = 2\pi kn + 2\pi. \tag{5.55}$$

For example, if trajectory *1* would have the precise equality $\varphi_a = \pi/3$, instead of $\varphi_a = 59{,}8^\circ$, that according to (5.54) is expressed by factors n = 0 and k = 3, then according to (5.55) the full period is $\varphi_P = 2\pi$. That is three periods of a change $\overline{R}(\varphi)$ would lead a particle to the initial point of space. For trajectory *4*, if $\varphi_a = (3\pi + \pi/2)57.3^\circ = 630^\circ$, instead of $626{,}8^\circ$, that corresponds to factors n = 3; k = 2 and full period, according to (5.55), $\varphi_P = 14\pi$. Here particle comes to the initial point of space after two periods of a change $\overline{R}(\varphi)$.

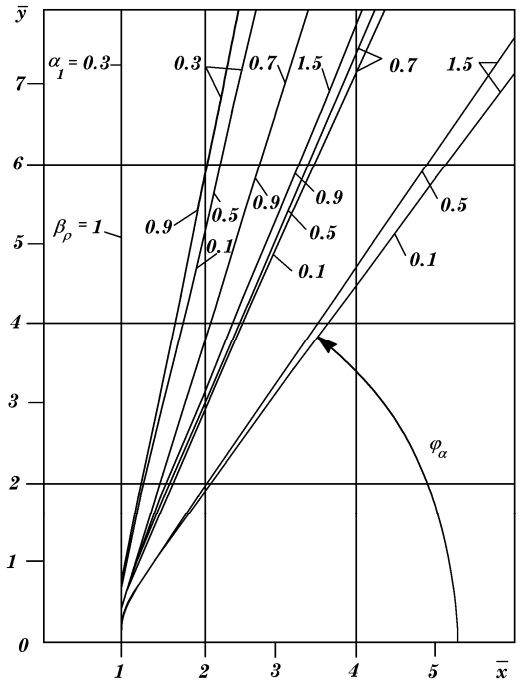


Fig. 5.8. Trajectories of a repulsion.

α_1	0.3	0.3	0.3	0.3	0.7	0.7	0.7	1.5	1.5	1.5
β_p	0.1	0.5	0.9	1.0	0.1	0.5	0.9	0.1	0.5	0.9
α	0.006	0.15	0.486	0.6	0.014	0.35	1.134	0.03	0.75	2.43
$\beta_{r\infty}$	0.126	0.61	0.968	1.0	0.1543	0.707	0.993	0.198	0.827	0.9999
φ_a°	76.7	76.9	79.1	90	65.7	66.6	73.2	53.2	55.6	69.0

As the trajectories and the angle φ_a are determined in parameters α_1 and β, then discrete values α_1 and β correspond to values n and k. These results about

discrete (quantum) parameters of periodic movements can have the important value for atom physics and a radioactivity.

We will consider one more exotic trajectory in Fig. 5.9 - trajectory *6*. It intersects it at a significant distance from the attracting centre. When define the parameters of attracting centre according to the trajectory characteristics and using the classical law of interaction one can make mistakes. For example, the size of the attracting centre can be overestimated or the interaction of attraction is perceived as repulsion: and in both cases the particle is reflected from centre.

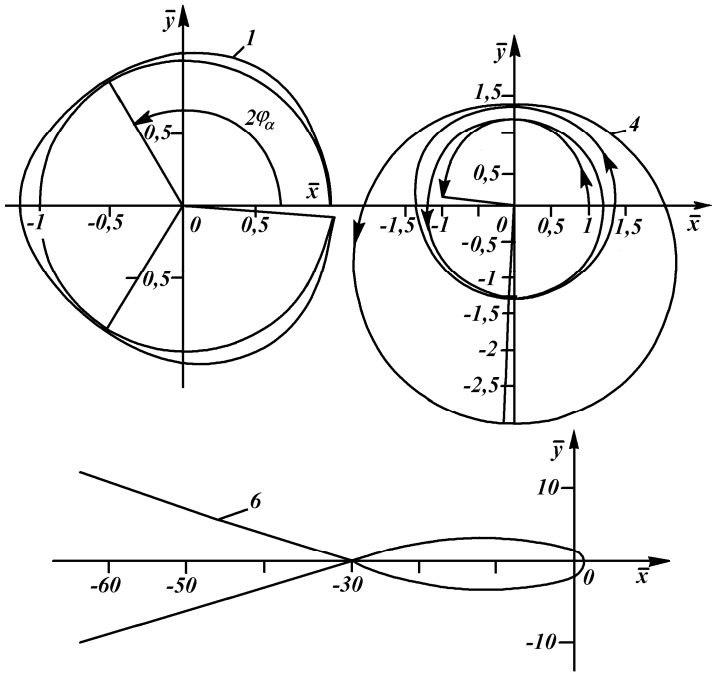


Fig. 5.9. Trajectories for full period in case of light velocity in pericentres. Numbers of trajectories *1, 4, 6* correspond to a Fig. 5.3.

The obtained results require more careful understanding and analysis. We can already see now that they can provide new mechanisms for understanding natural phenomena. Probably, these trajectories can explain availability of steady orbits of electrons in an atom, the transition of an electron from one orbit to an other, the capture by the kernel or atom of a particle having determined of kinematic parameters, etc.

With reference to gravitation the results say that many phenomena have an exact opposite effect of that predicted by GTR. For example, if gravitational action propagates at light speed, “the black holes” should not exist. If astronomers discovered this, they would likely conclude gravity propagates with in finite speed or much greater than the speed of light in vacuum.

5.7. ASYMPTOTIC SOLUTIONS

1. Solution close $\overline{R}=1$. A radicand in (5.39) we designate as $f\left(\overline{R}^2\right)$. One can see, that $f(1) = 0$. Let's decompose $f\left(\overline{R}^2\right)$ in the neighbourhood $\overline{R}^2=1$ in to Teilor's series

$$f\left(\overline{R}^2\right)\approx f(1)+f'(1)\left(\overline{R}^2-1\right)+f''(1)\frac{\left(\overline{R}^2-1\right)^2}{2}+\ldots \qquad (5.56)$$

and as $\overline{R}^2$ does not different from 1 greatly, we shall take only the first two addends.

According to (5.39), the derivative has a kind

$$\frac{\mathbf{d}f}{\mathbf{d}(\overline{R}^2)}=\frac{\beta_p^2}{\overline{R}^4}+\left(1-\beta_p^2\right)\exp\left[2\alpha_1\beta_p^2\left(\frac{1}{\sqrt{\overline{R}^2-\beta_p^2}}-\frac{1}{\sqrt{1-\beta_p^2}}\right)\right]\times$$

$$\times\frac{\alpha_1\beta_p^2}{\left(\overline{R}^2-\beta_p^2\right)^{3/2}}\Bigg|_{\overline{R}=1}=\beta_p^2\left(\frac{\sqrt{1-\beta_p^2}+\alpha_1}{\sqrt{1-\beta_p^2}}\right).$$

After a substitution of a derivative in (5.56), and $f\left(\overline{R}^2\right)$ in (5.39) radial velocities we have

$$\overline{v}_r=\sqrt{\left(1+\alpha_1\Big/\sqrt{1-\beta_p^2}\right)\sqrt{\overline{R}^2-1}}\,. \qquad (5.57)$$

Allowing for (5.57) the trajectory equation (5.2) will be recorded

$$\varphi=\frac{1}{\sqrt{1+\alpha_1\Big/\sqrt{1-\beta_p^2}}}\int_1^{\overline{R}}\frac{\mathbf{d}\overline{R}}{\overline{R}\sqrt{\overline{R}^2-1}}\,.$$

As the result of integration the expression (5.40) is obtained. Since the exact solution when $\beta_p = 0$ is shown by the equation (5.3), they were compared at different α_1. As the result it was established, that when $\overline{R}=1.001$ there is no difference between them to the third decimal place after a comma inclusively.

In the limiting case, when $\beta_p\to\beta_{pc}$, the derivative is $f'(1)\to 0$. Therefore to specify (5.40) it is necessary to consider the second derivative. After differentially derivation the first derivative we get

$$\frac{\mathbf{d}^2 f}{\mathbf{d}(\overline{R}^2)^2} = \frac{-2\beta_p^2}{\overline{R}^6} - \frac{\left(1-\beta_p^2\right)\alpha_1\beta_p^2\left(\alpha_1\beta_p^2 + 1.5\sqrt{\overline{R}^2-\beta_p^2}\right)}{\left(1-\beta_p^2\right)^3 \exp\left[2\alpha_1\beta_p^2\left(\frac{1}{\sqrt{1-\beta_p^2}} - \frac{1}{\sqrt{\overline{R}^2-\beta_p^2}}\right)\right]}. \qquad (5.58)$$

At the singular point $\beta_p = \beta_{pc}$ when $\overline{R} = 1$ the second derivative will be

$$f''(1) = -\beta_{pc}^2\left(0.25 + 0.5\beta_{pc}^2\right),$$

i.e. it is final. Thus, in view of the higher order of a smallness in relation to the $\left(\overline{R}^2 - 1\right)$ the third term in (5.56) can be neglected and the expression (5.40) remains valid when $\beta_p = \beta_{pc}$.

Now we will define the approximation near the pericentre for time. In a dimensionless kind the integral for time (5.16) will be recorded

$$\bar{t} = \int \frac{\mathbf{d}\overline{R}}{\overline{v}_r}. \qquad (5.59)$$

Substituting the radial velocity (5.57) in (5.59) and integrating, we obtain approximation for time close $\overline{R} = 1$ as follows:

$$\bar{t} = \frac{\ln\left(\sqrt{\overline{R}^2 - 1} + \overline{R}\right)}{\sqrt{1 + \alpha_1 \Big/ \sqrt{1-\beta_p^2}}}. \qquad (5.60)$$

2. <u>Approximation at light speed.</u> When $\beta_p = 1$, the expression (5.40) has a singularity, therefore to find increments φ in the area $1 \le \overline{R} \le 1.001$ we will employ a direct line equation, which at $\alpha_1 = 0$ is determined by the equation (5.11). Hence

$$\varphi = \arccos\left(1/\overline{R}\right). \qquad (5.61)$$

With $\overline{R} = 1.001$ and $\varphi = 4.47 \cdot 10^{-2}$ or $\varphi^{\circ} = 2.56^{\circ}$, i.e. for a light particle with variation of $\overline{R}$ from 1 to 1.001 the polar angle changes from 2.56.

3. <u>Approximation for an apocentre.</u> (5.2) For ellipse-like trajectories the numerical integration calculation was performed with smalls $\overline{v}_{r\,min}$, which have

the order $1 \cdot 10^{-3}$ and less. It is necessary to estimate, the increment φ corresponding to a change of radial velocity from $\overline{v}_{r\,min}$ up to zero. With this purpose we will express a relative radius $\overline{R}$ from (5.1) through velocity $\overline{v}_{r\,min}$:

$$\overline{R} = \frac{1}{-\alpha_1 - \sqrt{(\alpha_1+1)^2 - \overline{v}_r^2}}.$$

Here when solving the quadratic equation we choose the sign corresponding to the apocentre. A radius $\vec{R}$ will be substituted into the classical trajectory equation (5.3), whence we obtained

$$\Delta\varphi = \arccos\frac{\sqrt{(\alpha_1+1)^2 - \overline{v}_r^2}}{\alpha_1 + 1}. \qquad (5.62)$$

To include the effect of relative velocity β_p, we use the asymptotics (5.40) for a pericentre. Let's include the influence coefficient *k* as ratio of the angle φ, according to (5.40), to the same expression with $\beta_p = 0$. In the total we have

$$k = \sqrt{\frac{1+\alpha_1}{1+\alpha_1 \Big/ \sqrt{1-\beta_p^2}}}. \qquad (5.63)$$

Multiplying (5.62) by (5.63) the approximations for the apocentre will be recorded as

$$\Delta\varphi \approx \sqrt{\frac{1+\alpha_1}{1+\alpha_1 \Big/ \sqrt{1-\beta_p^2}}}\arccos\frac{\sqrt{(\alpha_1+1)^2 - \overline{v}_r^2}}{\alpha_1 + 1}. \qquad (5.64)$$

Due to the approximate character of (5.64) for limiting trajectory $\beta_p \to \beta_{pc}$ the numerical calculations with subsequent decreasing $\overline{v}_{r\,min}$ have been done. They have shown, that the particle moves to the apocentre with the final angle φ.

Now we will define the approximation for time in apocentres. From integral ratio for φ (5.2) and time *t* (5.59) it follows, that $\mathbf{d}t = \overline{R}_a^2 \mathbf{d}\varphi$. Therefore, in the apocentres with small changes φ it is possible to record the approximation for time as

$$\Delta\bar{t} = \overline{R}_a^2 \Delta\varphi, \qquad (5.65)$$

where $\Delta\varphi$ is set by the expression (5.64).

CHAPTER 6

THE FORCES OF INTERACTION OF VARIOUS BODIES ON MOVING CHARGED PARTICLE

6.1. INTERACTION OF THE CHARGED RECTANGULAR PLATE ON A PARTICLE

Let's consider the interaction of a charged plate with a charge q_2 on a moved charged particle q_1 (Fig. 6.1). The sides of a plate are $2a$ and $2b$. The beginning of coordinates system x, e z, y is taken in the cntre of a plate; the plane yOz coincides a plane of a plate; the axis Oy is parallel to side $2b$, and axis Oz is parallel to side $2a$. The coordinates of a particle are x_q, y_q, z_q, and it moves with velocity $\vec{v} = \vec{i}v_x + \vec{j}v_y + \vec{k}v_z$.

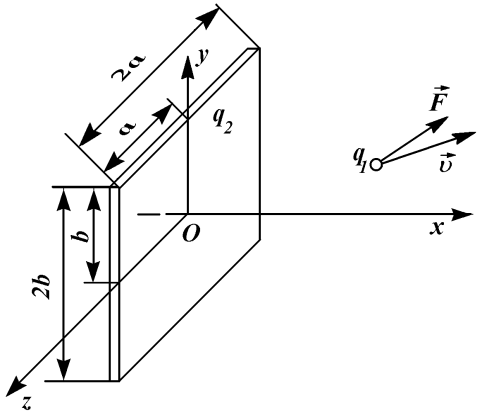


Fig. 6.1. The interaction of a charged rectangular plate on a moving particle with a charge q_1.

The element of a plate $\mathbf{d}y_s$ $\mathbf{d}z_s$ has a charge

$$dq = \sigma\, dy_s dz_s, \qquad (6.1)$$

where $\sigma = q_2/S$ is the area density of a charge; $S = 4ab$ is the area of a plate.

According to (4.58) we will record the expression for interaction force of an element $\mathbf{d}q$ and we summarize it on the whole surface of a plate:

$$\vec{F} = F_p\left(1-\beta^2\right)\int_{-a}^{a}\mathbf{d}z_s\int_{-b}^{b}\frac{\vec{R}\mathbf{d}y_s}{\left\{R^2-\left[\vec{\beta}\times\vec{R}\right]^2\right\}^{3/2}}, \qquad (6.2)$$

where

$$F_p = \frac{q_1 q_2}{\varepsilon S}, \qquad (6.3)$$

$$\vec{R} = \vec{i}x_q + \vec{j}\eta + \vec{k}\zeta, \qquad (6.4)$$

$$\eta = y_q - y_s, \quad \zeta = z_q - z_s, \qquad (6.5)$$

$x = x_q$, y_q, z_q are the coordinates of a moving particle q_1. As for a plate the coordinate is $x = 0$, the size x characterises a particle unequivocally. Introducing replacements

$$a_1 = x^2 y_x^{\,2} + 2x\zeta\beta_x\beta_z + \zeta^2\gamma_z^{\,2} > 0, \qquad (6.6)$$

$$b_1 = 2\beta_y(x\beta_x + \zeta\beta_z); \quad c_1 = \gamma_y^{\,2} > 0, \qquad (6.7)$$

$$\gamma_x = \sqrt{1-\beta_y^2-\beta_z^2}, \quad \gamma_y = \sqrt{1-\beta_x^2-\beta_z^2}, \quad \gamma_z = \sqrt{1-\beta_x^2-\beta_y^2}, \qquad (6.8)$$

let's rewrite the force equation (6.2) in projections on an axis of coordinates:

$$F_x = F_p(1-\beta^2)x\int_{z+a}^{z-a} J_1\mathbf{d}\zeta, \qquad (6.9)$$

$$F_y = F_p(1-\beta^2)x\int_{z+a}^{z-a} J_2\mathbf{d}\zeta, \qquad (6.10)$$

$$F_z = F_p(1-\beta^2)x\int_{z+a}^{z-a} J_1\mathbf{d}\zeta, \qquad (6.11)$$

where

$$J_1 = \int_{y+b}^{y-b}\frac{\mathbf{d}\eta}{(a_1+b_1\eta+c_1\eta^2)^{3/2}}, \quad J_2 = \int_{y+b}^{y-b}\frac{\eta\,\mathbf{d}\eta}{(a_1+b_1\eta+c_1\eta^2)^{3/2}}. \qquad (6.12)$$

After integrating the first integral in (6.12) we get

$$J_1 = \frac{1}{\gamma_z(1-\beta_x^2)(1-\beta^2)}\left\{\left.\frac{\zeta M+N}{(a_2+b_2\zeta+\zeta^2)\sqrt{a_3+b_3\zeta+\zeta^2}}\right|^{\eta=y-b} - \left.\frac{\zeta M+N}{(a_2+b_2\zeta+\zeta^2)\sqrt{a_3+b_3\zeta+\zeta^2}}\right|^{\eta=y+b}\right\}, \qquad (6.13)$$

where

$$a_2 = \frac{x^2(1-\beta_z^2)}{1-\beta_x^2} > 0, \quad b_2 = \frac{2x\beta_x\beta_z}{1-\beta_x^2}, \qquad (6.14)$$

$$a_3 = \frac{x^2\gamma_x^2 + 2x\eta\beta_x\beta_y + \eta^2\gamma_y^2}{\gamma_z^2} > 0\,, \quad b_3 = \frac{2\beta_z(x\beta_x + \eta\beta_y)}{\gamma_z^2}\,, \tag{6.15}$$

$$N = \eta\gamma_y^2 + x\beta_x\beta_y\,, \qquad M = \beta_y\beta_z\,. \tag{6.16}$$

After integrating the indexes "g" at coordinates are omitted.

After substituting the integral (6.13) in expressions for F_x (6.9) and F_z (6.11), accordingly we have

$$F_x = \frac{F_p x}{\gamma_z(1-\beta_x^2)} J_3\Big|_{\eta=y+b}^{\eta=y-b}\,, \tag{6.17}$$

$$F_z = \frac{F_p}{\gamma_z(1-\beta_x^2)} \int_{z+a}^{z-a} \zeta \mathbf{d} J_3\Big|_{\eta=y+b}^{\eta=y-b}\,, \tag{6.18}$$

where

$$J_3 = \int_{z+a}^{z-a} \frac{(\zeta M + N)\mathbf{d}\zeta}{(a_2 + b_2\zeta + \zeta^2)\sqrt{a_3 + b_3\zeta + \zeta^2}^{\,2}}\,. \tag{6.19}$$

The parameters of integral (6.19) N, a_3 and b_3 depend on η, therefore in equations (6.17) and (6.18) the values of an integral and integrand expression $\mathbf{d}J_3$ with different values η are normalised.

The integrand numerator of integrals which are included in F_z, is possible to transform as follows:

$$(M\zeta + N)\zeta = M[(a^2 + b_2\zeta + \zeta^2) + M_2\zeta + N_2]\,,$$

where

$$N_2 = -a_2 = -x^2\frac{1-\beta_z^2}{1-\beta_x^2}\,, \tag{6.20}$$

$$M_2 = \frac{N}{M} - b_2 = \frac{\eta(1-\beta_x^2)\gamma_y^2 + x\beta_x\beta_y(\gamma_y^2 - \beta_x^2)}{\beta_y\beta_z(1-\beta_x^2)}. \tag{6.21}$$

Then the integrals in (6.19) become simpler and are reduced to integrals of a type J_3 as follows:

$$F_z = \frac{F_p\beta_y\beta_z}{\gamma_z(1-\beta_x^2)}\Big[\ln\gamma_z\left(R_v + x\beta_x\beta_z + \eta\beta_y\beta_z + \zeta\gamma_z^2\right) + J_3(M_2, N_2)\Big]\Big|_{\eta=y+b}^{\eta=y-b}\Big|_{\zeta=z+a}^{\zeta=z-a}\,, \tag{6.22}$$

where

$$R_v = \sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2} = \sqrt{R^2(1-\beta^2) + (\vec{\beta}\vec{R})^2}\,. \tag{6.23}$$

Here in an integral $J_3(M_2, N_2)$ the factors M and N in the equation (6.19) are substituted for parameters M_2 and N_2 accordingly.

As we see, the expressions (6.17) and (6.22) for forces F_x and F_z are determined by integral (6.19). This integral is not in manuals and is calculated:

$$J_3 = \frac{M(a_3 - a_2 - A) - N(b_3 - b_2)}{A\sqrt{2A - B}}\operatorname{arctg}\frac{Z^-}{\sqrt{2A-B}\sqrt{a_3 + b_3\zeta + \zeta^2}} -$$

$$-\frac{M(a_3 - a_2 + A) - N(b_3 - b_2)}{2A\sqrt{2A + B}}\ln\frac{\sqrt{2A+B}\sqrt{a_3 + b_3\xi + \xi^2} + Z^+}{\sqrt{2A+B}\sqrt{a_3 + b_3\xi + \xi^2} - Z^+}, \tag{6.24}$$

where

$$A = \sqrt{(a_3 - a_2)^2 - (a_3b_2 - a_2b_3)(b_3 - b_2)}\,, \tag{6.25}$$

$$B = 2(a_3 - a_2) - b_2(b_3 - b_2)\,, \tag{6.26}$$

$$Z^\pm = A \pm [(a_3 - a_2) + \zeta(b_3 - b_2)]\,. \tag{6.27}$$

After substituting the integral in equation (6.17) and (6.22) and making great transformations we get the expression for interacting forces of a charged plane on a moving particle as follows:

$$F_x = F_p\operatorname{arctg}\frac{x^2\beta_y\beta_z - x\eta\beta_x\beta_z - \zeta(1-\beta_x^2) - x\zeta\beta_x\beta_y}{xR_v}\Big|_{\eta=y+b}^{\eta=y-b}\Big|_{\zeta=z-a}^{\zeta=z+a}\,, \tag{6.28}$$

$$F_z = \frac{F_p}{1-\beta_x^2}\Big[\,\gamma_y\,ln\Big(\gamma_y R_v + x\beta_x\beta_y + \eta\gamma_y^2 + \zeta\beta_y\beta_z\Big) -$$

$$-\frac{\beta_y\beta_z}{\gamma_z}ln\Big(\gamma_z R_v + x\beta_x\beta_z + \eta\beta_y\beta_z + \zeta\gamma_z^2\Big)\Big]\Big|_{\eta=y+b}^{\eta=y-b}\Big|_{\zeta=z-a}^{\zeta=z+a} - \frac{\beta_x\beta_z}{1-\beta_x^2}F_x\,, \tag{6.29}$$

$$F_y = \frac{F_p}{1-\beta_x^2}\Big[\,\gamma_z\,ln\Big(\gamma_z R_v + x\beta_x\beta_z + \eta\beta_y\beta_z + \zeta\gamma_z^2\Big) -$$

$$-\frac{\beta_y\beta_z}{\gamma_y}ln\Big(\gamma_y R_v + x\beta_x\beta_y + \eta\gamma_y^2 + \zeta\beta_y\beta_z\Big)\Big]\Big|_{\eta=y+b}^{\eta=y-b}\Big|_{\zeta=z-a}^{\zeta=z+a} - \frac{\beta_x\beta_y}{1-\beta_x^2}F_x\,, \tag{6.30}$$

where

$$R_v = \sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2} = \sqrt{\gamma_x^2x^2 + \gamma_y^2\eta^2 + \gamma_z^2\zeta^2 + 2\beta_x\beta_y x\eta + 2\beta_x\beta_z x\zeta + 2\beta_y\beta_z\eta\zeta}\,; \tag{6.31}$$

x, y, z are particle coordinates. Here the expression for force F_y is recorded by analogy with F_z, as the y and z are identical for a plate (see Fig. 6.1).

When the movement velocity of a point body approaches to the value c_1, i.e. $\beta \to 1$, the expressions (6.28) - (6.30), and (4.58) tend towards zero. It means that when a moving charged body reaches the velocity equal to velocity of electrical action propagation, the charged plate terminates the influence on a body, and it will move without acceleration. The given conclusion refers to the charged bodies of any form, as the expression for interaction force from them can be obtained by summation of expressions (4.58) on all elements of these bodies.

The expressions (6.28) - (6.30) describe the force interaction value of one plate on a moving body. If there are any plates, it is possible to write the expression for force from each plate and to integrate them. In case of two parallel identical plates (Fig. 6.2), located at distance $2d$ from each other, the obtained expression will describe the interaction force from the flat capacitor. The plate centre with a charge $(-q_2)$ is on the axis x, where $x = -d$. Relatively this plate the distance to a particle q_1 along the axis x will be $(x + d)$. Similarly, the plate with a charge q_2 is remote from a particle at the distance $(x - d)$. Therefore, using expressions for forces

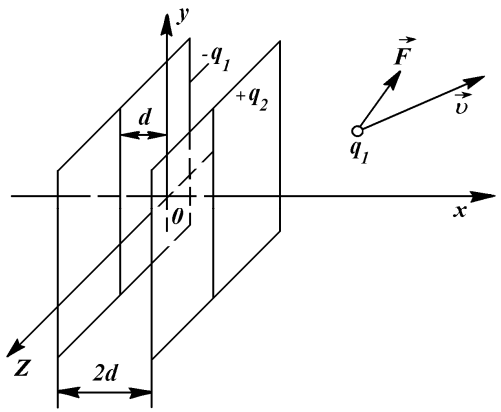


Fig. 6.2. The interaction of the flat rectangular capacitor on a moving charged particle q_1.

(6.28) - (6.30), but substituting the coordinate x to corresponding distances, in them with allowance for a sign of a plate charge we get the expression for interaction force of the flat capacitor on a moving particle

$$\vec{F}_k = \vec{F}(x-d) - \vec{F}(x+d), \quad (6.32)$$

where the components of vectors in the right part are determined (4.17) - (4.19) when x is equal to $(x - d)$ and $(x + d)$, accordingly.

If to direct the sizes of the capacitor a and b to infinity with $\beta < 1$, then the expression for force will be saved for a motionless particle with a charge q_1 only inside the capacitor, which has a kind

$$F_x = -\frac{4\pi\sigma}{\varepsilon} q_1, \quad (6.33)$$

where $\sigma = q_2/S$ is the density of a charge on plates of the capacitor. This limit corresponds to the capacitor, at which the distance between plates is significantly less than their sizes. As follows from the equation, the force is directed perpendicularly to plates ($F_y = F_z = 0$). The expression (6.33) is widely applied to electrostatics. They can be used at small velocity of charged particles moving between

the capacitor plates comparing with their sizes. But if the indicated conditions are not saved, the interaction of the capacitor on a particle is necessary to determine according to (6.32).

6.2. PARTICLE MOVEMENT VELOCITY IN THE CAPACITOR

Let's consider the movement of a charged particle in that specific case, when it moves along an axis x perpendicularly to a plate and along its centre, i.e. $y = z = \beta_y = \beta_z = 0$. In this case the lateral forces will not act on a particle and its movement will be linear. With mass m and charge q_1 the acceleration of a particle, according to expressions (6.28) - (6.30) and the second Newton's law (2.4), will be following:

$$w = w_p \operatorname{arctg} \frac{(1-\beta^2)ab}{x\sqrt{x^2 + (1-\beta^2)(a^2+b^2)}}, \quad (6.34)$$

where $w_p = 4\dfrac{q_1 q_2}{\varepsilon m S}$.

Let us consider the obtained differential equation in two approximations. In the first case we take a small velocity υ of a particle movement, i.e. $\beta = \upsilon/c_1 \to 0$. As the acceleration $w = \dfrac{\mathbf{d}\upsilon}{\mathbf{d}t} = \dfrac{1}{2}\dfrac{\mathbf{d}\upsilon^2}{\mathbf{d}x}$, (6.34) in a boundary condition $\upsilon(x_0) = \upsilon_0$ can be written as an integral

$$\upsilon^2 - \upsilon_0^2 = 2w_p \int \operatorname{arctg} \frac{ab}{x\sqrt{x^2 + a^2 + b^2}} \mathbf{d}x. \quad (6.35)$$

As the result of integrating by parts the equation (6.35) is reduced to the expression

$$\upsilon^2 = \upsilon_0^2 + 2w_p \left\{ x \operatorname{arctg} \frac{ab}{x\sqrt{x^2 + a^2 + b^2}} + ab \int \frac{(2z^2 - (a^2+b^2))\mathbf{d}x}{z^4 - (a^2+b^2)z^2 + a^2 b^2} \right\} \Bigg|_{x_0}^{x}$$

where $z = \sqrt{x^2 + a^2 + b^2}$.

As the denominator of an integral is decomposed on multiplicands

$$z^4 - (a^2+b^2)z^2 + a^2b^2 = (z-b)\,(z+b)\,(z-a)\,(z+a),$$

then after the integration we will record the expression for a particle velocity moving at the interaction of a charged plane (see Fig. 6.1) along x:

$$v^2 = v_0^2 + 2w_p\left(x\text{arctg}\,\frac{ab}{xR_s} + \frac{b}{2}\ln\frac{R_s - a}{R_s + a} + \frac{a}{2}\ln\frac{R_s - b}{R_s + b}\right)\Bigg|_{x_0}^{x}, \qquad (6.36)$$

where $R_s = \sqrt{x^2 + (a^2 + b^2)}$, v_0 is velocity of a particle at a point $x = x_0$.

The expression (6.36) describes a particle movement of, where action on it by the plate will be expressed by the electrostatic forces independent of velocity. This approach in case of two point bodies corresponds to Coulomb's law.

In the second case we will take a particle with velocity close to the size c_1. Let's present a symbol $u = \frac{(1-\beta^2)}{x^2}$, and then the acceleration will be written

$$w = 0{,}5c_1^2\frac{\mathbf{d}\beta^2}{\mathbf{d}x} = -0{,}5c_1^2\left(x^2\frac{\mathbf{d}u}{\mathbf{d}x} + 2xu\right).$$

In these symbols the equation (6.34) will be

$$x^2\frac{\mathbf{d}u}{\mathbf{d}x} + 2xu + 2w_p c_1^2\text{arctg}\,\frac{uab\,\text{sign}(x)}{\sqrt{1+u(a^2+b^2)}} = 0.$$

When $\beta \to 1$ variables $u \to 0$, therefore, decomposing arctg in a series of Tailor and limiting by terms of the first order, we obtain

$$x^2\frac{\mathbf{d}u}{\mathbf{d}x} + 2xu + \nu abu = 0, \qquad (6.37)$$

where

$$\nu = \frac{8q_1q_2\,\text{siqn}(x)}{\varepsilon c_1^2 mS}, \qquad (6.38)$$

where sign(x) = x/| x | is sign x.

The differential equation (6.37) is the equation with dividing variables. After its integration at the boundary condition $\beta(x_0)=\beta_0$ we discover

$$\beta^2 = 1 - (1-\beta_0^2)\exp\left[\nu ab\left(\frac{1}{x} - \frac{1}{x_0}\right)\right]. \qquad (6.39)$$

Here the integration is conducted with one sign x in the whole range of its change. Changing the sign x the equation (6.39) should be applied separately on the negative and positive segments x and connected on their boundary. Hereinafter we will introduce a symbol

$$\frac{\text{sign}(x)}{x} = \frac{1}{\sqrt{x^2}}. \qquad (6.40)$$

After the substitution in (6.39) the corresponding symbols of a particle movement velocity along the axis of a plate will be written as:

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\left[\frac{2q_1q_2}{\varepsilon mc_1^2}\left\{\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x_0{}^2}}\right\}\right]. \qquad (6.41)$$

For the capacitor, where the distance between plates is equal to $2d$ and the centre of coordinate system is located in the centre of the capacitor (see Fig. 6.2), the force of action on a particle will be defined according to (6.32) and (6.34). The equation of movement in that approach will be reduced to a differential equation similar to (6.37). As the result of its solution the approaching velocity of a particle movement for $\beta \to 1$ will be defined by the following expression:

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\left[\frac{2q_1q_2}{\varepsilon mc_1^2}\left(\frac{1}{\sqrt{(x-d)^2}} - \frac{1}{\sqrt{(x+d)^2}} + \frac{1}{\sqrt{(x_0+d)^2}} - \frac{1}{\sqrt{(x_0-d)^2}}\right)\right]. \qquad (6.42)$$

It is possible to use equation (6.42), as well as (6.41), with constant signs (x-d) and (x+d). The main difference of expressions (6.41) and (6.42) from expression (6.36) is a dependence relation of increments of a particle velocity square upon its initial velocity. We will return to this property, when we consider the energy of a particle movement.

The other difference of these expressions is that from equations (6.41) and (6.42) we come to the conclusion that the particle cannot reach the velocity, which is greater c_1. And its movement does not depend on a ratio of a plate sizes or the capacitor, and the equation (6.36) does not superimpose any limitations on size of a particle velocity: it can be as large as the factor w_p.

And the last difference is that in expression (6.36) the square of incremental velocity depends on the size of a charge and mass in a linear ratio, while in expressions (6.41) and (6.42) the given relation is submitted by more complex function.

All indicated differences are stipulated by the relation of force (acceleration) to a velocity and are unusual to classical mechanics. Because of them the application it of energetic methods is impossible. So, the increment of a body square velocity in mechanics is the increment of energy of its mass unit. However from expressions (6.41) and (6.42) it is clear, during the electrical interaction the increment of velocity square depends upon the mass of the whole body, for example (6.41)

$$\Delta v^2 = v^2 - v_0^2 = c_1^2(1-\beta_0^2)\exp\left[\frac{2q_1q_2}{\varepsilon mc_1^2}\left\{\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x_0{}^2}}\right\}\right]. \qquad (6.43)$$

6.3. INTERACTION FORCE OF THE CHARGED LINE SEGMENT AND VELOCITY OF PARTICLE

This problem can arise at consideration interaction of a charged line segment or charged wire with a particle, when the distance between them is more significant than the diameter of a wire. In this case, the wire can be described as the section of a straight line (Fig. 6.3) that is located along the axis *z*. If the length of the segment is 2*a*, and the centre of a coordinate system *xyz* is located in the middle of the section, then by substituting $\beta \to 0$ in expressions (6.28) - (6.30), describing the action from the plate with width 2*b* and length 2*a*, we will receive the expressions for forces from the section of a straight line.

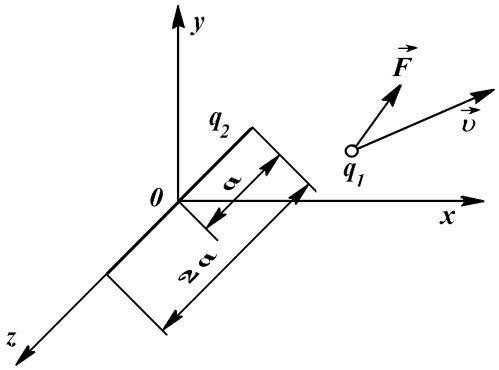


Fig. 6.3. Interaction of the charged section on the moved charged particle q_1.

After transformation of expressions the force on a moved particle with a charge q_1 will be written in a vector view:

$$\vec{F} = \frac{Qq_1}{\varepsilon R_v} \frac{\vec{R}(x\beta_x\beta_z + y\beta_y\beta_z + \gamma_z^2\zeta) - \vec{k}R_v^2}{(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2}\Bigg|_{\zeta=z-a}^{\zeta=z+a}, \qquad (6.44)$$

where

$$R_v = \sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2} = \sqrt{\gamma_x^2x^2 + \gamma_y^2y^2 + \gamma_z^2\zeta^2 + 2\beta_x\beta_y xy + 2\beta_x\beta_z x\zeta + 2\beta_y\beta_z y\zeta};$$

$$\vec{R} = \vec{i}\,x + \vec{j}\,y + \vec{k}\,\zeta;$$

$Q = q_2/2a$ is a linear density of a charge of the segment, and q_2 is its charge.

From expression (6.44) it is seen, that the size of the segment *a* and the arrangement of a particle in relation to the segment can considerably change a direction of force on a particle at the same distance between them. When the segment has a large length this influence decreases. Let's consider force (6.44) in a limit when $a \to \infty$, and the velocity of a particle is $\beta < 1$. In this case, when length of a charged wire is considerably exceeding a distance to a particle, the expressions for projections of the force to a particle look like:

$$F_x = \frac{2Qq_1\gamma_z}{\varepsilon} \frac{x}{(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2}, \qquad (6.45)$$

$$F_y = \frac{2Qq_1\gamma_z}{\varepsilon} \frac{y}{(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2}, \qquad (6.46)$$

$$F_z = \frac{2Qq_1\beta_z}{\varepsilon} \frac{-x\beta_x - y\beta_y}{(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2}. \qquad (6.47)$$

In formulas (6.45) - (6.47), as well as in the expression of force (6.33) in the flat capacitor with indefinitely large plates, limiting transition on the sizes is carried out at $\beta \neq 1$, therefore they can be applied at small velocities of a particle movement, i.e. $\beta < 1$. In that specific case of radial movement ($y = z = \beta_y = \beta_z = 0$), according to (6.45) - (6.47), the force along an axis *x* acts only:

$$F_x = \frac{2Qq_1\sqrt{1-\beta_x^2}}{\varepsilon x}. \qquad (6.48)$$

In this case when the particle approaches the velocity c_1 the force tends to zero as $\sqrt{1-\beta^2}$, i.e. it is weaker, than during the interaction of two dot particles. However it can be stipulated by the error of a passage to the limit at $a \to \infty$.

With small particle velocities $\beta << 1$ the projections of the force size from a long charged wire on a particle, moving with small velocity, according to expressions (6.45) - (6.47), will be written as

$$F_x = \frac{2Qq_1}{\varepsilon} \frac{x}{x^2 + y^2}; \qquad (6.49)$$

$$F_y = \frac{2Qq_1}{\varepsilon} \frac{y}{x^2 + y^2}. \qquad (6.50)$$

The above-mentioned expressions, as it is known, are used in electrostatics.

Now we will consider the movement of a charged particle along an axis *x*, which is located to a perpendicularly charged section and begins in its middle (see Fig. 6.3). In conditions $y = z = \beta_y = \beta_z = 0$ and according to (6.44) the size of force on a particle will be

$$F_x = \frac{2Qq_1a}{\varepsilon}(1-\beta_x^2) \Big/ \left(x\sqrt{x^2 + (1-\beta_x^2)a^2}\right) \qquad (6.51)$$

The precisions expression (6.51) differs from approximate (6.48), obtained for an infinite section.

If *m* - mass of a particle, according to the second Newton's law (2.4) its acceleration is

$$w = q_1 q_2 (1-\beta_x^2) \Big/ \left(\varepsilon m x \sqrt{x^2 + (1-\beta_x^2) a^2} \right) \qquad (6.52)$$

To define the velocity we will decide approximately this differential equation. Let's consider the case of a particle velocity close to the size c_1, i.e. $\beta_x \sim 1$. Neglecting $(1-\beta_x^2)$ in a denominator (6.52), we will rewrite it in this way

$$\frac{\mathbf{d}v^2}{\mathbf{d}x} = \frac{2q_1 q_2 \, sign(x)}{\varepsilon m c_1^2} \frac{c_1^2 - v^2}{x^2}.$$

The obtained differential equation with apportioned variables at limiting condition $v(x_0) = v_0$ gives the expression for velocity of a particle located under interaction of the segment, as

$$v^2 = c_1^2 - (c_1^2 - v_0^2) \exp \frac{2q_1 q_2}{\varepsilon m c_1^2} \left(\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x_0^2}} \right). \qquad (6.53)$$

As we can see, the last expression completely coincides with (6.41) for a plane. Those conclusions, which follow from expression (6.41), are also true for the segment.

We have calculated the force of action on a moving charged particle, and also its velocity of movement, if the other particle: plate or wire act on it. Using the offered method it is possible to define the influence from charged bodies of any form, and also from the combination of such bodies. The obtained expressions at small velocities of a particle coincide with known in Physics. On the other hand, from the obtained expressions at large velocities it follows, that as the result of an acceleration of a charged particle by any bodies it cannot exceed velocities c_1.

6.4. FORCE OF MOVING CHARGED PARTICLE ACTION ON MAGNET

The motionless charged body does not interact with magnet or with a conductor, along which the current flows. But during its movement such interaction is observed. We will calculate its force in some single cases.

The interaction of a dot charged body on magnet is determined by d'Alembert's differential equation (3.34). In projections round coordinates axes it will be written so:

$$H_x = -\frac{4\pi}{c} \left(\frac{\partial \rho}{\partial y} v_z - \frac{\partial \rho}{\partial z} v_y \right);$$

$$H_y = -\frac{4\pi}{c} \left(\frac{\partial \rho}{\partial z} v_x - \frac{\partial \rho}{\partial x} v_z \right); \qquad (6.54)$$

$$H_z = -\frac{4\pi}{c} \left(\frac{\partial \rho}{\partial x} v_y - \frac{\partial \rho}{\partial y} v_x \right).$$

Let's compare the right members of d'Alembert's equations (6.54) and (4.6) for E. If we multiply the right members of equations (4.6) by the components of velocity and we make differences of a type

$$\Box E_y v_z - \Box E_z v_y = \frac{4\pi}{\varepsilon} \left(\frac{\partial \rho}{\partial y} v_z - \frac{\partial \rho}{\partial z} v_y \right) = -c_1 \sqrt{\frac{\mu}{\varepsilon}} \Box H_x,$$

then we will receive

$$\Box \left[H_x + \sqrt{\frac{\varepsilon}{\mu}} (\beta_z E_y - \beta_y E_z) \right] = 0.$$

As this expression is fair at any values of variables,

$$H_x = -\sqrt{\frac{\varepsilon}{\mu}} (\beta_z E_y - \beta_y E_z). \qquad (6.55)$$

We shall receive the similar expressions for remaining components $\vec{H}$, therefore in a vector view we can record

$$\vec{H} = \sqrt{\frac{\varepsilon}{\mu}} [\vec{\beta} \times \vec{E}]. \qquad (6.56)$$

So, the solutions of equations (4.6) and (3.34) or (6.54) are connected with expression (6.56). From here, substituting in (6.56) solutions of an equation (4.6) for electrical interaction of a moving charge (4.55), we discover the force of action on a unit magnetic pole

$$\vec{H} = \frac{q_1 (1-\beta^2) [\vec{\beta} \times \vec{R}]}{\sqrt{\mu\varepsilon} \left\{ R^2 - [\vec{\beta} \times \vec{R}]^2 \right\}^{3/2}}, \qquad (6.57)$$

where $\vec{R}$ is a position vector from a moving body with a charge q_1 up to a unit magnetic pole, and $\vec{\beta} = \frac{\vec{v}}{c_1}$ is a normalised velocity relatively this pole. As follows from expression (6.57), the charged body does not influence on magnet at rest ($\beta = 0$) and with movement velocities, equal to c_1.

Knowing magnetic intensity $\vec{H}$ and using a expression $\vec{F} = M\vec{H}$, it is easy to define the force on any small magnetic pole with a magnetic charge M. If the magnetic pole cannot be accepted for dot, it can be divided into elementary segments with a magnetic charge ΔM and, by integrating the force on all the segments, we can calculate it onto the whole magnetized body.

6.5. FORCE OF ACTION OF POINT MAGNET ON A CHARGED PARTICLE

The force of interaction of a point pole *M* on a particle is opposite to the a direction of interaction force of this particle on a point pole and, according to (6.57), in case of $\vec{R}_{Mq} = -\vec{R}$ looks like

$$\vec{F}_q = \frac{Mq_1(1-\beta^2)[\vec{\beta}\times\vec{R}_{Mq}]}{\sqrt{\mu\varepsilon}\left\{R_{Mq}{}^2 - \left[\vec{\beta}\times\vec{R}_{Mq}\right]^2\right\}^{3/2}}, \qquad (6.58)$$

where $\vec{R}_{Mq}$ is a position vector from magnet *M* up to a particle q_1.

But if in a particle q_1 place there was other point pole M_1, the force of interaction of a pole *M* on it would be expressed as Coulomb's law:

$$\vec{F}_M = \frac{M\cdot M_1\vec{R}_{Mq}}{R_{Mq}{}^3} = \vec{H}M_1, \qquad (6.59)$$

where

$$\vec{H} = \frac{M\vec{R}_{Mq}}{R_{Mq}{}^3}; \qquad (6.60)$$

$\vec{H}$ is an interaction force of a point magnet *M* on a unit magnetic pole, located in a particle place. The value $\vec{H}$ will be called as magnetic intensity. Substituting $M\vec{R}_{Mq}$ from (6.60) in (6.58), we obtain the expression for interaction force of a point magnetic pole on a moved particle q_1:

$$\vec{F}_q = \frac{q_1(1-\beta^2)R_{Mq}^3[\vec{\beta}\times\vec{H}]}{\sqrt{\mu\varepsilon}\left\{R_{Mq}^2 - [\vec{\beta}\times\vec{R}_{Mq}]^2\right\}^{3/2}}. \qquad (6.61)$$

6.6. FORCE OF ACTION OF CONDUCTOR WITH CURRENT ON A CHARGED PARTICLE

As everybody know, the conductor with current acts on a magnet, for example, on the magnetized hand. We have already used expression (3.3), which expresses the interaction force of a conductor segment Δl with current *I* on a unit magnetic pole. The formula (3.3) characterizes magnetic intensity. If instead of magnet in the same point in relation to a conductor Δl there is a moving charged particle q_1, the force of action on it, stipulated by intensity *H*, is determined by

expression (6.61). Excluding *H* from these two expressions, we obtain the interaction force of a segment Δl conductor with current *I* on charged particle q_1 as:

$$\Delta\vec{F}_q = \frac{\mu I q_1(1-\beta^2)[\vec{v}\times[\Delta\vec{l}\times\vec{R}]]}{c^2\{R^2 - [\vec{\beta}\times\vec{R}]^2\}^{3/2}} \qquad (6.62)$$

Here $\vec{v} = \vec{\beta}\cdot c_1$ is a velocity of a particle relatively a conductor, and $\vec{R}$ is a position vector from a conductor $\Delta\vec{l}$ up to a particle. The expression (6.62) allows defining the interaction force of a direct conductor with current on a moved particle. With this purpose we will consider a conductor with length 2*a*, in the middle of which a centre of coordinates is selected, and the axis *z* is directed along a conductor in a direction of current *I*. If coordinates of a particle are *x, y, z*, the distance from the element $\Delta\vec{l}$ up to a particle q_1 will be

$$\vec{R} = \vec{i}x + \vec{j}y + \vec{k}\xi, \qquad (6.63)$$

where $\zeta = z - l$ is a length *l* which is counted from zero in a direction of an axis *z*. The double vector product in (6.62) can be written so:

$$\left[\vec{\beta}\times\left[\mathbf{d}\vec{l}\times\vec{R}\right]\right] = \mathbf{d}\vec{l}\,(\vec{\beta}\vec{R}) - \vec{R}(\vec{\beta}\mathbf{d}\vec{l}) = \mathbf{d}l\left[\vec{k}(x\beta_x + y\beta_y) - \beta_z(\vec{i}x + \vec{j}y)\right].$$

By integrating force (6.62) on all segments of a conductor, we can record

$$\vec{F} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{Iq_1(1-\beta^2)\left|\vec{k}(x\beta_x + y\beta_y) - \beta_z(\vec{i}x + \vec{j}y)\right|}{c}\int_{-a}^{a}\frac{\mathbf{d}l}{(a + b\zeta + \zeta^2)^{3\backslash 2}}, \qquad (6.64)$$

where

$$a = \gamma_x^2x^2 + \gamma_y^2y^2 + 2\beta_x\beta_y xy, \quad b = 2(\beta_x\beta_y x + \beta_y\beta_z y), \quad c = \gamma_z^2. \qquad (6.65)$$

As $\mathbf{d}l = -\mathbf{d}\zeta$ (, and the integral, according to [24], easily it is authorized, the force (6.64) will be

$$\vec{F} = -\sqrt{\frac{m}{\varepsilon}}\,\frac{Iq_1\left[\vec{k}(x\beta_x + y\beta_y) - \beta_z(\vec{i}x + \vec{j}y)\right](\beta_x\beta_z + y\beta_y\beta_z + \gamma_z^2\zeta)}{c[(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2]R_v}\Bigg|_{\zeta=z+a}^{\zeta=z-a}, \qquad (6.66)$$

where $R_v = \sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2}$, $\quad R = \vec{i}x + \vec{j}y + \vec{k}\zeta$.

After introduction of a current vector $\vec{I} = I\vec{k}$ and fulfilment of return vectorial transformations, the force of interaction of direct current on a moved charged particle is obtained as follows:

$$\vec{F} = \frac{\mu q_1}{c^2 R_\upsilon} \frac{(x\beta_x\beta_z + y\beta_y\beta_z + \gamma_z^2\zeta)[\vec{\upsilon}\times[\vec{I}\times\vec{R}]]}{(1-\beta_y^2)x^2 + 2\beta_x\beta_y xy + (1-\beta_x^2)y^2}\Bigg|_{\zeta=z-a}^{\zeta=z+a}. \qquad (6.67)$$

From this expression it follows, that the force does not act on a particle, when the particle rests ($\beta = 0$) and when it moves with velocity c_1, i.e. $\beta = 1$.

If the particle q_1 moves perpendicular to a conductor ($y = \beta_y = \beta_z = 0$) according to (6.66), the transversal force will act on it

$$F_z = \frac{\mu I q_1 \upsilon_x (1-\beta_x^2)}{c^2 x}\left(\frac{z+a}{\sqrt{x^2+(1-\beta_x^2)(z+a)^2}} - \frac{z-a}{\sqrt{x^2+(1-\beta_x^2)(z-a)^2}}\right). \qquad (6.68)$$

When the particle is in a mean plane ($z = 0$) the force of action on it, according to (6.68), is directed to the same side, as current, i.e. as though a positively charged particle is move by current.

If the velocity of a particle concerns a circle, in which centre and perpendicular to its plane the current is located, the double vector product in (6.67) is equal to zero, i.e. the current does not act on a particle, rotated round a conductor.

When the particle moves parallel to conductor ($y = \beta_y = \beta_x = 0$) forces is directed on it perpendicularly to a conductor and has a kind

$$F_x = -\frac{\mu I q_1 \upsilon_z}{c^2 x}\left(\frac{z+a}{\sqrt{x^2(1-\beta_z^2)+(z+a)^2}} - \frac{z-a}{\sqrt{x^2(1-\beta_z^2)+(z-a)^2}}\right) \qquad (6.69)$$

In a mean plane ($z = 0$) forces (6.69) will be written so:

$$F_x = -\frac{2\mu I q_1 a \upsilon_z}{c^2 x\sqrt{(1-\beta_z^2)x^2 + a^2}}, \qquad (6.70)$$

i.e. positive charged particle, which moves in a direction of current, is attracted to a conductor. Approaching to velocity c_1 (in case of $\beta_z \to 1$) the force of attraction grows. Such an action is also observed during the interaction of two particles, when the relative velocity of a particle $\bar{\upsilon}$ is perpendicular to a distance $\vec{R}$ between them (see Formula (4.58)).

If the conductor is long ($a >> x$) and the particle moves at distance $x = d$ from it, the expression for force, according to (6.70), will be

$$F_x = -\frac{2\mu I q_1 \upsilon_z}{c^2 d}. \qquad (6.71)$$

This equation reminds the expression for the interaction force of the conductor with current I on the other conductor with current I_1, located parallel to the first one:

$$F_x = -\frac{2 I I_1 \mu l}{c^2 d}, \qquad (6.72)$$

where l is a length of a conductor with current I_1, on which force F_x. acts. In expressions (6.71) and (6.72) the forces coincide at the direction, and also in size, if $q_1\upsilon_z = I_1 l$.

The interaction of a conductor with current on a particle differs from the interaction of a charged conductor. So, when the particle moves parallel to the charged conductor ($y = \beta_y = \beta_x = 0$), according to (6.44) the conductor acts on it by a transversal force

$$F_x = \frac{Q q_1}{\varepsilon x}\left(\frac{z+a}{\sqrt{x^2(1-\beta_z^2)+(z+a)^2}} - \frac{z-a}{\sqrt{x^2(1-\beta_z^2)+(z-a)^2}}\right) \qquad (6.73)$$

and a longitudinal one

$$F_z = -\frac{Q q_1}{\varepsilon}\left(\frac{1-\beta_z^2}{\sqrt{x^2(1-\beta_z^2)+(z+a)^2}} - \frac{1-\beta_z^2}{\sqrt{x^2(1-\beta_z^2)+(z-a)^2}}\right). \qquad (6.74)$$

In a mean plane ($z = 0$) when the length of a conductor is greater ($a \to \infty$) the longitudinal force disappears, while it exists for current (see (6.68)). The expression for transversal force when $x = d$ and at small velocities

$$F_x = \frac{2 Q q_1}{\varepsilon x} \qquad (6.75)$$

is similar to expression (6.71) for transversal interaction force of a conductor with current, but there is an essential difference: the charged conductor repels of the same name charged particle.

The solenoidal magnets in which the spool is derived by many layers of a ring-type coil conductor, find the greatest application. Let us investigate the interaction force of a ring-type coil conductor with current I on a moving particle. In view of mathematical complexity of this problem, we will consider a particular case, when the particle is located in the centre of a ring. If the axis z is directed perpendicularly to a plane of a ring, the centre of a coordinate system is located in centre of a ring so that the current is directed from an axis x to an axis y, the integration of expression (6.62) will lead to the value of force action on a particle as

$$\vec{F} = \frac{\mu I q_1}{c^2 R} \frac{[\vec{\upsilon} \times \vec{k}]}{\sqrt{1-\beta_z^2}} \int_0^{2\pi} \sqrt{1 - n^2 \sin^2 \gamma}\, \mathbf{d}\gamma, \qquad (6.76)$$

where $\vec{k}$ is a unit vector of an axis *z*; *R* is a radius of a ring,

$$n^2 = \frac{\beta_x^2 + \beta_y^2}{1-\beta_z^2}.$$

As it is seen from expression (6.76), the force is directed perpendicularly to velocity of a particle in the centre of a ring-type coil conductor. The expression (6.62) can be rewritten as following

$$\vec{F} = 2p \frac{\mu I q_1}{c^2 R} \frac{[\vec{\upsilon} \times \vec{k}]}{\sqrt{1-\beta_z^2}}, \qquad (6.77)$$

where *p* varies in limits from $p = \pi$ at $n^2 = 0$ up to $p = 2$ at $n^2 = 1$, i.e. the factor p varies unsignificantly in the whole interval of a change β from 0 up to 1 (just so a change *n* corresponds to a change β).

For small velocities ($\beta << 1$; $1 - \beta_z^2 \approx 1$; $p = \pi$) the expression (6.77) gives the same result, which is used in Physics and Engineering to define the interaction force of an orbit with current on a moved particle. Really, as for a circular orbit magnetic intensity is

$$\vec{H} = \frac{2\pi I \vec{k}}{cR}, \qquad (6.78)$$

and Lorentz's force

$$\vec{F} = \frac{\mu q_1 [\vec{\upsilon} \times \vec{H}]}{c} \qquad (6.79)$$

determines a magnetic action on a particle, then excluding $\vec{H}$, we obtain the interaction force of a ring-type coil conductor on a particle

$$\vec{F} = 2\pi \frac{\mu I q_1}{c^2 R} [\vec{\upsilon} \times \vec{k}]. \qquad (6.80)$$

This expression is fair at small velocity of particles. At subluminal velocities instead of a factor 2π it is necessary, according to (6.77) to use a factor $2p = 4$, and also the relation to an axial velocity β_z. For particles located not on an axis of ring-type coil current, it is necessary to integrate (6.76) numerical methods.

6.7. SIMPLIFIED FORCES DISREGARDING FORMS OF ACTING BODIES

Now in electrodynamics the method of calculation of actions on a moving particle, which does not depend on the form of an acting body, is applied. They consider values $\vec{E}$ and $\vec{H}$ in a point where there is a particle and depending on them determine force of action on a particle. It is considered that an electrical field $\vec{E}$ and magnetic $\vec{H}$ are created by the charged and magnetized bodies, and these fields act a moving particle. Therefore the same values $\vec{E}$ and $\vec{H}$, stipulated by bodies of different forms, act on a moving particle in the same way.

Let us research ratio, obtained by us, for example during the interaction of a point magnet. The force, according to (6.61), will be written

$$\vec{F} = k_\upsilon \frac{\mu q_1 (1-\beta^2)}{c} [\vec{\upsilon} \times \vec{H}], \qquad (6.81)$$

where

$$k_\upsilon = \frac{R^3}{R_\upsilon^3} = \frac{1}{\{1 - [\vec{\beta} \times \vec{R}/R]^2\}^{3/2}}. \qquad (6.82)$$

Here k_υ is determined particle directions of arrangement and movement relatively a point magnet.

In case of a ring-type coil conductor with current *I*, the interaction forces of action on a particle (6.77) with allowance for (6.78) will be

$$\vec{F} = k_{\upsilon 1} \frac{\mu q_1}{c\sqrt{1-\beta_z^2}} [\vec{\upsilon} \times \vec{H}], \qquad (6.83)$$

where $k_{\upsilon 1} = P/\pi$ is determined by a particle directions of arrangement and movement relatively an annular conductor with current. In spite of a particle being in a point, these bodies create identical magnetic intensity $\vec{H}$, the forces of their action on a moving particle (6.81) and (6.83) differ.

The similar situation is observed for the interaction of charged bodies on a moving particle. For example, the interaction force of a particle q_2 on a q_1 particle, moving relatively it, according to (4.55), can be recorded as

$$\vec{F} = k_\upsilon q_1 (1-\beta^2) \vec{E}, \qquad (6.84)$$

where

$$\vec{E} = \frac{q_2 \vec{R}}{\varepsilon R^3}. \qquad (6.85)$$

Here $\vec{E}$ is an electrical intensity, created by the particle q_2 in a place particle being q_1. In case of action to a particle of another form charged bodies for example,

charged of a thread, according to (6.44), the expression the force depending on intensity $\vec{E}$ will differ from (6.84) another value of a geometric factor k_υ and, probably, additional multiplicands.

The mentioned examples (6.81), (6.83), (6.84) show, that in case of interaction of any form body for precisions calculation of forces it is necessary to conduct the summation of all its elements actions under the law of interaction of two point bodies. In case of magnetic action the force, according to (6.81) and (6.82), will be written so:

$$\vec{F} = \mu \frac{q_1(1-\beta^2)}{c}\left[\vec{\upsilon} \times \int\limits_M \frac{R^3}{R_\upsilon^3}\mathbf{d}\vec{H}\right], \qquad (6.86)$$

where M shows the integration on the whole parameter size of action M. In case of interaction of a charged body q_2, according to (6.84) and (6.82), the force

$$\vec{F} = q_1(1-\beta^2)\int\limits_E \frac{R^3\mathbf{d}\vec{E}}{R_\upsilon^3}. \qquad (6.87)$$

The value $k_\upsilon = (R/R_\upsilon)^3$, included in integrals, at small velocity is equal to a unit. Parameter k_υ can much differ greatly from the unit at large velocity ($\beta \to 1$), if it is perpendicular to the line connecting the interacting particles. During the integration of action from all elements of a body such situation arises only for a limited number of elements $\mathbf{d}\vec{E}$ or $\mathbf{d}\vec{H}$. Therefore it will not influence greatly on the total interaction. Therefore, the integrals in (6.86) and (6.87) can be replaced by the approximate expressions

$$\int\limits_M k_\upsilon \mathbf{d}\vec{H} \approx \vec{H} \text{ and } \int\limits_E k_\upsilon \mathbf{d}\vec{E} \approx \vec{E}, \qquad (6.88)$$

where both magnetic $\vec{E}$ and electrical intensity $\vec{H}$ are created accordingly by a magnet and a charged body in a point of a moving particle being. Then the approximate expressions for magnetic and electrical actions on a moved particle without dependence from the form of acting bodies according to (6.86) and (6.87) will accept a form

$$\vec{F} \approx \frac{\mu q_1(1-\beta^2)}{c}[\vec{\upsilon} \times \vec{H}], \qquad (6.89)$$

$$\vec{F} \approx q_1(1-\beta^2)\vec{E}. \qquad (6.90)$$

These expressions can give large errors when $\beta \to 1$, if the velocity of a particle is perpendicular to greater part of points of acting body. In that case exact expressions (6.86) and (6.87) should be integrated. The expressions (6.89) and (6.90) differ from used in electrodynamics by availability of a multiplicand $(1 - \beta^2)$.

When a particle velocities approach to light speed ($\beta \to 1$), the obtained expressions show that the force of action on a particle tends to zero.

The expressions, deduced by us, for forces of action on a moving charged body are based on the results of two groups of measurements. The first measurements pertain to evaluation of force between the charged motionless bodies. Their generalization is Coulomb's law (3.1) for electrical charges.

The second group of measurements pertains to the evaluation of interaction between a conductor with current and magnet. Expressing their results as the interaction force of a conductor on magnet Biot-Savart - Laplace's ratio (3.3) follows. The experiments on rotation of the charged disk (Ahenvald's and others experiment) are used for the substantiation that, that the moving charged body during its action on a magnet is equivalent to a conductor with current. Taking it into account in expression (3.3), we come to the interaction force of a moving charged body on the magnet circumscribed by the second Maxwell's equation (3.22). If to express the results of the second group of measurements through the interaction force of a magnet on current, and current - through movement of a charged body, we will receive a ratio for interaction of the magnet on a charged body, moving relatively to it, as the 1-th Maxwell's equation (3.24) and as Faraday law of an induction (3.5). It is necessary to mark, that originally equation (3.5) was calculated by measurement of EMF, induced in a closed loop during a magnet movement.

Thus, the expressions for force of action on a moving particle are deduced from the experimental facts. There is a long way of one ratio into the other one transformations, a transition from some values to the other from the results of measurement to the final expression. It is possible, that during the intermediate operations any assumptions were made. In this connection we could name absolutely valid such relations for forces of action on a moving particle, which would be defined as the result of direct measurements of interaction forces of a magnet and a charged body on a particle or its measurements of an acceleration.

CHAPTER 7

APPLICATION OF THE METHOD OF FORCES FOR CALCULATION OF DIFFERENT INTERACTIONS

7.1. ENERGY OF INTERACTION, WHICH DEPENDS ON VELOCITY

A characteristic peculiarity of the charged and magnetized bodies action forces on a moving particle is their relation from a particle velocity. The calculated particle movement velocities, as it is seen from equations (4.80), (6.41), (6.42), (6.53), can be recorded so:

$$\upsilon^2 = c_1^2 - (c_1^2 - \upsilon_0^2)\exp\frac{2q_1q_2}{\varepsilon m_{re}c_1^2}\left(\frac{1}{\sqrt{r^2}} - \frac{1}{\sqrt{r_0^2}}\right), \qquad (7.1)$$

where m_{re} and r have a certain kind for a particular acting body. For example, during the action of a point body with a charge q_2 and a mass m_2

$$r = \sqrt{R^2 - \frac{h^2}{c_1^2}}, \qquad m_{re} = \frac{m_1 \cdot m_2}{m_1 + m_2}. \qquad (7.2)$$

The kinetic energy of a moving particle at the point r_0 is equal to $E_{c0} = m_1\upsilon_0^2/2$, and in a point r $E_c = m_1\upsilon^2/2$. According to (2.24), the work A of the action force in increment of a particle kinetic energy is equal to a loss of potential energy U at a point r in relation to a point r_0:

$$A = \Delta E_c = E_c - E_{co} = -U = -\frac{m_1c_1^2}{2}(1-\beta_0^2)\left[\exp\frac{2q_1q_2}{\varepsilon m_{re}c_1^2}\left(\frac{1}{\sqrt{r^2}} - \frac{1}{\sqrt{r_0^2}}\right) - 1\right]. \qquad (7.3)$$

It follows from here that the potential particle energy depends on its initial velocity. Thus, each point in a neighbourhood of an acting body r_0 has not ability to give a located here body the same energy, as it is accepted consider in physics. During the movement from this point the different energy will be given to a particle depending on velocity β_0 coming exactly to a point r_0 of a particle. Besides, the relation of energy to mass and value of a charge is unusual. The full energy of a particle, according (7.1) and (7.3), will be

$$E_f = E_c + U = \frac{m\upsilon_0^2}{2} = \text{const}, \qquad (7.4)$$

i.e. the full energy of a particle in relation to a point r_0 is equal to a kinetic energy at this point and remains constant during a particle movement.

For forces dependent only from a distance between interacting bodies, the potential energy according to expression (4.83) when $h = 0$, will be recorded so:

$$U = -A = -\Delta E_c = \frac{m_1q_1q_2}{\varepsilon m_{re}}\left(\frac{1}{R} - \frac{1}{R_0}\right), \qquad (7.5)$$

i.e. the potential energy is characterized only by a particle position. Taking into account the difference between a potential energy (7.3) of a moving charged body and potential energy of a body (7.5), caused by Coulomb's force, the energetic methods of mechanics for calculation of charged bodies interactions are necessary to conclude with caution. The omission of these peculiarities in the Theory of Relativity and some other sections of modern physics have led to the emergence of

contradictory conclusions, including such phenomena, as infinite mass and energy, imaginary energy and mass etc.

In modern physics the energetic method of interactions has practically completely superseded the method of forces. This consideration was greatly promoted by the notion about the energy of substance. In the Theory of Relativity the expression for energy of object with mass m and velocity υ is used

$$E = \frac{mc^2}{\sqrt{1-\beta^2}},$$

whence with $\beta = 0$ the energy of resting object follows

$$E_0 = mc^2. \qquad (7.6)$$

This expression has not physical explanation. Let's show that it follows from main rules positions of a classical mechanics.

It is considered that the relation (7.6) is experimentally confirmed during the research of nuclear transformations and processes of annihilation. It is supposed that the collision of an electron and positron, proton and antiproton conduces to radiation of photons with velocity c. If there was an antimatter, the contact lead with substance would lead it to in collision of particles and antiparticles and, therefore, to transformation of two such objects in a flow of photons moving with the velocity c. Therefore the internal energy $E_0 = mc^2$ of an object is possible to understand as its ability to radiate the substance with light speed.

Let the object with the initial mass m_0 reject the substance with a constant on velocity at the value and direction u of a relatively remaining part with mass m. If the consumption of the thrown off mass is $\mathbf{d}m/\mathbf{d}t$, the remaining part m ia acted by an implying jet with the jet force

$$R = u\frac{\mathbf{d}m}{\mathbf{d}t}. \qquad (7.7)$$

Then the equation of a remaining part of object movement m, according to the second Newton's law (2.4), will be recorded

$$m\frac{\mathbf{d}\upsilon}{\mathbf{d}t} = R. \qquad (7.8)$$

After a substitution of the jet force R the differential equation is the following:

$$m\mathbf{d}\upsilon = u\mathbf{d}m, \qquad (7.9)$$

known as Mesthersky's equation. Deciding (7.9) on entry conditions $m = m_0$ when $\upsilon = 0$, we will receive Ciolkovsky's formula

$$\upsilon = u\ln\frac{m_0}{m}, \qquad (7.10)$$

which determines the velocity of a decomposed part of an object.

Now we will consider what energy has the object m_0, if it completely breaks up with velocity u. During decay the force R will act on each element, which makes work

$$\mathbf{d}A = R\mathbf{d}s.$$

The energy of the object will be equal to the work along the whole path of the decay:

$$E_0 = A = \int R\mathbf{d}s = \int R\upsilon\mathbf{d}t .$$

After a substitution of jet force R (7.7), we obtained

$$E_0 = u\int_0^{m_0} \upsilon\mathbf{d}m .$$

Now with allowance for (7.10) we can record

$$E_0 = u^2\int_0^{m_0} ln\frac{m_0}{m}\mathbf{d}m = m_0u^2 .$$

If the decay is made with light speed $u = c$, the energy

$$E_0 = m_0c^2 . \qquad (7.11)$$

7.2. NEW INTEGRALS OF MOVEMENT

The expression for velocity (7.1) with allowance for (7.2) can be rewritten as

$$(c_1^2 - \upsilon_0^2)\exp\left(\frac{-2q_1q_2}{\varepsilon m_{re}c_1^2\sqrt{R_0^2 - (h/c_1)^2}}\right) = (c_1^2 - \upsilon^2)\exp\left(\frac{-2q_1q_2}{\varepsilon m_{re}c_1^2\sqrt{R^2 - (h/c_1)^2}}\right). \qquad (7.12)$$

In left and right parts of equality the expressions are identical, but relate to different points of a body trajectory, i.e. the numerical value of this expression is identical to all points of trajectory and is saved during the movement of a body, i.e. the parameter of movement

$$S = (c_1^2 - \upsilon^2)\exp\left(-\frac{2\mu_1}{c_1^2\sqrt{R^2 - (h/c_1)^2}}\right) = \text{const} . \qquad (7.13)$$

For Coulomb's forces, according to (4.83), it is also possible to record an identity

$$\upsilon_r^2 + \frac{2\mu_1}{R} + \frac{h^2}{R^2} = \upsilon_{r0}^2 + \frac{2\mu_1}{R_0} + \frac{h^2}{R_0^2} ,$$

which after introduction of full velocity $\upsilon^2 = \upsilon^2_r + \upsilon^2_t$ receives the following form:

$$\upsilon_r^2 + \frac{2\mu_1}{R} = \upsilon_0^2 + \frac{2\mu_1}{R_0} . \qquad (7.14)$$

As in the right and left parts (7.14) the expressions are identical, but relate different points of trajectory of a body, then similarly to (7.13) it is possible to enter the parameter of movement for Coulomb's forces

$$S_k = \upsilon^2 + \frac{2\mu_1}{R} = \text{const} . \qquad (7.15)$$

Let's consider the connection of integral parameter S_k with the conventional parameters of movement. According to (7.5), the potential energy of a particle m_1 in relation to an indefinitely remote point ($R_0 \to \infty$) will be

$$U_\infty = \frac{m_1\mu_1}{R} . \qquad (7.16)$$

Then the full energy of a particle will be recorded

$$E_{f\infty} = E_c + U_\infty = \frac{m_1\upsilon^2}{2} + \frac{m_1\mu_1}{R} = \frac{S_km_1}{2} .$$

From here we obtain

$$S_k = \frac{2E_{f\infty}}{m_1} , \qquad (7.17)$$

i.e. the integral of movement of a particle S_k is the double specific energy of a particle in relation to an indefinitely remote point. This value, as well as the full energy of a particle remains constant during movement. Forces of action dependent on velocity the parameter of movement S, defined by ratio (7.8) according to (4.58), cannot be expressed through the energy as (7.17). Nevertheless in a physical sense it possible to compare it with the parameter S_k.

In a number of cases the action on a charged body is possible to describe by the forces, which are not dependent on its velocity in a considered point [54]. With this purpose the action of a point body is researched. The acceleration, created by a charged particle is described by expression (4.61). Due to its solution the radial velocity of a particle (4.80) and transversal one $\upsilon_t = h/R$ depending on its distance up to an acting body and a velocity in an initial point was determined. Let's differentiate (4.80) on time and, taking into account, that $\mathbf{d}R/\mathbf{d}t = \upsilon_r$, we receive

$$\frac{\mathbf{d}\upsilon_r}{\mathbf{d}t} = \frac{\mathbf{d}^2R}{\mathbf{d}t^2} = \frac{h^2}{R^3} + \frac{\mu_1R(1-\beta_0^2)}{(R^2 - (h/c_1)^2)^{3/2}}\exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2 - (h/c_1)^2}} - \frac{1}{\sqrt{R_0^2 - (h/c_1)^2}}\right).$$

Then the radial acceleration in a polar coordinate system will be recorded

$$\vec{w}_r = \frac{\vec{R}}{R}\left(\frac{\mathbf{d}^2R}{\mathbf{d}t^2} - R\omega^2\right) = \frac{\mu_1(1-\beta_0^2)\vec{R}}{(R^2 - (h^2/c_1^2))^{3/2}}\exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2 - (h/c_1)^2}} - \frac{1}{\sqrt{R_0{}^2 - (h/c_1)^2}}\right), \qquad (7.18)$$

and the transversal one, when $\omega R^2 = h$, will be

$$w_t = \frac{1}{R}\frac{\mathbf{d}(\omega R^2)}{\mathbf{d}t} = 0 .$$

Therefore, the full acceleration of a particle is directed on a radius, which is going out the attracting centre, and is equal to $\vec{w} = \vec{w}_r$. Multiplying (7.18) by a particle mass m_1, we will receive a new expression for the force as

$$\vec{F} = \frac{\mu_1 m_1 (1-\beta_0^2)\vec{R}}{(R^2 - (h^2/c_1^2))^{3/2}} \exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2 - (h^2/c_1^2)}} - \frac{1}{\sqrt{R_0{}^2 - (h^2/c_1^2)}}\right). \quad (7.19)$$

The equation (7.19) represents a point body force of action with a charge q_2 on other point body with a charge q_1 and the mass m_1, which moves relatively the first one and at any initial distance R_0 from it has full velocity $\upsilon_0 = \beta_0 c_1$ and perpendicular to R_0 it has a transversal velocity equal to $\upsilon_t = h/R_0$.

Expression (7.19) includes only the initial velocity of a particle movement, which is constant. Therefore the given expression is possible to consider dependent only from a distance. As well as to any forces dependent on a distance, force (7.19) should correspond to a potential energy in the equation

$$\vec{F} = -\text{grad}\, U . \quad (7.20)$$

Indeed, the operation (-grad) conduces to a equation (7.19) for force from expression (7.3) for potential energy of a particle.

The force (7.19) facilitates a problem of movements calculation by the fact that the solution of difficult differential equations is substituted by the integration. However, the kinematic angular momentum of a moving particle $h = \upsilon_t R = \text{const}$, which concerns only the interaction of two point bodies, is included in expression (7.19). During the action of the other objects it is necessary to consider h to each element of such an object. As at the angle φ between υ and R the transversal velocity $\upsilon_t = \upsilon \cdot \sin(\varphi) = \upsilon\left|[\vec{v}\times\vec{R}]\right| / \upsilon R$, then

$$R^2 - (h^2/c_1^2) = R^2 - [\vec{\beta}\times\vec{R}]^2 . \quad (7.21)$$

With allowance for (7.21) expressions for force (7.19) become more universal:

$$\vec{F} = \frac{\mu_1 m_1 (1-\beta_0^2)\vec{R}}{\left\{R^2 - [\vec{\beta}\times\vec{R}]^2\right\}^{3/2}} \exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2}} - \frac{1}{\sqrt{R_0{}^2 - [\vec{\beta}\times\vec{R}_0]^2}}\right), \quad (7.22)$$

however it depends on a particle movement velocity.

7.3. METHOD OF VELOCITY SUMMATION

Let's consider a method of calculation of a particle movement velocity, when the force is not used. It allows defining the velocity of a body given to it by any

device with the similar character of action having known the expression for velocity, which one point body due to action gives to other. We will demonstrate this method on the example of two charged bodies interaction. According to expressions (4.14) and (7.2), the central body gives the other point body with mass m_1 and electrical charge q_1 the velocity, which is described by expression

$$\upsilon^2 = c_1^2 - (c_1^2 - \upsilon_0^2)\exp\frac{2\mu_1}{c_1^2}\left(\frac{1}{\sqrt{R^2 - (h^2/c_1^2)}} - \frac{1}{\sqrt{R_0{}^2 - (h^2/c_1^2)}}\right). \quad (7.23)$$

If to designate

$$\lambda = \frac{2\mu_1}{c_1^2}\frac{1}{\sqrt{R^2 - (h^2/c_1^2)}}, \quad (7.24)$$

that value of velocity will be expressed by the equation

$$\upsilon^2 = c_1^2 - (c_1^2 - \upsilon_0^2)\exp(\lambda - \lambda_0) . \quad (7.25)$$

In order the symbol λ has not depended on a kinematic angular momentum h, we use the replacement (7.21)

$$\lambda = \frac{2\mu_1}{c_1^2}\frac{1}{\sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2}} . \quad (7.26)$$

The expression (7.25) determines a particle movement velocity depending on the difference in the values of a function λ in final and initial points of trajectory. Apparently, if to calculate values λ, created by any body, then using (7.25) it is possible to find out what velocity it will give to a particle in case of movement from one point to other.

As an example, we will calculate the velocity of a charged particle, which is forced by the rectangular charged plate. It is possible to record symbol λ, according to expression (7.26), for an elementary part of a plate with a charge $\mathbf{d}q_2$ in the following way:

$$\mathbf{d}\lambda = \frac{2q_1}{\varepsilon c_1^2 m_1}\frac{\mathbf{d}q_2}{\sqrt{R^2 - [\vec{\beta}\times\vec{R}]^2}} , \quad (7.27)$$

where R is a distance from an element with a charge $\mathbf{d}q_2$ up to a particle with a charge q_1, which velocity we compute. The movement of a particle with mass m_1 relatively a plate with mass m_2, therefore $m_1 << m_2$ is considered here. We select the axes of coordinates in the same way, as in case of evaluation of expressions (6.28) - (6.30) for forces (see Fig. 6.1): a beginning of coordinates in centre of a plate, the axis x is perpendicular to it, and remaining parameters we will designate: $\eta = y - y_s$; $\zeta = z - z_s$; y_s, z_s are the coordinates of an element of a plate $\mathbf{d}q_2$; x, y, z are the coordinates of a particle q_1; $\vec{R} = \vec{i}\,x + \vec{j}\,\eta + \vec{k}\,\zeta$.

Charge of an element of a plate $\mathbf{d}q_2 = \sigma\,\mathbf{d}\eta\mathbf{d}\zeta$, where ($-\mathbf{d}\eta$) and ($-\mathbf{d}\zeta$) are the elements of the area in a direction of axes y and z accordingly; $\sigma = q_2/4ab$ is the

area density of a plate charge. Then the symbol λ, created by the whole plate, will be equal to

$$\lambda = \frac{2q_1\sigma}{\varepsilon c_1^2 m_1}\int_{\eta=y+b}^{\eta=y-b}\int_{\zeta=z-a}^{\zeta=z+a}\frac{\mathbf{d}\eta\mathbf{d}\zeta}{\sqrt{R^2-[\vec{\beta}\times\vec{R}]^2}}. \qquad (7.28)$$

After a deployment of a denominator the integral multiplicand in (7.28) has a kind

$$I = \int_{z+a}^{z-a}\mathbf{d}\zeta\int_{y+a}^{y-a}\frac{\mathbf{d}\eta}{\sqrt{\gamma_x^2x^2+\gamma_z^2\zeta^2+2\beta_x\beta_z\zeta+2\beta_y(\beta_xx+\beta_z\zeta)\eta+\gamma_y^2\eta^2}}.$$

The integral on η is a tabular integral. After transformations and substitution in (7.28) we have

$$\lambda = \frac{2q_1\sigma}{\varepsilon c_1^2 m_1\gamma_y\gamma_z}\int ln\left[A+BZ+\sqrt{C+Z^2}\right]\mathbf{d}Z\Big|_{\eta=y+a}^{\eta=y-a}, \qquad (7.29)$$

where

$$Z = \left[\zeta+\frac{\beta_z}{\gamma_z^2}(\beta_xx+\beta_y\eta)\right]\gamma_z,$$

$$C = \gamma_x^2x^2+\gamma_y^2\eta^2+2\beta_x\beta_yx\eta-\left[\frac{\beta_z}{\gamma_z}(\beta_xx+\beta_y\eta)\right]^2,$$

$$A = \gamma_y\left[\eta+\frac{\beta_x\beta_y}{\gamma_y^2}x-\frac{\beta_y\beta_z^2(\beta_xx+\beta_y\eta)}{\gamma_y^2\gamma_z^2}\right],$$

$$B = \frac{\beta_y\beta_z}{\gamma_y\gamma_z}.$$

The integral on z, included in (7.29), is reduced to an integral (6.19) after integration by parts. After fulfilment of transformations its solution will be recorded as

$$\int ln\left[A+BZ+\sqrt{C+Z^2}\right]\mathbf{d}Z = -Z-\frac{A}{B^2-1}ln\left[Z+\sqrt{C+Z^2}\right]$$
$$+Zln\left[A+BZ+\sqrt{C+Z^2}\right]-\frac{BA}{1-B^2}ln\left[A+BZ+\sqrt{C+Z^2}\right]+ \qquad (7.30)$$
$$+\frac{\sqrt{C(1-B^2)-A^2}}{1-B^2}\left\{\text{arctg}\frac{CB-AZ}{\sqrt{C(1-B^2)-A^2}\sqrt{C+Z^2}}-\text{arctg}\frac{AB+(B^2-1)Z}{\sqrt{C(1-B^2)-A^2}}\right\}.$$

After a substitution of a value of an integral (7.30) in (7.29) and change of variables is obtained the following expression for λ is obtained:

$$\lambda = \frac{2q_1\sigma}{\varepsilon c_1^2 m_1(1-\beta_x^2)}\left[\frac{x\beta_x\beta_y+\eta(1-\beta_x^2)}{\gamma_z}\ln\left(\gamma_zR_v+x\beta_x\beta_z+\eta\beta_y\beta_z+\zeta\gamma_z^2\right)+\right.$$
$$+\frac{x\beta_x\beta_z+\zeta(1-\beta_x^2)}{\gamma_y}\ln\left(\gamma_yR_v+x\beta_x\beta_y+\eta\gamma_y^2+\zeta\beta_y\beta_z\right)+ \qquad (7.31)$$
$$\left.+x\,\text{arctg}\frac{x^2\beta_y\beta_z-x\eta\beta_x\beta_z-\zeta\eta(1-\beta_x^2)-x\beta_x\beta_y}{xR_\upsilon}\right]\Bigg|_{\eta=y+b}^{\eta=y-b}\Bigg|_{\zeta=z+a}^{\zeta=z-a},$$

where $R_\upsilon = \sqrt{R^2-[\vec{\beta}\times\vec{R}]^2}$.

The expressions (7.25) and (7.31) determine the velocity of a charged particle, which the charged plate affects. λ_0 is calculated according to (7.31) in an initial point x_0, y_0, z_0 and with initial velocity υ_{x0}, υ_{y0}, υ_{z0}. However, here there is a number of difficulties. At first, the expression (7.25) determines only the module of velocity in a point x, y, z, but the direction of velocity is not known. Secondly, equation (7.25) does not express the velocity in an obvious kind, as the included in the right member symbol λ depends itself on the velocity in a point x, y, z, as it is seen from (7.31). Besides, the knowledge of a velocity component is required which is unknown. So, in a common case the velocity of a particle is not determined by this method.

Let's consider a particular case of a particle movement perpendicularly to the plate along an axis x: $y = z = \beta_y = \beta_z = 0$ and $\beta = \beta_x$. Under such circumstances the parameter λ, according to (7.31), after a substitution of limits on η and ζ accepts a kind

$$\lambda = -\frac{8q_1\sigma}{\varepsilon c_1^2 m_1(1-\beta^2)}\left[x\,\text{arctg}\frac{ab(1-\beta^2)}{xR_\upsilon}+\right.$$
$$\left.+\frac{b\sqrt{1-\beta^2}}{2}ln\frac{R_v-a\sqrt{1-\beta^2}}{R_v+a\sqrt{1-\beta^2}}+\frac{a\sqrt{1-\beta^2}}{2}ln\frac{R_v-b\sqrt{1-\beta^2}}{R_v+b\sqrt{1-\beta^2}}\right], \qquad (7.32)$$

where $R_v = \sqrt{x^2+(1-\beta^2)(a^2+b^2)}$.

At small velocity of movement or when $c_1 \to \infty$, as you can easily be convinced, the equation for velocity (7.25) and (7.32) give expression (6.36), which is obtained due to the solution of a differential equation of a particle movement (6.34) when a plate acts on it at a small velocity of a particle movement. At large

velocities $\beta \to 1$ the limit of expression (7.32) gives $\lambda = \frac{2q_1q_2}{\varepsilon c_1^2 m_1 \sqrt{x^2}}$. Then the equation (7.25) is transformed in (6.41), which is the solution of an equation of movement (6.34) when $\beta \to 1$. Thus, expressions (7.25) and (7.32) are the solution of an equation (6.34) in two limiting cases. From here it is possible to assume, that they will be the solutions (6.34) at other values β, too. By the considered method the solution is obtained not in an obvious kind: the required value - velocity v - enters in a left-part and a right part of the equation (7.25). Nevertheless, it is not obviously possible to decide an equation (6.34) by other method.

Let's consider now some new results. Let particle move from infinity ($x_0 \to \infty$) to a plate ($x \to 0$). Then it follows from the equation (7.32) when $x = x_0$, that the $\lim\limits_{x_0 \to \infty} \lambda_0 = 0$. It is necessary to consider a limiting passage λ at $x \to 0$, considering that $x < (1-\beta^2)(a^2+b^2)$, i.e. it will probably, will be unfair at $\beta \to 1$. Then from (7.32) it follows

$$\lim_{x \to 0} \lambda = -\frac{q_1q_2}{\varepsilon c_1^2 m_1 ab\sqrt{1-\beta^2}} \left[b \ln \frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}+a} + a \ln \frac{\sqrt{a^2+b^2}-b}{\sqrt{a^2+b^2}+b} \right]. \quad (7.33)$$

For a square plate ($a = b$) with allowance for (7.33) the expression for velocity of a particle, according to (7.31), will be recorded

$$v^2 = c_1^2 - (c_1^2 - v_0^2) \exp \frac{2q_1q_2 \ln(3+2\sqrt{2})}{\varepsilon m_1 c_1^2 a \sqrt{1-\beta^2}}. \quad (7.34)$$

During the attraction $q_1q_2 < 0$, therefore at $\beta \to 1$ the second addend tends at zero, i.e. $v^2 \to c_1^2$. Let's introduce a symbol for the parameter of interaction

$$B_1 = -\frac{2q_1q_2 \ln(3+2\sqrt{2})}{\varepsilon c_1^2 m_1 a}. \quad (7.35)$$

Then the adduced velocity of a particle moving to a plate from infinity, according to (7.34) can be recorded as

$$\beta^2 = 1 - (1-\beta_0^2) \exp\left(-\frac{B_1}{\sqrt{1-\beta^2}} \right). \quad (7.36)$$

In these symbols for Coulomb's interaction, as follows from (6.36) at $x_0 \to \infty$ and $x \to 0$, the velocity of a particle at a plate will be

$$\beta^2 = \beta_0^2 + B_1, \quad (7.37)$$

i.e. from here one more physical sense follows: B_1 is a square of a particle increment adduced of velocity during Coulomb's interaction. From (7.37) it is seen, that at $\beta_0 = 0$ the value is $\beta = \sqrt{B_1}$. At $B_1 = 1$ the particle moving from infinity with initial zero velocity approaching a plate will gain a light speed.

Except implicit expression (7.36) the asymptotic solution (6.39) at $\beta \to 1$ was obtained for the action of a charged plate on a particle. The expression (6.39) has a peculiarity at $x = 0$ and gives a light speed when a particle contacts with a plate at any charge q_2 of a plate. It is connected with the peculiarities of a limiting passage of an addend $x^2/(1-\beta^2)$ during simultaneous rushing $x \to 1$ and $\beta \to 1$. To compare the results of the equation (7.36) to the results (6.39), we will consider the movement of a particle from infinity up to a distance in the latter case

$$x_a = \frac{a}{\ln(3+2\sqrt{2})} = 0{,}567a. \quad (7.38)$$

Then, according to (6.39), with allowance for (7.35) the adduced velocity of a particle reaching a distance x_a will be recorded as

$$\beta_{xa}^2 = 1 - \left(1-\beta_0^2\right) \exp\left(-B_1\right). \quad (7.39)$$

In Fig. 7.1,*a* the results of the equation (7.36) solution at different values of adduced velocity is shown. It is seen, that with increase of initial velocity β_0 the same value of the parameter of interaction B_1 leads to decreasing increments of velocity. At $\beta_0 = 1$ the schedule is shown by a horizontal line, i.e. the velocity of a particle is not increased.

All curves in a Fig. 7.1,*a* do not reach the value $\beta = 1$. In the area from some value β up to $\beta = 1$ the solutions of the equation (7.36) have singularities: there can be some solutions or there cannot be any one. However, all curves have the general solution $\beta = 1$ at $B_1 = 1$. That is at this value of the interaction parameter, the particles reaching the plates, will have light speed.

In Fig. 7.1,b the velocities of the particle, approaching the plate from infinity at the distance x_a are shown. The velocities are calculated by the formula (7.39) depending on the parameter of interaction B_1 at different β_0. Comparing Fig. 7.1,*a* with Fig 7.1,b, here of incremental velocity is less in case of same values B_1, and the light velocity is reached in case of $B_1 \to \infty$. It is necessary to note, that the velocities in a point x_a can be calculated also by first method, but for it is necessary to take advantage of more complex expression (7.32) for λ.

We have calculated the velocity of a particle during one plate action. Combining values λ, it is possible to find the velocity during the action of any number of plates. So, when two plates located at distance 2*d* from each other (see Fig. 6.2), the expression for velocity of a particle will be

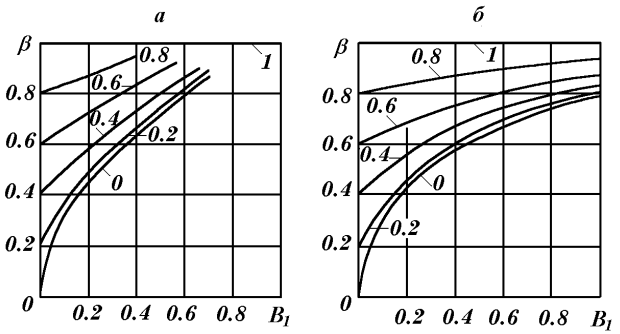


Fig. 7.1. The, adduced velocities of a particle, moving from infinity with the different initial velocities β_0 (are given on the schedules) and located at the action of a charged plate are the results of the solution of an equation (7.36) during the contact of a plate; *б*- asymptotic solution (7.39) during approach of a plate at the distance $x_0 = 0.567a$.

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\left[\lambda(x+d) - \lambda(x-d) - \lambda_0(x_0+d) + \lambda_0(x_0-d)\right]. \quad (7.40)$$

Here the centre of coordinates is located in the centre of the capacitor, the axis x is perpendicular to the plates, and the value λ is determined from expression (7.31) or (7.32) by the substitution $x \pm d$ and $x_0 \pm d$ instead of x and x_0, accordingly.

Let's consider a particular case of movement of a particle in such a flat rectangular capacitor, when the particle moves along an axis from one plate ($x_0 = -d$) with a charge q_2 to another ($x = d$) with a charge ($-q_2$). Then with allowance for (7.32) the expression (7.40) receives a sight

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\left[\lambda(2d) - \lambda(0)\right], \quad (7.41)$$

where $\lambda(2d)$ is determined from (7.32) when $x = 2d$, and $\lambda(0)$ - at x=0.
As the distance between the plates of the capacitor $2d$ is much less than its sizes, the expression (7.41), together with (7.32), will be simplified at $2d/\sqrt{(1-\beta^2)(a^2+b^2)} \to 0$ will be written as

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\left[-\frac{2q_1 u}{c_1^2 m_1 (1-\beta^2)}\right]. \quad (7.42)$$

where $u = 4\pi(2d)\sigma/\varepsilon$ - residual of potentials between the plates of the capacitor. When $u \to \infty$, $v \to c_1$ follows from (7.42). That is, even at a very large voltage in the plates of the capacitor the velocity of accelerated particles will not exceed the light speed. At small velocities, i.e. at $c_1 \to \infty$ ` from (7.42) we get the known in physics equation

$$\frac{m_1 v^2}{2} = \frac{m_1 v_0^2}{2} + q_1 u, \quad (7.43)$$

It follows from (7.43), that the initial kinetic energy ($E = m_1 v_0^2/2$) of the particle q_1 after its passing the accelerating potential u will be increased by value $q_1 u$. Taking into account (7.43) the unit of energy of particles - electron - volt is entered in physics. It is equal to energy of a particle with a charge of an electron equal to e, which it gains passing the potential difference u = 1V. So, when $u = 10^6$V this energy is called 1 MeV, at $u = 10^9$V = 1 GeV.

But actually, as it is seen from (7.43), at the acceleration the charged particle gains the energy $e \cdot u$ only at small velocity of a particle. With increase of velocity the gained energy in the capacitor decreases. Due to an acceleration of such a particle, as electron, its energy will not exceed $m_e c^2/2$=0.256 MeV ($m_e = 9.108 \cdot 10^{-31}$ kg, $c = 3 \cdot 10^8$m/sec, 1eV = $1.602 \cdot 10^{-19}$J) at voltage in the plates $u \to \infty$. Therefore assigning the energy to a particle in 1 MeV, in 1 GV etc. is fictitious.

Let's record the expression for λ during the action of a charged section directed along the axis z (see Fig. 6.3). By a passage to the limit $b \to 0$ we obtain from expression (7.31)

$$\lambda = -\frac{2q_1 Q}{\varepsilon c_1^2 m_1 \gamma_z} \ln\left(\gamma_z R_V + x\beta_x\beta_z + y\beta_y\beta_z + \zeta\gamma_z^2\right)\Bigg|_{\zeta=z-a}^{\zeta=z+a} \quad (7.44)$$

For expression (7.44) all conclusions, which were adduced for expression (7.31), are fair.

7.4 CALCULATION OF PARTICLES MOVEMENT IN ACCELERATORS

In the majority of modern boosters the acceleration of charged particles implements the action on them of charged bodies. The device is created so, that the accelerating force was directed along the velocity of a particle. During the action of a dot object with mass $m_2 >> m_1$ the acceleration according to expression (4.58) and law of a Newton (2.4) will be

$$w = \frac{q_1 q_2 (1-\beta^2)}{\varepsilon m_1 x^2} c, \quad (7.45)$$

where x is a distance from an object up to a particle. Deciding a differential equation (7.45), we will receive the expression for the velocity of a particle

$$v^2 = c_1^2 - (c_1^2 - v_0^2)\exp\frac{2q_1}{m_1 c_1^2}\left(\frac{q_2}{\varepsilon x} - \frac{q_2}{\varepsilon x_0}\right) \quad (7.46)$$

depending on a distance from the object, if the particle had the velocity v_0 at the initial distance x_0. The q_2 value, included in this expression, $q_2/\varepsilon x = V$ represents

an electrical potential of a final point, and $q_2/\varepsilon x_0 = V_0$ is a potential initial. Then the residual of potentials of these two points can be recorded

$$U = V - V_0 = \frac{q_2}{\varepsilon x} - \frac{q_2}{\varepsilon x_0}.$$

And the expression for velocity accept a kind

$$\upsilon^2 = c_1^2 - (c_1^2 - \upsilon_0^2)\exp\frac{2q_1 U}{m_1 c_1^2}. \qquad (7.47)$$

The residual of the acting object potentials enters only in this expression, which a particle passes there. In case of not dot acting objects, the bought velocity of a particle at a potential difference U, which it will pass, can also be defined by expressions (7.47). As the calculations show a velocity calculated so differs a little from that, received by the evaluation of force from this object. Therefore, the expression (7.47) with an adequate accuracy can be applied to a calculation of the action on a particle of any objects.

Let's consider some singularities of expression (7.47). In case of attraction q_1 the value is $U < 0$. When $U \to \infty$ the velocity of a particle is $\upsilon \to c_1$, i.e. any large residual of potentials U the particle has not passed, its velocity will not be more than c1. The energy of a particle after an acceleration is the kinetic energy $E = E_c = m\upsilon^2/2$. If to define the energy of a particle as charge q its product on the passed residual of potentials U, i.e. $E_{rel} = qU$, this value does not correspond to a real energy of a particle. For example, in case of indefinitely large potential difference ($U \to \infty$), the energy of a particle, according to (7.47), will not be infinite, and only comes nearer to value $E \to mc_1^2/2$.

For an electron the value $m g^2/2$ is equal to 0.256 MeV. As the electron in existing boosters can not gain the velocities greater than c, then it can not exceed the energy $m g^2/2$. As energy, appropriated to an electron, in 1 MeV, 100 MeV, 1 GeV characterizes only sum of potential differences 10^6, 10^8, 10^9 V, accordingly, which the particle has passed, hereinafter we call it, in difference from real energy, relativistic.

The second singularity of expression (7.47) is that the particle, passing the same residual of potentials, will receive a different incremental velocity, and also the increment of energy. The value of increment depends on the initial velocity the particle is accelerated. The particle is accelerated better, when its initial velocity is equal to zero, and it is not accelerated at all, when the initial velocity comes nearer to c_1. Thus, the relativistic energy does not correspond to a real one because at different initial velocities the particle receives different increases of velocity.

We research the movement of a particle, which is acted by a magnet. If the velocity of a particle $\vec{\upsilon}$ is perpendicular to magnetic intensity $\vec{H}$, homogeneously distributed in space, and other bodies do not force on a particle, the particle will move along a circle. Really, by substituting force (6.81) in equation (2.4), we obtain an acceleration of a particle

$$\vec{w} = \frac{\mu q}{m_1 c}(1-\beta^2)[\vec{\upsilon} \times \vec{H}], \qquad (7.48)$$

which shows, that it is perpendicular to a velocity and a magnetic intensity. Let's write acceleration (7.48) in polar coordinate system as two components: tangential in a direction of velocity

$$\frac{1}{R}\frac{\mathbf{d}(\upsilon R)}{\mathbf{d}t} = 0, \qquad \upsilon R = h = \text{const} \qquad (7.49)$$

and a radial, perpendicular velocity,

$$\frac{\mathbf{d}^2 R}{\mathbf{d}t^2} - \frac{\upsilon^2}{R} = \frac{\mu q(1-\beta^2)\upsilon H}{m_1 c}. \qquad (7.50)$$

In steady-state case $\mathbf{d}^2R/\mathbf{d}t^2 = 0$, and it follows from here, that a particle will move along the circle with a radius

$$R = \frac{m_1 \upsilon c}{\mu q H(1-\beta^2)}. \qquad (7.51)$$

We drop a minus, considering the modules of values. As the particle movement velocity along the circle is $\upsilon = \omega R$, the angular velocity of a particlemovement round an axis, and which is separated from it at the distance R, will be

$$\omega = \frac{\mu q H(1-\beta^2)}{m_1 c}. \qquad (7.52)$$

If to substitute of the Lorentz's force (6.79), in (2.4) we receive the angular rate a particle of rotation

$$\omega_L = \frac{\mu q H}{m_1 c}. \qquad (7.53)$$

The angular rate, unlike calculated by Lorenz's force as it is seen from (7.52) decreases with the increase of a particle velocity and reaching the velocity of propagation of electromagnetic action ($\beta \to 1$) tends to zero, i.e. at any large intensity $\vec{H}$ magnet can not bend the trajectory of a particle.

The angular rate (7.52) is shown in an implicit kind. Expressing β through $\omega R/c$ and deciding a quadratic equation, we will record the angular of a particle movement moving along a circular trajectory in an obvious kind:

$$\omega = \frac{-1 \pm \sqrt{1 + 4\omega_L^2 R^2 / c_1^2}}{2\omega_L R^2 / c_1^2}. \qquad (7.54)$$

Using a sign a plus, we can see, that at small values of intensity H, and consequently, ω_L the angular velocity $\omega = \omega_L$. At large values of intensity H, that is at $\omega_L^2 R^2 / c_1^2 >> 1$, neglecting the unit in a numerator, we obtain, that the angular rate tends to a limit $\omega = c_1/R$. In this case, the velocity of a particle is equal the

light speed and according to (7.51) radiuses of orbit tends to infinity, i.e. the trajectory of a particle will be the linear.

Disposing the description of a particle movement, when the electromagnetic devices as (7.47) force on it and when magnetic such as (7.52) or (7.54) we can consider the processes in boosters of particles.

In the high-voltage electrostatic booster the acceleration of charged particles is made by the charged parts of installation. A series number of electrodes plates with orifices or tubes are usually used, which potential is increased from the previous to consequent. Passing a distance between the first electrodes, the particle is accelerated under an action of a potential difference U_1 between them. If the initial velocity of a particle is υ_0, after passing of the second electrode its velocity, according to (7.47), will be

$$\upsilon_1 = \sqrt{c_1^2 - (c_1^2 - \upsilon_0^2)\exp\frac{2q_1U_1}{m_1c_1^2}}\,. \qquad (7.55)$$

After passing a distance between the second and third electrode with a potential difference U_2 the particle has the velocity

$$\upsilon_2 = \sqrt{c_1^2 - (c_1^2 - \upsilon_1^2)\exp\frac{2q_1U_2}{m_1c_1^2}}\,. \qquad (7.56)$$

Substituting υ_1 in (7.56) of (7.55), we will receive

$$\upsilon_2 = \sqrt{c_1^2 - (c_1^2 - \upsilon_0^2)\exp\frac{2q_1U}{m_1c_1^2}}\,, \qquad (7.57)$$

where $U = U_1 + U_2$.

Thus, we can see, that only the residual of potentials between the initial and the final electrodes or the total voltage passed by a particle is included in the expression for the velocity of an accelerated particle. If the particle passes $n + 1$ electrode and the full voltage will be $U = \sum_{i=1}^{n} U_i$, its velocity will be expressed by the equation (7.47). Therefore even after passing infinite number of electrodes the velocity of a particle will not exceed value c_1.

In a cyclotron the particle moves along a circle or along the other cyclical curve forced by magnets with perpendicular to velocity magnetic intensity $\vec{H}$. The pairs of electrodes are located along a circle, the acceleration of a particle happens when the potential difference acts on it. Usually in cyclotrons there are two electrodes - dee, representing two parts of a cylindrical box, cut on a diameter, with small altitude. The particles rotate inside dees and pass slots between them. The high-frequency voltage is brought to dees so that at the moment of passing a slot the particle was accelerated. At phase φ at this moment the acceleration will implement voltage $\upsilon = U_m \cos\varphi$, where U_m is a voltage excursion.

In such cyclotron the phase of voltage in a slot varies with each revolution. With growing velocity of a particle its angular rate, according to (7.52), decreases, therefore during each consequent revolution the particle delays in relation to the previous phase of voltage. During n turns particle will pass the total voltage $U = \sum_{i=1}^{n} u = U_m \sum_{i=1}^{n} \cos\varphi_i$. The final velocity of a particle will be defined by expression (7.47). As the phase grows, the acceleration usually begin at negative phase $\varphi > -\pi/2$, and finishes at positive $\varphi < \pi/2$. The final velocity of a particle can precisely be defined, carrying conducting series of calculations after each particle passing a slot. If we knew the initial velocity υ_0 of input particles in the accelerator and initial phase φ_0 voltage, that we may, according to (7.47), find the velocity υ_1 of a particle after the first slot.

It is possible to calculate angular rate of a particle ω_p on υ_1 by (7.53). The phase of voltage φ_2 in the second slot is determined by the difference ω_p and the cyclical frequency of voltage $\omega = 2\pi f$. By φ_2 and υ_1 again with the help of (7.47) we can calculate a velocity after the second slot. Continuing the further calculation in this way, we can calculate the final velocity of a particle. These evaluations are convenient for conducting on the computer. Let's note, that in this process the radius of trajectory will vary, therefore for a precise calculation of process of acceleration in a cyclotron it is necessary to use the precise solution of an equation (7.50).

The synchrocyclotron, or phasotron, differs from a cyclotron because it changes a frequency of accelerating voltage in accordance with a change of the angular rate of a particle. With allowance for (7.52) the frequency should be changed in the following way:

$$f = \frac{\omega_p}{2\pi} = \frac{\mu q_1 H}{2\pi m_1 c}(1 - \beta^2)\,. \qquad (7.58)$$

In existing boosters in view of a uncontrollable change ω_P the acceleration of a particle happens according to the oscillation of a phase.

In a cyclotron and phasotron, as it is seen from (7.51), with an acceleration of a particle the radius of trajectory grows. The radius of a trajectory is maintained constant in a synchrotron and synchrophasotron not to create magnet on the whole interval of a trajectory radius change. With this purpose it is necessary to increase the magnetic intensity as it is seen from (7.51), with growth of velocity of a particle by the law

$$H = \frac{m_1 c}{\mu q_1 R_a}\frac{\upsilon}{(1 - \beta^2)}\,, \qquad (7.59)$$

where R_a - radius of the booster. But as the velocity of a particle grows, and the radius of its trajectory does not vary, the angular rate of a particle begins to increase, and the phase of accelerating voltage decreases. To avoid this in a synchrophasotron unlike synchrotron changes the frequency of accelerating voltage.

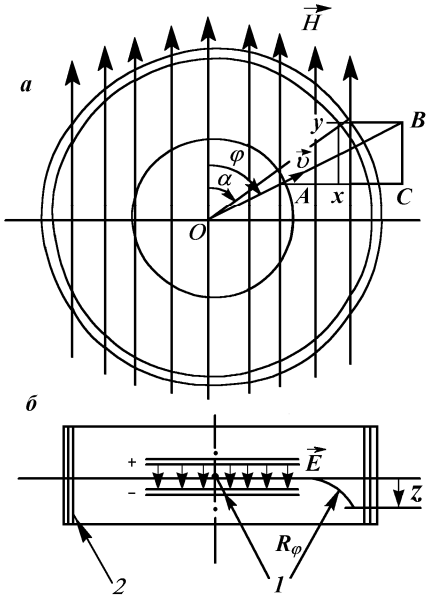
From the listed cyclical boosters everything, except a cyclotron, works in a pulse mode. In the linear booster the acceleration of particles is made along a direct line. The electromagnetic high-frequency field create the difference of potentials in the tubular electrodes (drift tubes) or in the waveguide. The final velocity is determined by expression (7.47), where U represents a total residual of potentials passed by a particle.

7.5. CALCULATION OF RESULTS OF THE BUCHERER'S EXPERIMENT

The direct measurements of forces between the moving charged particles were not conducted. Apparently, for the first time by the results of deviation of particles forced by the charged and magnetized bodies V. Kauphman determined in 1902 that the attitude of a charge to a mass of a moving electron depends on its velocity, i.e. the interaction between moving charged objects differs from Coulomb's law. Hereinafter in the Theory of Relativity it was treated as increase of a particle mass during its movement. In 1908 A.G. Bucherer [86] conducted more precise experiments showing the action on an electron by the charged and magnetized bodies, in which he confirmed the dependence of the attitude of a charge to an electron mass from its velocity. The experiments were executed during an electron change of velocity in a large range, including the velocities approaching to a light speed. It helped to check up the various theoretical explanations. Most likely, the obtained results were widely considered, but we knew only one publication 1919 [74].

Let's apply to professor Shaposhnikov description of Bucherer's experiments [74]. The grain of radium, located in the centre of the flat circular capacitor (Fig. 7.2), emitted β - rays in all possible directions. The capacitor was placed between the poles of a magnet with homogeneous magnetic intensity $\vec{H}$ located in a plane of the capacitor. On a cylindrical surface the photographic film coaxial to capacitor (see Fig. 7.2, *b*) was settled down. Through a narrow slot of the capacitor passed mainly those electrons, for which an electrical action of the capacitor and perpendicular to it action of a magnet were mutually compensated. " The Compensated electrons "coming from the capacitor were forced only by magnet and deviated by it from a mean plane of the capacitor (Fig. 7.2, *b*). Then they reached a photographic film pasted on an internal side coaxial with the capacitor of the cylinder and made it blackening.

Fig. 7.2. The scheme of Bucherer's experiment.
a - view along an axis of the capacitor;
b - view in the diametrical section of the capacitor.
1 - source of g-rays; *2* - photographic film.



Inside the capacitor (see Fig. 7.2, *b*) the force acts on the electron from the capacitor up, and from a magnetic system - downwards. Therefore from the radioactive source, located in centre, only those particles will take off, for which the magnetic and electrical force will be counterbalanced. The electrical force $\vec{F}_E$ on a moving particle is rather exactly described by a equation (6.90). According to expression (6.89) the magnetic force at $\mu = 1$ is the following

$$\vec{F}_M = \frac{\vec{z}}{z} q\beta(1-\beta^2)H\sin\varphi\,, \qquad (7.60)$$

where an axis z and polar angle φ are determined in a Fig. 7.2. Then the condition of compensations of forces on an electron $\vec{F}_E = \vec{F}_M$ will be

$$qE(1-\beta^2) = q\beta(1-\beta^2)H\sin\varphi$$

or

$$\beta\sin\varphi = \frac{E}{H} = \text{const}\,. \qquad (7.61)$$

If in the installation any constant attitude E/H is selected, at $\varphi = \pi/2$ the value β is the least, i.e. on a horizontal axis (see Fig. 7.2, *a*) the particles will go out from the capacitor with the least velocity. With the reduction of the angle φ from $\pi/2$ the velocities of particles will be increased and approaching the angle $\varphi = \arcsin(E/H)$ the particles with velocity close, to light speed will go out from the capacitor. Outside the capacitor, the particles, being only under magnetic action, and moving on a circular helixes round the vector of magnetic intensity $\vec{H}$, will deviate from a mean plane of the capacitor (see Fig. 7.2, *b*) and fall on a photographic film. As with growth of a particle velocity the action on it decreases, the particles will receive the greatest deviation on a horizontal axis, i.e. when $\varphi = \pi/2$ and least one with the angle $\varphi = \arcsin(E/H)$. According to Shaposhnikov, all the experiments

by Bucherer gave an approximately identical picture shown in a Fig. 7.3. The main difference of the Theory of Relativity from the experiment is that the curve, calculated according to the Theory of Relativity, of deviation, coming nearer to angles $|\varphi|$ = arcsin (E/H), should go vertically, as shown in Fig. 7.4.

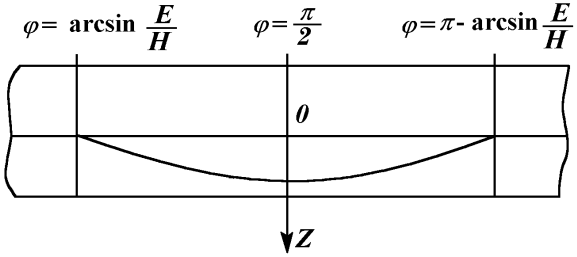


Fig. 7.3. A curve of electrons deviation at different velocities on a photographic film in the Bucherer's experiment.

Repeating a technique of calculation explained by the Shaposhnikov, let's imagine, that the electron moving from the capacitor in Є. *A* (see Fig. 7.2, *a*) with velocity υ will go along a screw trajectory up to a contact with a photographic film. The trajectory can be received if to bend a triangle *ABC* on a cylindrical surface by a radius R_φ of an electron movement trajectory (see Fig. 7.2, *b*). Then the hypotenuse *AB* will be the trajectory of an electron, and the radius of a curvature will be defined from the equality of magnetic and centrifugal forces $\vec{F}_M = \vec{F}_R$:

$$q\beta(1-\beta^2)H\sin\varphi = m\upsilon^2\sin^2\varphi / R_\varphi ,$$

whence

$$R_\varphi = \frac{mc^2}{q\beta\cdot\sin\varphi}\frac{\beta^2\sin\varphi}{H(1-\beta^2)} = \frac{mc^2\beta\sin\varphi}{qH}\frac{1}{1-\beta^2} . \qquad (7.62)$$

But as for the given experiment according to (7.61) $\beta\sin\varphi$ = const, the radius of a curvature (7.62) can be recorded

$$R_\varphi = \frac{A}{1-\beta^2} , \qquad (7.63)$$

where

$$\frac{mc^2\beta\sin\varphi}{qH} = A = \text{const} . \qquad (7.64)$$

After bending a triangle *ABC* p. *B* falls in p. *y* of a photographic film, which will defend on *z* from a median plane and be with the angle α to the intensity $\vec{H}$.
By designating $Ax = x$, the value *z* can be expressed through R_φ and *x* if to take into account, that the deviation of a point *C* in case of curvature of a triangle too is equal to *z* too. Then $z = R_\varphi - \sqrt{R_\varphi^2 - x^2}$. After differentiating this expression at the angle α, after transformation we will receive

$$\frac{\mathbf{d}z}{\mathbf{d}\alpha} = -\frac{z}{R_\varphi - z}\frac{\mathbf{d}R_\varphi}{\mathbf{d}\alpha} + \frac{x}{R_\varphi - z}\frac{\mathbf{d}x}{\mathbf{d}\alpha} . \qquad (7.65)$$

From expression (7.63) after derivation we have

$$\frac{\mathbf{d}R_\varphi}{\mathbf{d}\alpha} = \frac{2A\beta}{(1-\beta^2)^2}\frac{\mathbf{d}\beta}{\mathbf{d}\alpha} = \frac{2\beta R_\varphi^2}{A}\frac{\mathbf{d}\beta}{\mathbf{d}\alpha} . \qquad (7.66)$$

The derivative from β is defined, differentiating (7.61):

$$\frac{\mathbf{d}\beta}{\mathbf{d}\alpha} = -\frac{\beta}{\mathrm{tg}\varphi}\cdot\frac{\mathbf{d}\varphi}{\mathbf{d}\alpha} . \qquad (7.67)$$

After a substitution (7.66) and (7.67), the expression (7.65) will be recorded as

$$\frac{\mathbf{d}z}{\mathbf{d}\alpha} = \frac{2\beta^2 zR_\varphi^2\mathrm{ctg}\varphi}{A(R_\varphi - z)}\frac{\mathbf{d}\varphi}{\mathbf{d}\alpha} + \frac{x}{R_\varphi - z}\frac{\mathbf{d}x}{\mathbf{d}\alpha} . \qquad (7.68)$$

Let's consider behaviour of the tangent to a curve at $\beta \to 1$. As it is seen from (7.63), at $\beta \to 1$ the radius $R_\varphi \to \infty$, therefore from the expression $z = R_\varphi - \sqrt{R_\varphi^2 - x^2} \approx R_\varphi - R_\varphi(1 - 0.5x^2 / R_\varphi^2)$ it follows that $zR_\varphi = x^2/2$. Substituting the value z in a right member (7.68), we will consider behaviour of the tangent to a curve with large velocities of particles

$$\lim_{\beta\to 1}\frac{\mathbf{d}z}{\mathbf{d}\alpha} = \lim_{R_\varphi\to\infty}\left(\frac{x^2\beta\mathrm{ctg}\varphi}{A}\frac{R_\varphi}{R_\varphi - z}\frac{\mathbf{d}\varphi}{\mathbf{d}\alpha} + \frac{x}{R_\varphi - z}\frac{\mathbf{d}x}{\mathbf{d}\alpha}\right) = \frac{x^2\beta\mathrm{ctg}\varphi}{A}\frac{\mathbf{d}\varphi}{\mathbf{d}\alpha} . \qquad (7.69)$$

As we can see, the limit of this expression is a final value. As $\mathbf{d}z/\mathbf{d}\alpha$ is a tangent of a curve declination, it will come nearer to a horizontal line at any acute angle, as shown in Fig. 7.3.

Shaposhnikov and Kasterin considered Bucherer's experiment of the basis of theTheory of Relativity. According to Shaposhnikov [74], the condition of compensation in the Theory of Relativity is expressed by the same equation (7.61). For a radius of a curvature he received [56] the following expression:

$$R_\varphi^{'} = \frac{C_0}{\sqrt{1-\beta^2}} \quad , \qquad (7.70)$$

where $C_0 = A$ = const. K.N. Shaposhnikov obtains the dependence for tangent as follows:

$$\frac{\mathbf{d}z}{\mathbf{d}\alpha} = \frac{x^2\beta\ \mathbf{ctg}\varphi}{A}\frac{R_\varphi^2}{R_\varphi - z}\frac{\mathbf{d}\varphi}{\mathbf{d}\alpha} + \frac{x}{R_\varphi - z}\frac{\mathbf{d}x}{\mathbf{d}\alpha} . \qquad (7.71)$$

The first member in (7.71), and consequently, and $\mathbf{d}z/\mathbf{d}\alpha$ tends to infinity at $R_\varphi \to \infty$, i.e. the curve will approach a horizontal axis α at the right angle (see Fig. 7.4). It is possible to be convinced, that the superfluous multiplicand R_φ in the first addend of expression (7.71), which the tangent of declination conduces to infinity, is stipulated by relation of a radius of a curvature (7.70) from $1/\sqrt{1-\beta^2}$ unlike

$1/(1-\beta^2)$ in our expressions (7.63), (6.90) and (7.60). Thus, both ratio of the Theory of Relativity and ours give at $\beta \to 1$ the identical qualitative outcome - the action of a charged body and magnet on a moving charged particle decreases and ceases absolutely in case of approach to a light speed.

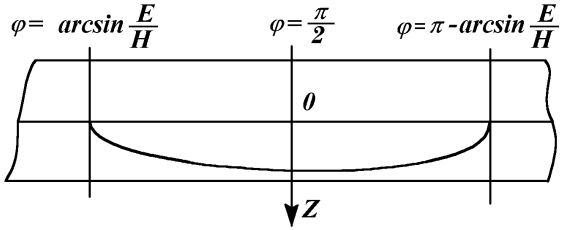


Fig. 7.4. Calculated by the Shaposhnikov, according to the Theory of Relativity, character of deviation of electrons with different velocities.

However, the quantitative responses differ with velocities close to the light speed, i.e. β = 1. The relation for a radius of a curvature (7.63) from (1-β^2) is received from the experimental laws of an electromagnetism. Relation (7.70) from $\sqrt{1-\beta^2}$ is obtained in the Theory of Relativity due to acceptance of a number of hypothesises, including the hypothesis of a mass change of a particle during its movement.

In Bucherer's experiment a track of a particle with the velocity close to the light speed remains on a photographic film, that allows to see a divergence between the Theory of Relativity and reality. Bucherer conducted experiments in case of two directions $\vec{E}$ and $\vec{H}$ [86], therefore on the photograph there are upper and lower branches of a curve. They incorporate under sharp corners, forming lentil. However, the connection of branches, as in Fig. 7.4, according to the Theory of Relativity should happen at the right angles, representing a figure as an ellipse. All results of Bucherer's experiments in 1908 give a lens-shaped curve, thereby rejecting the results of the Theory of Relativity.

CHAPTER 8

APPROACH OF FORCES AND RELATIVISTIC METHOD

8.1. ETHER, THEORY OF RELATIVITY AND LORENTZ'S TRANSFORMATIONS

The knowledge of the world by the person happened so, that it among the uncountable a mount of moving objects he always tried to find the one, which rested, and all remaining ones moved relatively him. Before appearing Copernic's system such central system was the Earth.

Then it was transferred to the Sun. But when by observations it was determined, that the Sun, as well as remaining stars, moves, such an absolute system of the world began connected the hypothetical global media the ether. The people thought, that the light from distant stars should be spread in the ether, and as it has an electromagnetic origin, then the electromagnetic action should be spread in relation to this ether. Here as A. Einstein in his work "The Principle of Relativity and its Consequences in Modern Physics" [77] summed up at the beginning of the XX century the representations at that time about the necessity of the ether: "When it was found, that there is a steep analogy between the elastic oscillations of a ponderable matter and the interference and diffraction of light rays, they decided, that light is necessary to consider as the oscillatory condition of any special substance. As light can be spread in space, where the ponderable substance is absent, to explain it, it was necessary to admit the existence of a singular substation distinguished from a ponderable matter; this substation was named as ether. And as the bodies distinguished by a small density, as, for example, in gases the velocity of propagation of light is approximately equal to the velocity in vacuum, it was necessary to assume, that in these bodies the ether also is the main carrier of light phenomena. At last, the hypothesis, according to which the ether is inside liquid and rigid bodies, in turn became necessary to understand the propagation of light inside these bodies, for with the help of only elastic properties of a ponderable substance it was impossible to explain the vast velocity of light rays propagation".

It is necessary to underline the method of knowledge, which seemed true at that time. All happening should have explanation. With this purpose it is necessary to put forward hypothesises and on their basis to explain the world around. Let's follow the explanation of the world, based on the hypothesis of the ether.

As the Earth moves in the prospective ether, the light at a surface of the Earth, being spread with certain velocity in relation to the ether, should have other velocity relatively the earth surface in a direction of the Earth motion, an in a perpendicular direction. By the experimentators headed by A. Michelson many experiments were made, but the calculated residual of velocity was not confirmed experimentally.

Then there were attempts to explain theoretically that fact, that the light has identical velocity as in relation to ether, and in relation to the Earth, which moves in ether. G.A. Lorentz and other independent resechers stated a hypothesis, that in case of movement of the Earth in the ether, its respective decreasing it in the direction of movement happens, so it is impossible to measure a change of light speed in relation to the Earth. As for electromagnetic action there were equations, describing its behaviour during movement, these equations of electrodynamics are used. With their help the put forward hypothesis was justified by the mathematical transformations of coordinates *x*, which reduced a field of a moving system of charges to a field of motionless charges [53].

Let's underline, that these transformations arise from a condition: the field of moving charges is the same, as the resting ones. It is based on fancying a field as some essence, which must not depend on movement. This condition is mistaken, as the moving charges create current, which acts on a magnet, but the motionless

charges do not act on a magnet. That is the field of moving charges should differ from a field of motionless ones. But as the hypothesis of abbreviation of the values is adopted, then from its positions the transformation $x = x'\sqrt{1-\beta^2}$ is considered as the abbreviation of the values in a moving system of charges in a direction of movement. It was supposed, that we don't find any differences in propagation of light in a moving or resting medium in relation to the ether. Later G.A. Lorentz refuses the given hypothesis, as in introduction of all new hypothesises "there can be a necessity every time, when we know the new facts" he notices in the work "Electromagnetic phenomena in a system, moving with any velocity, that is less than the light speed" [33]. He writes here too: "A position of things would be satisfactory, if it was possible by of certain main assumptions to show, that many electromagnetic phenomena, do not depend on the movement of a system". In these words said in 1904, the physicists of those years show their ambitions of physics and general direction is expressed, namely: the electrical field of a moving system will be expressed by the same rates, as the electrical field of a motionless system. The physicists will not search any more reasons explaining simultaneous existence of firm ether with the same velocity of light in it and in a reference system, which moves with any velocity relatively to the ether.

Let's give of Einstein's words from his work [77]: " By such way have reached understanding of these fields in emptiness, as the special statuses of an ether, which are not demanding for deeper analysis". And Einstein, not resolving this inconsistent problem of existence of identical speed of light on air and in system moving in it, and only accepting it for one of the principles, creates the Theory of Relativity. Originally given principle in "To Electrodynamics of Moving Bodies" [33] was formulated so: "Each ray of light moves in "resting" coordinate system with certain velocity v irrespective of, whether this ray of light is emitted by a resting or a moving body ". Afterwards this cautious formulation will be replaced by more direct: "the velocity of propagation of interaction is identical in all inertial systems of a reference" [26]. A. Einstein executes a principle of relativity by G.A. Lorentz's transformations G.A. the Lorentz, and he notes in the same way the equations of electrodynamics for a moving and resting system. For this, the transformation of time is introduced additionally. A sense of Einstein's operations is the following: if in vacuum there is any electrical system of actions and is in the given coordinate system x, y, z it is described by Maxwell's equations (3.22) and (3.24) for vectors $\vec{E}$ and $\vec{H}$, then relatively moving with velocity v of coordinate system x', y', z' the same system of electrical actions is described by the same Maxwell's equations, but for the other vectors $\vec{E}'$ and $\vec{H}'$. The transformations $\vec{E}$ and $\vec{H}$ in $\vec{E}'$ and $\vec{H}'$, and also x, y, z, t in x', y', z', t' follow from the given condition. This approach also makes the second principle of the Theory of Relativity: "The Laws, by which the condition of physical systems vary, do not depend on that, to which two coordinate systems, located relatively one another in a uniform transitional movement, these changes of a condition belong" [33].

So, the following link of hypothesises and conclusions following from them, is traced.

1. A hypothesis about the ether as the media, relatively which all movements take place.

2. The experiments (Michelson etc.) do not confirm availability of such ether.

3. The conclusion is received: the movement relatively ether cannot be defined, as the bodies in a direction of movement are reduced.

4. The corollaries follow from it. The laws of mechanics do not depend on movement. The fields (with which the action of bodies are expressed) nor depend on movement. The forces of action of moving bodies are the same, as the motionless ones.

5. The transformations of space and time follow from a condition of invariance of fields during movement.

6. The experiments (by Kaufman, Bucherer) testify that the action on a moving particle decreases with increase of its velocity.

7. The explanation is put forward: the mass of a particle grows with the increase of its velocity.

We have enumerated only a part of the hypothetical suppositions and conclusions following from them, on which the Theory of Relativity was developed.

Further, the obtained transformation of coordinates is received for real communications of the ambient world. The light speed is identified with the velocity of propagation of any action, and as Lorentz's transformations do not exist at the velocities that are, greater than the light speed, it is received for limiting velocity. The coordinates and time are considered as an independent object – four-dimensional space-time.

8.2 ESSENCE OF TRANSFORMATIONS

Many works are devoted to the analysis of Lorentz's transformations [41.43,79,87,92,103,127] and the principle of relativity [71, 93]. Instead one hypothesis the others ones are often introduced. If to discard all hypothesises, on which the Theory of Relativity is based, the essence of its method of describing the actions is that the action of moving bodies is determined through a action of motionless bodies, which parameters vary according to Lorentz's transformations. However action of moving bodies differs from the action of motionless ones. They also are described differently. Therefore, equating of certainly unequal equations is a formal acceptance, which can be applied, but these transformations have not a physical sense, i.e. the parameters of appearances and objects are not connected by these rates.

Let's consider such formal equating of rates describing the electrical actions of moving and motionless charged bodies. The charge of bodies (Fig. 8.1, *a*) is

described density of charge $\rho(x, y, z)$, and they move with the velocity υ along an axis x_υ. During movement we write all the variables with indexes "υ": a density of charges $\rho(x, y, z)$ in a coordinate system x_υ, y_υ, z_υ (see Fig. 8.1, *b*). Let's consider the force of action of charged bodies on a unit charge, which rests in this system, i.e. the electrical intensity $\vec{E}$. All further will not vary, if the unit charge moves with velocity ($-\upsilon$) relatively a system of charges. As it was found by the experiment, the electrical action depends only on a relative velocity of interacting objects, therefore these cases are equivalent.

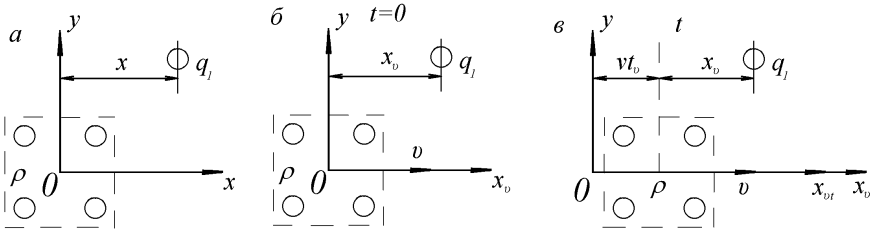


Fig. 8.1. The schemes of action of motionless charged bodies (*a*) and moving (*b*) and (*c*) on a motionless body with a unit charge ($q_1 = 1$).

The force of action of a moving system of charged bodies with a density ρ_υ is determined by d'Alember's equation (4.6), which in a projection on the axis x_υ and y_υ will be written so:

$$\frac{\partial^2 E_{\upsilon x}}{\partial x_\upsilon^2}+\frac{\partial^2 E_{\upsilon x}}{\partial y_\upsilon^2}+\frac{\partial^2 E_{\upsilon x}}{\partial z_\upsilon^2}-\frac{1}{c_1^2}\frac{\partial^2 E_{\upsilon x}}{\partial t_\upsilon^2}=\frac{4\pi}{\varepsilon}\frac{\partial}{\partial x_\upsilon}\rho_\upsilon+\frac{4\pi}{\varepsilon c_1^2}\frac{\partial}{\partial t_\upsilon}\rho_\upsilon, \quad (8.1)$$

$$\frac{\partial^2 E_{\upsilon y}}{\partial x_\upsilon^2}+\frac{\partial^2 E_{\upsilon y}}{\partial y_\upsilon^2}+\frac{\partial^2 E_{\upsilon y}}{\partial z_\upsilon^2}-\frac{1}{c_1^2}\frac{\partial^2 E_{\upsilon y}}{\partial t_\upsilon^2}=\frac{4\pi}{\varepsilon}\frac{\partial}{\partial y_\upsilon}\rho_\upsilon. \quad (8.2)$$

As the projection of force to the axis z_υ is similar to a projection to the axis y_υ, the last is not mentioned.

As the system of charges is constant, its action in a moment t_υ and at a point $x_{\upsilon t} = x_\upsilon + \upsilon t_\upsilon$, y_υ, z_υ (see Fig. 8.1, *c*) will be equal to the action in a moment $t_\upsilon = 0$ at a point with coordinates x_υ, y_υ, z_υ, where t_υ is any interval of time. The full derivatives on time from variable set of equations (8.1) - (8.2) will be equal to zero:

$$\frac{\mathbf{d}}{\mathbf{d}t_\upsilon}=\frac{\partial}{\partial x_\upsilon}\frac{\partial x_\upsilon}{\partial t_\upsilon}+\frac{\partial}{\partial t_\upsilon}=0. \quad (8.3)$$

Let's mark, that G.A. Lorentz used expression (8.3) during deducing the transformation of coordinates and time [26,32], called afterwards by his name. With allowance for (8.3) the partial derivatives

$$\frac{\partial}{\partial t_\upsilon}=-\upsilon\frac{\partial}{\partial x_\upsilon}, \quad (8.4)$$

$$\frac{\partial^2}{\partial t_\upsilon^2}=\upsilon^2\frac{\partial^2}{\partial x_\upsilon^2}. \quad (8.5)$$

Then the action of a moving system of charges (8.1) and (8.2) after replacement of derivatives on time, according to (8.4) and (8.5), will be described so:

$$(1-\beta^2)\frac{\partial^2 E_{\upsilon x}}{\partial x_\upsilon^2}+\frac{\partial^2 E_{\upsilon x}}{\partial y_\upsilon^2}+\frac{\partial^2 E_{\upsilon x}}{\partial z_\upsilon^2}=\frac{4\pi}{\varepsilon}\frac{\partial}{\partial x_\upsilon}\rho_\upsilon(1-\beta^2), \quad (8.6)$$

$$(1-\beta^2)\frac{\partial^2 E_{\upsilon y}}{\partial x_\upsilon^2}+\frac{\partial^2 E_{\upsilon y}}{\partial y_\upsilon^2}+\frac{\partial^2 E_{\upsilon y}}{\partial z_\upsilon^2}=\frac{4\pi}{\varepsilon}\frac{\partial}{\partial y_\upsilon}\rho_\upsilon. \quad (8.7)$$

The expressions (8.6) and (8.7) are certain at a point, which at any moment of time uniformly withdraws from the centre of moving systems of charges, but does not participate in a movement of charges.

The force of action created by the same charged bodies, is determined by Laplace's equation (3.15), which has a sight

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}=\frac{4\pi}{\varepsilon}\frac{\partial\rho}{\partial x}, \quad (8.8)$$

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial z^2}=\frac{4\pi}{\varepsilon}\frac{\partial\rho}{\partial y}. \quad (8.9)$$

The set of equations (8.6), (8.7) and (8.8), (8.9) describe the forces of action of the same charged system ρ on a unit charge located at the same distance. Despite of identical parameters of action, the equations (8.6), (8.7) differ from equations (8.8), (8.9). The difference is stipulated by the fact that the forces of action vary during movement. If we wish to reduce the description of the action of a moving system to the description of action of a motionless system, we should replace the values, which are included in (8.6) and (8.7), through values, which are included in (8.8) and (8.9). So, comparing (8.6) and (8.8), we can see, that two replacements are enough:

$$x_\upsilon = x\sqrt{1-\beta^2}, \qquad \rho_\upsilon = \rho/\sqrt{1-\beta^2}, \quad (8.10)$$

and comparing (8.7) and (8.9), we discover

$$E_{\upsilon y} = E_y/\sqrt{1-\beta^2}. \quad (8.11)$$

Similarly, we receive for a projection to the axis *z*

$$E_{\upsilon z} = E_z/\sqrt{1-\beta^2}. \quad (8.12)$$

Thus, by the change of variables (8.10) - (8.12) we can reduce d'Alembert's equation of the (4.6) to the equation (3.15), in spite of the fact that the equations determine different processes. The capability of such replacement does not mean, that these processes are identical. Therefore, the transformations of values (8.10) - (8.12) do not reflect a real character of their change.

It is possible to show, that the equation (4.6) is reduced to (3.15) and by the other system of transformations, namely

$$E_{\upsilon x} = E_x\sqrt{1-\beta^2}\,, \qquad x_\upsilon = x\sqrt{1-\beta^2}\,. \tag{8.13}$$

The first system of transformations (8.10) - (8.12) leaves charge constant:

$$q_\upsilon = \iiint \rho_\upsilon \mathbf{d}x_\upsilon \mathbf{d}y_\upsilon \mathbf{d}z_\upsilon = \iiint \frac{\rho \mathbf{d}x\sqrt{1-\beta^2}\,\mathbf{d}y\mathbf{d}z}{\sqrt{1-\beta^2}} = q_\upsilon\,.$$

Other system of transformations (8.13) will also transform the charge:

$$q_\upsilon = \iiint \rho_\upsilon \mathbf{d}x'_\upsilon \mathbf{d}y_\upsilon \mathbf{d}z_\upsilon = \sqrt{1-\beta^2}\,q\,.$$

By the transformations and proceeding from their sense we can receive the solution for action from moving charges from the solution for action of motionless charged bodies of a system in such order:

1. There are components of electrical intensity from the given motionless system of charges at a point *x, y, z* (the beginning of coordinates - in the centre of a system) as

$$E_x(x, y, z), \quad E_y(x, y, z), \quad E_z(x, y, z).$$

2. Instead of coordinates *x* we write everywhere $(x_{\upsilon t} - \upsilon t_\upsilon)/\sqrt{1-\beta^2}$, where υt_υ determines the centre of a moving system of charges, and $x_{\upsilon t}$ - coordinates of a point in a coordinate system x_υ, y_υ, z_υ at the moment t_υ (hereinafter we lower index *t*). The axis *x* is directed on the velocity of charges system. Then the components of the electrical intensity will be:

$$E_x\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \qquad E_y\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \qquad E_z\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right).$$

Applying the second system of transformations, it is necessary to multiply the value of a charge by $1/\sqrt{1-\beta^2}$, i.e. to replace *q* on $q_\upsilon/\sqrt{1-\beta^2}$.

3. To record components of the electrical intensity from a moving system of charges for the first system of transformations according to (8.11), (8.12), as:

$$E_{\upsilon x} = E_x\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \tag{8.14}$$

$$E_{\upsilon y} = \frac{1}{\sqrt{1-\beta^2}} \cdot E_y\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \tag{8.15}$$

$$E_{\upsilon z} = \frac{1}{\sqrt{1-\beta^2}} \cdot E_z\left(\frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \tag{8.16}$$

and for the second system of transformations, according to (8.13), so:

$$E_{\upsilon x} = \sqrt{1-\beta^2} \cdot E_x\left(\frac{q_\upsilon}{\sqrt{1-\beta^2}}, \frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \tag{8.17}$$

$$E_{\upsilon y} = E_y\left(\frac{q_\upsilon}{\sqrt{1-\beta^2}}, \frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right), \tag{8.18}$$

$$E_{\upsilon z} = E_z\left(\frac{q_\upsilon}{\sqrt{1-\beta^2}}, \frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}}, y_\upsilon, z_\upsilon\right). \tag{8.19}$$

It is uneasy to be convinced, that using this method, it is possible from expression for electrical intensity of a dot charge

$$E_x = \frac{xq}{\varepsilon(x^2+y^2+z^2)^{3/2}}, \quad E_y = \frac{yq}{\varepsilon(x^2+y^2+z^2)^{3/2}}, \quad E_z = \frac{zq}{\varepsilon(x^2+y^2+z^2)^{3/2}}$$

to receive in the projections on the axis of coordinates the expression for intensity of a moving point charge with velocity υ:

$$E_{\upsilon x} = \frac{(1-\beta^2)(x_\upsilon - \upsilon t_\upsilon)q_\upsilon}{\varepsilon\left\{(x_\upsilon - \upsilon t_\upsilon)^2 + (1-\beta^2)(y_\upsilon{}^2 + z_\upsilon{}^2)\right\}^{3/2}}, \tag{8.20}$$

$$E_{\upsilon y} = \frac{(1-\beta^2)y_\upsilon q_\upsilon}{\varepsilon\left\{(x_\upsilon - \upsilon t_\upsilon)^2 + (1-\beta^2)(y_\upsilon{}^2 + z_\upsilon{}^2)\right\}^{3/2}}, \tag{8.21}$$

$$E_{\upsilon z} = \frac{(1-\beta^2)z_\upsilon q_\upsilon}{\varepsilon\left\{(x_\upsilon - \upsilon t_\upsilon)^2 + (1-\beta^2)(y_\upsilon{}^2 + z_\upsilon{}^2)\right\}^{3/2}}. \tag{8.22}$$

The expressions (8.20) - (8.22) represent the projections of electrical intensity (4.54), created by a moving point charge.

So, both obtained systems of transformations reduce the expression for a motionless system of charges to the expression for the action of the same moving

charged bodies. And the actions of both moving bodies and motionless ones differ, and the expressions give different results at the same input data. For example, for the first system of transformations (8.14) - (8.16) communications between the forces are following: if the motionless system of charges acts on a unit resting charge, separated from its centre at distances $x = a$, $y = b$, $z =$ by the force with projections $E_x = A$, $E_y = B$, $E_z = C$, during the movement it, will be acted by the force $E_{\upsilon x} = A, E_{\upsilon y} = B/\sqrt{1-\beta^2}, E_{\upsilon z} = C/\sqrt{1-\beta^2}$ on a unit charge, separated from the centre at the on other distances: $x_\upsilon = a\sqrt{1-\beta^2}$, $y_\upsilon = b$, $z_\upsilon = c$.

The similar sense is expressed also by the transformations of the second system (8.17), (8.18).

8.3. DESCRIPTION OF INTERACTION OF MOVING NON-STATIONARY SYSTEMS OF BODIES WITH HELP OF THE TRANSFORMATIONS

The previous transformations were obtained for a motionless system of charges. If the electrical system varies in a due course, a ratio between the coordinates of points, in which the force of action of a motionless system and moving are in certain conformity, will depend on an instant t, in which this conformity is considered. Therefore, the time t will be determined in coordinates along movement x_υ, in which the action of a moving system, and also moment t_υ, i.e. $t = t(t_\upsilon, x_\upsilon)$ is considered.

So, there is a variable electromagnetic system with a density of an electricity ρ and density of currents $\rho\vec{u}$. Its action on a motionless unit charge is determined by vector $\vec{E}$, and on a moving charge, current or magnet with unit parameters – vector $\vec{H}$, which are deduced from Maxwell's equations (3.22), (3.28), (3.19), (3.14). Let's copy these equations

$$\mathrm{rot}\vec{H} = \frac{4\pi}{c}\rho\vec{u} + \frac{\varepsilon}{c}\cdot\frac{\partial\vec{E}}{\partial t}, \tag{8.23}$$

$$\mathrm{rot}\vec{E} = -\frac{\mu}{c}\frac{\partial\vec{H}}{\partial t}, \tag{8.24}$$

$$\mathrm{div}\vec{E} = \frac{4\pi}{\varepsilon}\rho, \tag{8.25}$$

$$\mathrm{div}\vec{H} = 0. \tag{8.26}$$

In a chapter 3 and 4 by eliminating $\vec{H}$ from a set of equations (8.23), (8.24) we received a d'Alembert's equation (3.26) for the action of moving charges on a motionless charge, and by eliminating $\vec{E}$ - for the action of a moving magnet on motionless objects.

Now we will consider a method of describing the action of a moving electromagnetic system by the transformations of variables. Let it move with velocity υ along the axis x relatively resting charged body and magnet, having units of electrical and magnetic charges. The action of an electromagnetic system on these bodies will be determined by the same set of equations (8.23) - (8.26), but with other current density:

$$\vec{J} = \rho[(u_{\upsilon x} - \upsilon)\vec{i} + u_{\upsilon y}\vec{j} + u_{\upsilon z}\vec{k}]. \tag{8.27}$$

Let's copy this set of equations, by replacing in (8.23.) a current density $\rho\vec{u}$ on (8.27):

$$\frac{\varepsilon}{c}\frac{\partial E_{\upsilon x}}{\partial t_\upsilon} + \frac{4\pi}{c}\rho_\upsilon(u_{\upsilon x} + \upsilon) = \frac{\partial H_{\upsilon z}}{\partial y_y} - \frac{\partial H_{\upsilon y}}{\partial z_\upsilon}, \tag{8.23a}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{\upsilon y}}{\partial t_\upsilon} + \frac{4\pi}{c}\rho_\upsilon u_{\upsilon y} = \frac{\partial H_{\upsilon x}}{\partial z_\upsilon} - \frac{\partial H_{\upsilon z}}{\partial x_\upsilon}, \tag{8.23b}$$

$$\frac{\varepsilon}{c}\frac{\partial E_{\upsilon z}}{\partial t_\upsilon} + \frac{4\pi}{c}\rho_\upsilon u_{\upsilon z} = \frac{\partial H_{\upsilon y}}{\partial x_\upsilon} - \frac{\partial H_{\upsilon x}}{\partial y_\upsilon}, \tag{8.23c}$$

$$\frac{\mu}{c}\frac{\partial H_{\upsilon x}}{\partial t_\upsilon} = -\frac{\partial E_{\upsilon z}}{\partial y_\upsilon} + \frac{\partial E_{\upsilon y}}{\partial z_\upsilon}, \tag{8.24a}$$

$$\frac{\mu}{c}\frac{\partial H_{\upsilon y}}{\partial t_\upsilon} = -\frac{\partial E_{\upsilon x}}{\partial z_\upsilon} + \frac{\partial E_{\upsilon z}}{\partial x_\upsilon}, \tag{8.24b}$$

$$\frac{\mu}{c}\frac{\partial H_{\upsilon z}}{\partial t_\upsilon} = -\frac{\partial E_{\upsilon y}}{\partial x_\upsilon} + \frac{\partial E_{\upsilon x}}{\partial y_\upsilon}, \tag{8.24c}$$

$$\frac{\partial E_{\upsilon x}}{\partial x_\upsilon} + \frac{\partial E_{\upsilon y}}{\partial y_\upsilon} + \frac{\partial E_{\upsilon z}}{\partial z_\upsilon} = \frac{4\pi}{c}\rho_\upsilon, \tag{8.25a}$$

$$\frac{\partial H_{\upsilon x}}{\partial x_\upsilon} + \frac{\partial H_{\upsilon y}}{\partial y_\upsilon} + \frac{\partial H_{\upsilon z}}{\partial z_\upsilon} = 0. \tag{8.26a}$$

As these equations describe the action of moving with velocity υ of the electromagnetic bodies, all the values are recorded with an index υ. This set of equations differs from a set of equations (8.23), (8.24), (8.25), (8.26), which describes the action of a motionless electromagnetic system, by the fact that the addend with velocity υ enters in equation (8.23a). Let's search for transformations of variables,

which would result a set of equations during the movement (with an index υ) in equations for the case of rest. As the communication between explanatory variables is known:

$$x = \frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta^2}},\ y = y_\upsilon,\ z = z_\upsilon \quad \text{и} \quad t=t(t_\upsilon, x_\upsilon, \upsilon), \tag{8.28}$$

it is possible to record derivatives:

$$\frac{\partial}{\partial x_\upsilon} = \frac{\partial t}{\partial x_\upsilon}\cdot\frac{\partial}{\partial t} + \frac{1}{\sqrt{1-\beta^2}}\frac{\partial}{\partial x};\quad \frac{\partial}{\partial y_\upsilon} = \frac{\partial}{\partial y};\quad \frac{\partial}{\partial z_\upsilon} = \frac{\partial}{\partial z};$$

$$\frac{\partial}{\partial t_\upsilon} = \frac{\partial t}{\partial t_\upsilon}\cdot\frac{\partial}{\partial t} - \frac{\upsilon}{\sqrt{1-\beta^2}}\cdot\frac{\partial}{\partial x}.$$

After their substitution in equations (8.23a), (8.24a), (8.25a), (8.26a) the system will be copied so:

$$\frac{\varepsilon}{c}\frac{\partial t}{\partial t_\upsilon}\frac{\partial E_{\upsilon x}}{\partial t} - \frac{\varepsilon}{c}\frac{\upsilon}{\sqrt{1-\beta^2}}\frac{\partial E_{\upsilon x}}{\partial x} + \frac{4\pi}{c}\rho_\upsilon(u_{\upsilon x}+\upsilon) = \frac{\partial H_{\upsilon z}}{\partial y} - \frac{\partial H_{\upsilon y}}{\partial z}, \tag{8.23'a}$$

$$\frac{\mu}{c}\frac{\partial t}{\partial t_\upsilon}\frac{\partial H_{\upsilon x}}{\partial t} - \frac{\mu}{c}\frac{\upsilon}{\sqrt{1-\beta^2}}\frac{\partial H_{\upsilon x}}{\partial x} = -\frac{\partial E_{\upsilon z}}{\partial y} + \frac{\partial E_{\upsilon y}}{\partial z}, \tag{8.24'a}$$

$$\frac{\partial t}{\partial x_\upsilon}\frac{\partial E_{\upsilon x}}{\partial t} + \frac{1}{\sqrt{1-\beta^2}}\frac{\partial E_{\upsilon x}}{\partial x} + \frac{\partial E_{\upsilon y}}{\partial y} + \frac{\partial E_{\upsilon z}}{\partial z} = \frac{4\pi}{\varepsilon}\rho_\upsilon, \tag{8.25'a}$$

$$\frac{\partial t}{\partial x_\upsilon}\frac{\partial H_{\upsilon x}}{\partial t} + \frac{1}{\sqrt{1-\beta^2}}\frac{\partial H_{\upsilon x}}{\partial x} + \frac{\partial H_{\upsilon y}}{\partial y} + \frac{\partial H_{\upsilon z}}{\partial z} = 0. \tag{8.26'a}$$

The remaining equations are noted similarly. Further, the equations (8.23a), (8.24a), (8.25a) and (8.26a) will be transformed with the use of relation between variables, which follow from other equations. The addition of expressions (8.23'a) with (8.25'a), multiplied by $\upsilon\varepsilon/c$, gives

$$\frac{\varepsilon}{c}\cdot\left(\frac{\partial t}{\partial t_\upsilon} + \upsilon\frac{\partial t}{\partial x_\upsilon}\right)\cdot\frac{\partial E_{\upsilon x}}{\partial t} + 4\pi\frac{\rho_\upsilon u_{\upsilon x}}{c} = \frac{\partial}{\partial y}\left(H_{\upsilon z} - \frac{\upsilon\varepsilon}{c}E_{\upsilon y}\right) - \frac{\partial}{\partial z}\left(H_{\upsilon y} + \frac{\upsilon\varepsilon}{c}E_{\upsilon z}\right). \tag{8.23"a}$$

The addition (8.25'a) with (8.23'a), multiplied by $\upsilon/(c\varepsilon)$, gives

$$\frac{\partial E_{\upsilon x}}{\partial t}\left(\frac{\upsilon}{c^2}\cdot\frac{\partial t}{\partial t_\upsilon} + \frac{\partial t}{\partial x_\upsilon}\right) + \frac{\partial E_{\upsilon x}}{\partial x}\cdot\frac{1-\beta_\circ^2}{\sqrt{1-\beta^2}} +$$

$$+\frac{\partial}{\partial y}\left(E_{\upsilon y} - \frac{\upsilon}{c\varepsilon}H_{\upsilon z}\right) + \frac{\partial}{\partial t}\left(E_{\upsilon z} + \frac{\upsilon}{c\varepsilon}H_{\upsilon y}\right) = \frac{4\pi}{\varepsilon}\rho_\upsilon\left(1 - \frac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}\right). \tag{8.25"a}$$

The addition (8.24'a) with (8.26'a), multiplied by $\upsilon\mu/c$, gives

$$\frac{\mu}{c}\cdot\left(\upsilon\frac{\partial t}{\partial x_\upsilon} + \frac{\partial t}{\partial t_\upsilon}\right)\frac{\partial H_{\upsilon x}}{\partial t} = \frac{\partial}{\partial z}\left(E_{\upsilon y} - \frac{\upsilon\mu}{c}H_{\upsilon z}\right) - \frac{\partial}{\partial y}\left(E_{\upsilon z} + \frac{\upsilon\mu}{c}H_{\upsilon y}\right). \tag{8.24"a}$$

The addition (8.26'a) with (8.24'a) multiplied by $\upsilon\mu/c$, gives

$$\left(\frac{\upsilon}{c^2}\cdot\frac{\partial t}{\partial x_\upsilon} + \frac{\partial t}{\partial t_\upsilon}\right)\cdot\frac{\partial H_{\upsilon x}}{\partial t} + \frac{1-\beta_\circ^2}{\sqrt{1-\beta^2}}\cdot\frac{\partial H_{\upsilon x}}{\partial x} +$$

$$+\frac{\partial}{\partial y}\left(H_{\upsilon y} + \frac{\upsilon}{c\mu}E_{\upsilon z}\right) + \frac{\partial}{\partial z}\left(H_{\upsilon z} - \frac{\upsilon}{c\mu}E_{\upsilon y}\right) = 0. \tag{8.26"a}$$

Here $\beta_\circ = \upsilon/c$. As to reduce these equations to corresponding equations for a motionless system in case of any ε and μ is inconvenient, we reduce them at first for $\varepsilon = 1$; $\mu = 1$. Then $\beta = \upsilon/c = \beta_\circ$.

Sequentially comparing (8.23"a), (8.25"a), (8.24"a), (8.26"a) with equations (8.23), (8.25), (8.24) and (8.26), written for corresponding projections, we will come to a conclusion, that the equations will be identical, if the following equalities are executed:

$$\frac{\partial t}{\partial t_\upsilon} + \upsilon\frac{\partial t}{\partial x_\upsilon} = \sqrt{1-\beta_\circ^2};\quad \frac{\upsilon}{c^2}\cdot\frac{\partial t}{\partial t_\upsilon} + \frac{\partial t}{\partial x_\upsilon} = 0. \tag{8.29}$$

Allowing equations (8.29) relatively unknown derivatives, we obtain:

$$\frac{\partial t}{\partial t_\upsilon} = \frac{1}{\sqrt{1-\beta_\circ^2}};\quad \frac{\partial t}{\partial x_\upsilon} = -\frac{\upsilon}{c^2\sqrt{1-\beta_\circ^2}}. \tag{8.30}$$

The integrating equations (8.30), we will record accordingly

$$t = \frac{t_\upsilon}{\sqrt{1-\beta_\circ^2}} + C(x_\upsilon);\quad t = -\frac{\upsilon x_\upsilon}{c^2\sqrt{1-\beta_\circ^2}} + C(t_\upsilon), \tag{8.31}$$

where $C(x_\upsilon)$ is a constant of integration on t_υ,, and $C(t_\upsilon,)$ - on x_υ. Equalities (8.31) are possible, if to exact constant the relation t from t_υ and x_υ will be the following:

$$t = \frac{t_\upsilon - (\upsilon/c^2)x_\upsilon}{\sqrt{1-\beta_\circ^2}}. \tag{8.32}$$

The expression (8.32) determines the transformation of time, which is necessary to describe the actions of a motionless system of bodies to transform to the description of action moving one. To find transformations for remaining variables, we will substitute the values of derivatives (8.30) in equations (8.23"a), (8.25"a), (8.24"a) and (8.26"a) and is comparable with corresponding equations (8.23), (8.25), (8.24) and (8.26). We define two systems of transformation of variables.

The first system of transformations

$$x = \frac{x_\upsilon - \upsilon t_\upsilon}{\sqrt{1-\beta_\circ^{\,2}}}; \quad t = \frac{t_\upsilon - (\upsilon / c^2)x_\upsilon}{\sqrt{1-\beta_\circ^{\,2}}}; \tag{8.33}$$

$$u_x = \frac{u_{\upsilon x}}{1-\dfrac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}}; \quad \rho = \rho_\upsilon \frac{1-\dfrac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}}{\sqrt{1-\beta_\circ^{\,2}}}; \tag{8.34}$$

$$u_y = \frac{u_{\upsilon y}\sqrt{1-\beta_\circ^{\,2}}}{1-\dfrac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}}; \quad u_z = \frac{u_{\upsilon z}\sqrt{1-\beta_\circ^{\,2}}}{1-\dfrac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}}; \tag{8.35}$$

$$E_x = E_{\upsilon x}, \quad H_x = H_{\upsilon x}; \tag{8.36}$$

$$E_y = \frac{E_{\upsilon y} - \dfrac{\upsilon}{c}\cdot H_{\upsilon z}}{\sqrt{1-\beta_\circ^{\,2}}}; \quad H_y = \frac{H_{\upsilon y} + \dfrac{\upsilon}{c}\cdot E_{\upsilon z}}{\sqrt{1-\beta_\circ^{\,2}}}; \tag{8.37}$$

$$E_z = \frac{E_{\upsilon z} + \dfrac{\upsilon}{c}\cdot H_{\upsilon y}}{\sqrt{1-\beta_\circ^{\,2}}}; \quad H_z = \frac{H_{\upsilon z} - \dfrac{\upsilon}{c}\cdot E_{\upsilon y}}{\sqrt{1-\beta_\circ^{\,2}}}. \tag{8.38}$$

The second system of transformations.

It actuates the transformations of the first system for coordinates *x* and time *t* (8.33) and the transformation for components of velocities. A density of a charge and the components of intensity will be transformed as follows:

$$\rho = \rho_\upsilon\left(1-\frac{\upsilon u_{\upsilon x} + \upsilon^2}{c^2}\right); \tag{8.39}$$

$$E_x = \sqrt{1-\beta_\circ^{\,2}}\,E_{\upsilon x}; \quad H_x = \sqrt{1-\beta_\circ^{\,2}}\,H_{\upsilon x}; \tag{8.40}$$

$$E_y = E_{\upsilon y} - \frac{\upsilon}{c}\cdot H_{\upsilon z}; \quad H_y = H_{\upsilon y} + \frac{\upsilon}{c}\cdot E_{vz}; \tag{8.41}$$

$$E_z = E_{\upsilon z} + \frac{\upsilon}{c}\cdot H_{\upsilon y}; \quad H_z = H_{\upsilon z} - \frac{\upsilon}{c}\cdot E_{\upsilon y}. \tag{8.42}$$

By these systems of transformations it would also be possible to transform electrodynamics the equations at ε and μ, distinct from the unit, if the units of measurement *H* and *E* were not selected for vacuum at $\varepsilon = \mu = 1$, but for media with ε and μ distinct from the units. Then the ratio between an electromagnetic system of units and system CGSE would be determined not by *c*, but $c_1 = c/\sqrt{\varepsilon\mu}$; c_1 would be included in electrodynamics equations, and in transformations $\beta_\circ$ would replace β.

Let's bring some totals.

1. If there is a system of charged bodies with a density function of charges ρ (*x*, *y*, *z*) and the velocity of movement $\vec{u}$ (*x*, *y*, *z*), it action *E* on a motionless unit charge and action *H* on a motionless unit magnet are described by the electrodynamics equations (8.23), (8.24), (8.25) and (8.26), which are made by measurement of forces (Coulomb's law, Biot-Savart-Laplace's law, Faraday's law of induction).

2. If this system of bodies moves with the velocity $\vec{\upsilon}$ relatively an object, the velocity of moving charges becomes equal to $\vec{u}_1(x,y,x) = \vec{u}\,(x,y,z) + \vec{\upsilon}$ and its act on the same object is described by the same electrodynamics equations, in which $\vec{u}$ is replaced by $\vec{u}_1$ (*x*, *y*, *z*).

3. The action of a moving system of electromagnetic bodies can be described by a set of equations (8.23), (8.24), (8.25) and (8.26), by replacing their variables according to transformations (8.33) - (8.38).

4. When we know the action created by a motionless system of bodies, it is possible to calculate the action from a moving system bodies by transformations.

5. The transformation of parameters, for example, distances, time, velocity etc. shows, how they should be changed, to describe the action of a moving charged system by the same equations, as the action of a motionless one.

8.4. ABOUT TRANSFORMATION OF A WAVE EQUATION

For a motionless system of charges the transition from d'Alembert's equations (8.1) and (8.2), of moving charges, describing the action, to Laplace's equations (8.8) and (8.9), describing the action of motionless charged bodies was realized. For a non-stationary electrical system such transition will already be between d'Alembert's equations distinguished by densities of currents.

In these two cases the intensity $\vec{E}$ described the forces stipulated by bodies, which charge is determined by a density of an electrization. Let's consider what the intensity is determined by, if the densities ρ are not obviously included in equations (8.1) and (8.2). D'Alembert's equation without a right member is named as a wave equation. If the right member is equal to zero, according to (4.6), the density of a charge ρ is determined by the following equations:

$$\frac{\partial \rho}{\partial x}+\frac{u_x}{c_1^2}\frac{\partial \rho}{\partial t}=0\,, \tag{8.43}$$

$$\frac{\partial \rho}{\partial y}+\frac{u_y}{c_1^2}\frac{\partial \rho}{\partial t}=0\,, \tag{8.44}$$

$$\frac{\partial \rho}{\partial z}+\frac{u_z}{c_1^2}\frac{\partial \rho}{\partial t}=0\,, \tag{8.45}$$

where u_x, u_y, u_z are the components of charges movement velocities. The equation (8.43) is possible at any values of explanatory variables, if

$$\frac{\partial \rho}{\partial x}=A_x;\quad \frac{u_x}{c_1^2}\frac{\partial \rho}{\partial t}=-A_x\,, \tag{8.46}$$

where $A_x = A_x$ (x, y, z, t). Let's consider a particular case A_x = const. Then the equations (8.46) are integrated and can be written accordingly

$$\rho=A_x x+B_x;\quad \rho=-A_x\frac{c_1^2}{u_x}t+C_x\,, \tag{8.47}$$

where constants of integration B_x do not depend on x, and C_x do not depend on t. Two solutions (8.47) can be written as one equation

$$\rho=A_x\left(x-\frac{c_1^2}{u_x}t\right)+D_x\,, \tag{8.48}$$

here D_x is a constant of integration, which do not depend on x and y. The similar solutions are obtained for equations (8.44) and (8.45) accordingly:

$$\rho=A_y\left(y-\frac{c_1^2}{u_y}t\right)+D_y\,, \tag{8.49}$$

$$\rho=A_z\left(z-\frac{c_1^2}{u_z}t\right)+D_z\,. \tag{8.50}$$

It is easy to be convinced, that the solutions (8.48) - (8.50) when $A_x = ku_x$, $A_y = ku_y$ and $A_z = ku_z$ are noted as one expression

$$\rho=k(\vec{r}\vec{u}-c_1^2 t)+\rho_0\,, \tag{8.51}$$

where ρ_0 = const.

If the electrical system is characterized by a density of a charge (8.51), that the force of its action on a unit charge it is described by d'Alembert's equation without a right member:

$$\frac{\partial^2 \vec{E}}{\partial x^2}+\frac{\partial^2 \vec{E}}{\partial y^2}+\frac{\partial^2 \vec{E}}{\partial z^2}-\frac{1}{c_1^2}\frac{\partial^2 \vec{E}}{\partial t^2}=0\,. \tag{8.52}$$

If this electrical system moves relatively a unit charge with velocity υ along the axis x, or it moves with velocity ($-\upsilon$) relatively the electrical system, its action on a charge will be the other. We can find it by transformations (8.33). Let's copy them, supplementing relations for axes y and z:

$$x=\frac{x_\upsilon-\upsilon t_\upsilon}{\sqrt{1-\beta^2}},\quad t=\frac{t_\upsilon-(\upsilon/c_1^2)x_\upsilon}{\sqrt{1-\beta^2}},\quad y=y_\upsilon,\quad z=z_\upsilon\,. \tag{8.53}$$

The transformations (8.53) differ from Lorentz's transformations (8.33) by the fact that they are recorded for the velocity of action propagation in media c_1, instead of the light speed in vacuum c. Let's express variables with an index "υ" through unindexed variables:

$$x_\upsilon=\frac{x+\upsilon t}{\sqrt{1-\beta^2}},\quad t_\upsilon=\frac{t+(\upsilon/c_1^2)x}{\sqrt{1-\beta^2}},\quad y_\upsilon=y,\quad z_\upsilon=z\,. \tag{8.54}$$

Let's pass to the indexed variables in a wave equation (8.52). According to (8.53), $x = x(x_\upsilon, t_\upsilon)$, and $t = t(x_\upsilon, t_\upsilon)$, we can pass from the derivation on a composite function to the derivation on arguments:

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial x_\upsilon}\cdot\frac{\partial x_\upsilon}{\partial x}+\frac{\partial}{\partial t_\upsilon}\cdot\frac{\partial t_\upsilon}{\partial x},\quad \frac{\partial}{\partial t}=\frac{\partial}{\partial x_\upsilon}\cdot\frac{\partial x_\upsilon}{\partial t}+\frac{\partial}{\partial t_\upsilon}\cdot\frac{\partial t_\upsilon}{\partial t}\,. \tag{8.55}$$

Differentiating expressions (8.54) substituting them in (8.55), we obtain the rates between derivatives:

$$\frac{\partial}{\partial x}=\frac{1}{\sqrt{1-\beta^2}}\left(\frac{\partial}{\partial x_\upsilon}+\frac{\upsilon}{c_1^2}\frac{\partial}{\partial t_\upsilon}\right),\quad \frac{\partial}{\partial t}=\frac{1}{\sqrt{1-\beta^2}}\left(\upsilon\frac{\partial}{\partial x_\upsilon}+\frac{\partial}{\partial t_\upsilon}\right). \tag{8.56}$$

Repeatedly differentiating (8.56), we discover the rates between the second derivatives:

$$\frac{\partial^2}{\partial x^2}=\frac{1}{1-\beta^2}\left(\frac{\partial^2}{\partial {x_\upsilon}^2}+2\frac{\upsilon}{c_1^2}\frac{\partial^2}{\partial t_\upsilon \partial x_\upsilon}+\frac{\upsilon^2}{c_1^4}\frac{\partial^2}{\partial {t_\upsilon}^2}\right), \tag{8.57}$$

$$\frac{\partial^2}{\partial t^2} = \frac{1}{1-\beta^2}\left(\upsilon^2 \frac{\partial^2}{\partial x_\upsilon{}^2} + 2\upsilon \frac{\upsilon}{c_1^2}\frac{\partial^2}{\partial t_\upsilon \partial x_\upsilon} + \frac{\partial^2}{\partial t_\upsilon{}^2}\right), \qquad (8.58)$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y_\upsilon{}^2}; \quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z_\upsilon{}^2}. \qquad (8.59)$$

After substitution the second derivatives in the equation (8.52) it will be transformed in the same wave equation:

$$\frac{\partial^2 \vec{E}_\upsilon}{\partial x_\upsilon{}^2} + \frac{\partial^2 \vec{E}_\upsilon}{\partial y_\upsilon{}^2} + \frac{\partial^2 \vec{E}_\upsilon}{\partial z_\upsilon{}^2} - \frac{1}{c_1^2}\frac{\partial^2 \vec{E}_\upsilon}{\partial t_\upsilon{}^2} = 0. \qquad (8.60)$$

The ratio between $\vec{E}$ and $\vec{E}_\upsilon$ can be anyone linear, independent from the variables of derivation. The action $\vec{E}_\upsilon$ of a charged system moving with velocity υ according to (8.60), the density of which charge is in that specific case determined by expression (8.51), will take place at points x_υ, y_υ, z_υ and in a moment, displaced in comparison with action of motionless system according to expression (8.54).

The wave equation (8.52) as the linear differential equation can have the solution [24] with dividing variables

$$\vec{E} = \vec{E}_0(x, y, z)e^{\pm i\omega t}. \qquad (8.61)$$

After its substitution in (8.52) we obtain

$$\Delta\vec{E}_0(x,y,z) + \frac{\omega^2}{c_1^2}\vec{E}_0(x,y,z) = 0.$$

This is Helmholtz's equation, which, depending on boundary conditions can have the different solutions. In that specific case it can have the solution

$$\vec{E}_0(x, y, z) = \vec{E}_0 e^{-i k r}, \qquad (8.62)$$

where $\vec{E}_0$ = const; $\vec{k} = \frac{\omega}{c}\vec{n}$ is a wave vector; $\vec{n} = \hat{\alpha}\vec{i} + \hat{\beta}\vec{j} + \hat{\gamma}\vec{k}$ is a normal to planes, which cosine directions are equal to the axes *x*, *y*, *z*: $\hat{\alpha} = \cos(n\hat{x})$; $\hat{\beta} = \cos(n\hat{y})$; $\hat{\gamma} = \cos(n\hat{z})$.

With allowance for (8.61) and (8.62) the solutions of a wave equation will be written as

$$\vec{E}(x, y, z, t) = \vec{E}_0 \cos[\omega(\pm t - (\hat{\alpha}x + \hat{\beta}y + \hat{\gamma}z)/c_1)]. \qquad (8.63)$$

The substitution (8.63) in (8.52) gives a ratio for the direction cosines

$$\hat{\alpha}^2 + \hat{\beta}^2 + \hat{\gamma}^2 = 1. \qquad (8.64)$$

From (8.63) it is seen, that when

$$\pm t - (\hat{\alpha}x + \hat{\beta}y + \hat{\gamma}z)/c_1 = \text{const} \qquad (8.65)$$

The values of intensity $\vec{E}$ are identical on a plane at any moment t_1

$$\hat{\alpha}x + \hat{\beta}y + \hat{\gamma}z = \text{const} \pm c_1 t_1. \qquad (8.66)$$

If the plane is perpendicular to the axis *x*, i.e. at $\hat{\alpha} = 1$ and $\hat{\beta} = \hat{\gamma} = 0$, at any moment its coordinates *x* will be

$$x = x_0 \pm c_1 t. \qquad (8.67)$$

In case of a sign "+" the plane of a constant of intensity *E* moves in a direction of the axis *x* (a direct wave) with velocity c_1. In case of sign "-" the plane moves in the opposite direction (a backward wave). Such moving plane is called a flat wave, and expression (8.63) is the equation of a flat wave.

As the period of a cosine is equal to 2π, it follows from (8.63), that simultaneously the action $\vec{E}$ will be identical on the planes, separated from each other at distances

$$\Delta x = \lambda = 2\pi c_1/\omega, \qquad (8.68)$$

where λ - wavelength.

At a fixed point according to (8.63) intensity

$$\vec{E} = \vec{E}_0 \cos\omega(\pm t - x/c_1) \qquad (8.69)$$

varies in time in the limits ($-E_0$) $\le E \le E_0$ with a period $T = 2\pi/\omega$. Thus, at the initial moment ($t = 0$) values $E = E_0 \cos\varphi_0$, where $\varphi_0 = -\omega x/c_1$ is called as an initial phase of a wave. As is seen, the phase of a wave along the axis *x* varies.

It is necessary to note, that though equation (8.63) is called the equation of a wave and say, that the waves are spread, it does not follow from it. We can see, that, at the moment $t = 0$ at all points of space there is an action $\vec{E}$. But this action is varied by the harmonic law and is had different phases φ, which, as follows from (8.69), depend on coordinates *x*. In that case the parameter c_1 represents the velocity of propagation of a phase. Therefore a wave is not the motion wave of any substance or substation, similar to waves in the water. The electromagnetic wave is a variable action at every point, which remote from a considered body. The other charged or magnetized body will test this action, which is located at this point.

The flat wave (8.63) is one of the possible solutions of a wave equation (8.52). The singularities and properties of wave process are well seen on its example. Other solutions of a wave equation are possible also. They are easily received, expressing (8.52) in cylindrical and spatial polar coordinates [24]. For example, in the spatial polar coordinates the equation receives a sight

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c_1^2 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \vec{E}}{\partial r} \right). \qquad (8.70)$$

At replacement $\vec{E} = \vec{A}/r$ it will be transformed in to one-dimensional along the spatial coordinates r

$$\frac{\partial^2 \vec{A}}{\partial t^2} = c_1^2 \frac{\partial^2 \vec{A}}{\partial r^2}, \qquad (8.71)$$

which solution

$$A = A\cos\left[\omega(t \pm \frac{r}{c_1}) \right], \qquad (8.72)$$

where $\vec{A}_0$ is a constant

Here vectors $\vec{E}$ are also $\vec{A}$ expressed in the spatial polar coordinates. With allowance for (8.72) the solution as spherical waves is obtained:

$$E_r = \frac{A_{ro}}{r}\cos\left[\omega(t \pm \frac{r}{c_1}) \right], E_\varphi = \frac{A_{\varphi o}}{r}\cos\left[\omega(t \pm \frac{r}{c_1}) \right], E_\Theta = \frac{A_{\Theta o}}{r}\cos\left[\omega(t \pm \frac{r}{c_1}) \right]. \qquad (8.73)$$

So, in unlike the flat waves the intensity decreases with increase of a distance r from centre of coordinates. The wavelength λ in this case will be the least distance between points, in which the intensity is in an identical phase.

8.5. ABERRATION AND DOPPLER'S EFFECT

We have considered the possible solutions of a wave equation. The electromagnetic waves are created by different ways, for example, at fast change of a charge on the plates of the capacitor. The plates of the capacitor are connected with the antenna and earth surface and, thus, generate radio waves. The radiator of the smaller value will radiate more short waves. The electromagnetic waves with a wavelength λ = 0,3 - 0,8 microns are light and are created as the result of a charge change within the limits of the atom sizes. The form of these waves differs from the flat ones (8.63) and the spherical ones (8.73). In particular case the precise expression for waves can be obtained as the result of d'Alembert's equation solution (4.6), at a task of the law of change is ρ or the velocity of charges motion is $\vec{u}(t)$, the initial values of intensity is $\vec{E}(x,y,z)$ and its velocity of change is $\partial \vec{E} / \partial t = f(x,y,z)$.

Created by a variable electrical system $\rho(x,y,z,t)$ the action $\vec{E}$, being spread in all space with identical speed c_1, at large distances r_s from an electrical system will have the forward front as a spherical surface. Therefore, solutions (8.73) for a spherical wave can be used here. At small distances from r_s the spherical surface

practically does not differ from a plane, therefore, the solution (8.63) for a flat wave in that case can be used.

We considered above the solution of a wave equation for the action of a motionless electrical system $\rho(x,y,z,t)$. Let's discover its action on the receiver, moving relatively it, in a direction of an axis with speed υ. Let's consider this act on the example of the solution for flat waves (8.63) by the method of transformations. Substituting replacements (8.53) in (8.63) and noting electrical intensity with an index "υ", we obtain the following expression:

$$\vec{E}_\upsilon = \vec{E}_{o\upsilon}\cos\left\{ \frac{1+\hat{\alpha}\beta}{\sqrt{1-\beta^2}}\omega\left[\pm t_\upsilon - \frac{x_\upsilon(\hat{\alpha}+\beta) + y_\upsilon\hat{\beta}\sqrt{1-\beta^2} + z_\upsilon\hat{\gamma}\sqrt{1-\beta^2}}{c_1(1+\hat{\alpha}\beta)} \right] \right\}. \qquad (8.74)$$

Let's apply the symbols:

$$\omega_\upsilon = \omega\frac{1+\hat{\alpha}\beta}{\sqrt{1-\beta^2}}, \qquad (8.75)$$

$$\hat{\alpha}_\upsilon = \frac{\hat{\alpha}+\beta}{1+\hat{\alpha}\beta}, \qquad (8.76)$$

$$\hat{\beta}_\upsilon = \frac{\sqrt{1-\beta^2}}{1+\hat{\alpha}\beta}\hat{\beta}, \qquad (8.77)$$

$$\hat{\gamma}_\upsilon = \frac{\sqrt{1-\beta^2}}{1+\hat{\alpha}\beta}\hat{\gamma}. \qquad (8.78)$$

It is uneasy to be convinced, that $\hat{\alpha}_\upsilon^2 + \hat{\beta}_\upsilon^2 + \hat{\gamma}_\upsilon^2 = 1$, i.e. $\hat{\alpha}_\upsilon, \hat{\beta}_\upsilon, \hat{\gamma}_\upsilon$ represent the direction cosines of normal to a plane. Then the equation for a flat wave (8.74), which expresses the action of a source on the receiver, moving relatively it, will be written as

$$\vec{E}_\upsilon(x_\upsilon, y_\upsilon, z_\upsilon, t_\upsilon) = \vec{E}_{o\upsilon}\cos\omega_\upsilon[\pm t_\upsilon - (\hat{\alpha}_\upsilon x_\upsilon + \hat{\beta}_\upsilon y_\upsilon + \hat{\gamma}_\upsilon z_\upsilon)/c_1], \qquad (8.79)$$

where the speed υ is positive at the approach of the receiver and the source. The receiver will also perceive the action, as a flat wave, but other frequency ω_υ, the angles of declination of a wave plane $\hat{\alpha}_\upsilon, \hat{\beta}_\upsilon, \hat{\gamma}_\upsilon$ will differ. The expression (8.75) describes a known Doppler's effect, and expression (8.76) - (8.78) describe the appearance of aberration. Value of action $\vec{E}_{o\upsilon}$, as follows from transformations (8.14) - (8.16) or (8.17) - (8.19), with allowance for transformations of a charge will vary as follows:

$$E_{ovx} = E_{ox},\ E_{ovy} = \frac{E_{oy}}{\sqrt{1-\beta^2}},\quad E_{ovz} = \frac{E_{oz}}{\sqrt{1-\beta^2}}, \qquad (8.80)$$

i.e. it will amplify across motion of the receiver.

The attempts of experimental check of Doppler's effect (8.75) are known in the literature [44]. However, this problem has been unclear until now in the theoretical plan: there are about ten different formulas, some of them give mutually opposite results.

On the example of rates for a flat wave we will consider the action of a moving source of spherical waves on the motionless receiver (Fig. 8.2). As it has already been noted, that at large distance from a source it is possible to take an advantage of the results for a flat wave. With the velocity of a source directed on the receiver (see Fig. 8.2, a), $\hat{\alpha}=1;\hat{\beta}=\hat{\gamma}=0$ according to (8.75) - (8.78) we have

$$\omega_\upsilon=\frac{1+\beta}{\sqrt{1-\beta^2}}\omega,\ \hat{\alpha}_\upsilon=1,\ \hat{\beta}_\upsilon=\hat{\gamma}_\upsilon=0\,. \tag{8.81}$$

As we can see, the direction of a source observation does not vary, only the frequency of perceived waves is increased. At the approach it tends at speed c_1 to the infinity. In case of removing a source from the receiver $\hat{\alpha}=-1$ the frequency also decreases:

$$\omega_\upsilon=\sqrt{(1-\beta)(1+\beta)}\omega\,, \tag{8.82}$$

and with the approach to speed c_1 it tends to zero.

At the speed of a source, perpendicular to the direction on the receiver (see Fig. 8.2, *b*) $\hat{\alpha}=\hat{\gamma}=0,\ \hat{\beta}=1$ also according to (8.75) - (8.76) we have

$$\omega_\upsilon=\omega/\sqrt{1-\beta^2}\,;\hat{\beta}_\upsilon=\sqrt{1-\beta^2}\,\hat{\beta}\,. \tag{8.83}$$

Here, the cyclical frequency of radiation is increased and with approach of υ to c_1 tends to infinity. It is called as the transversal Doppler's effect.

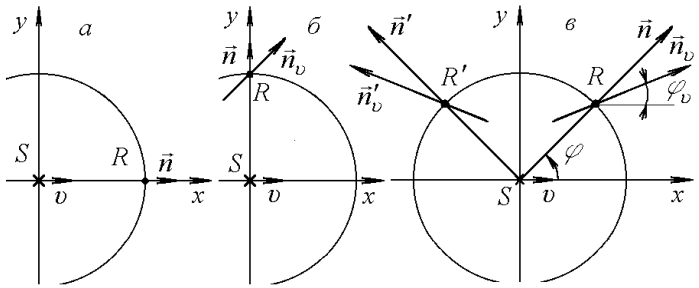


Fig. 8.2. Action of a moving source of radiation *S* on the motionless receiver *R*.
a - Motion on the receiver; *b* is the motion perpendicular to the receiver;
c - Motion at the angle to the receiver.

As $\hat{\beta}_\upsilon$ decreases with the increase of speed, the angle of declination of the normal $\vec{n}_\upsilon$ to the axis *y* is increased, i.e. the line-of-sight on a moving source deviates from a vertical.

In the inclined case of motion we will express the parameters of radiation of a moving source through the angle φ between a line-of-sight on the receiver and the speed of a source (see Fig. 8.2, *c*). As, $\hat{\alpha}=\cos\varphi,\ \hat{\beta}=\sin\varphi$ and $\hat{\gamma}=0$, according to (8.75) - (8.78) we record

$$\omega_\upsilon=\omega\frac{1+\beta\cos\varphi}{\sqrt{1-\beta^2}}\,, \tag{8.84}$$

$$\varphi_\upsilon=\arccos\hat{\alpha}_\upsilon=\arccos\frac{\beta+\cos\varphi}{1+\beta\cos\varphi}\,. \tag{8.85}$$

In Fig. 8.2,*c* we show the angles of observation φ_υ of a moving source and normal n_υ to the observable surfaces of radiation at two positions of the receiver: *R* and *R*`. With the approach υ to c_1 the normal of observable radiation n_υ removes from a vertical.

8.6 SPEED OF LIGHT BETWEEN MOVING BODIES

From d'Alembert's equation and the wave equations we obtain the solutions for the action, which depends on c_1. With velocity c_1 the phase of a wave moves in space. If the changes of an acting body will happen, they will be spread with velocity c_1. Therefore it is called as the velocity of propagation of electromagnetic action. The value c_1 depends from dielectric ε and magnetic μ permeabilities of a media between interacting bodies. If there are different environments between them, then each of them will have a velocity of propagation, defined by the properties ε and μ.

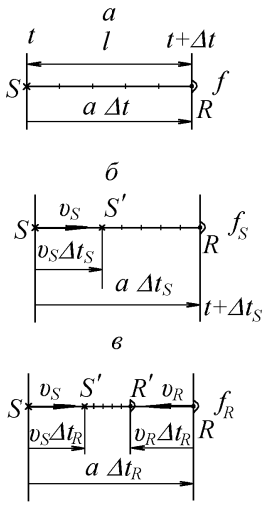


Fig. 8.3. The propagation of acoustic waves in media, in which the source *S* moves with speed υ_S, and the receiver *R* moves with speed υ_R in a direction of approach.

At the motion of the receiver and the source the interaction between them depends only on their relative speed, therefore electromagnetic waves differ from acoustic ones or the waves in water. The source of acoustic waves, for example, the hooter of the motionless locomotive, at p. *S* (Fig. 8.3,*a*) at the moment *t* produces sound oscillations by frequency *f*, which are spread in a motionless air with the velocity *a* = 340 m\s. Through $\Delta t = l/a$ the sound reaches the receiver *R* and for this time $n = f\,\Delta t$ of oscillations will be

produced. The distance between them, i.e. wavelength, is $\lambda = l/n = a/f$. As these oscillations are spread with velocity a, they will enter in the receiver with frequency $f_R = a/\lambda = f$.

If the locomotive S approaches the receiver with speed υ_R (see Fig. 8.3,*b*), the oscillations, produced by the hooter, being spread with speed a in media, also will be heard by the receiver through $\Delta t = l/a$. Let's underline, that the speed of a sound has not been changed, as it is spread in media and does not depend on the motion of a source. During Δt the source creates $n = \Delta t f$ of oscillations, will pass a distance $\upsilon_S \Delta t$ and will take a position S'. Thus, the oscillations will be arranged in length $l - \upsilon_S \Delta t = (a - \upsilon_S)\Delta t$. Then a distance between them is $\lambda = (a - \upsilon_S)\Delta t/n = (a - \upsilon_S)/f$. As the oscillations are spread with speed a relatively the receiver, the frequency of a sound perceived by the receiver, will be

$$f_s = \frac{a}{\lambda} = \frac{a}{a - \upsilon_S} f \ . \tag{8.86}$$

If the receiver (see Fig. 8.3,*c*) moves to a source with speed υ_R relatively media, it comes nearer towards a spread sound, i.e. the speed of a sound relatively the receiver will make

$$a_R = a + \upsilon_R \ . \tag{8.87}$$

The sound will reach the receiver during $\Delta t_R = l/(a + \upsilon_R)$, during Δt_R the source will create $n_R = \Delta t_R f$ of oscillations, will pass a distance $\upsilon_S \Delta t_R$ and will take a position R'. The oscillations will be in length $l - (\upsilon_S + \upsilon_R)\Delta t_R$ and the distance between them will be

$$\lambda_R = [l - (\upsilon_s + \upsilon_R)\Delta t_R]/(\Delta t_R f) \ ,$$

i.e.

$$\lambda_R = \frac{a - \upsilon_S}{f} \ . \tag{8.88}$$

Such lengths of waves are perceived with speed a_R, therefore, the frequency of a perceived sound

$$f_R = \frac{a_R}{\lambda_R} = \frac{a + \upsilon_R}{a - \upsilon_S} f \ . \tag{8.89}$$

These rates are fair to the motion of bodies in water, for example, at the motion of the ship, creating waves, and their action on a moving boat. According to (8.89) the actions of waves on a boat depend on the speed of the ship υ_S and the speed of a boat υ_R.

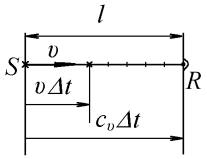


Fig. 8.4. Propagation of electromagnetic oscillations between a relatively moving source *S* and the receiver *R*.

The interaction of electromagnetic objects depends only on their relative speed. If the objects are motionless relatively one another,

the electromagnetic waves are spread between them with speed c_1, irrespective of the objects movement relatively media. It is testified by the experiments, from which d'Alembert's equation (3.28) follows. At first: the magnet induces electromotive force in a conductor only at their relative motion; secondly, the moving charge is equivalent to a current and creates the action on a magnet, which depends on their relative speed. The interaction of electromagnetic objects, for example spools with a magnet, does not depend on what is moving or what is resting. It is determined by their relative speed.

From these positions we will consider the propagation of light between a moving source and the receiver. Let's consider, that a relative distance l between a source S and the receiver R the electromagnetic action passes with speed c_υ, which depends on speed υ of their relative motion (Fig. 8.4). Then the time of propagation will be $\Delta t = l/c_\upsilon$. For this time the source creates a quantity of oscillations $n = f\,\Delta t = f\,l/c_\upsilon$, and the distance between the source and the receiver will become $l - \upsilon\Delta t$. At this distance there is n of oscillations, therefore, the distance between them, that is the wavelength, will be

$$\lambda = (l - \upsilon\Delta t)/n = (l - \upsilon\Delta t)c_\upsilon/(f\,l) = (1 - \beta_\upsilon)c_\upsilon / f \ ,$$

where

$$\beta_\upsilon = \upsilon / c_\upsilon \ . \tag{8.90}$$

As the oscillations passes a relative distance between the source and the receiver with speed c_υ, the receiver will perceive them with frequency

$$f_\upsilon = \frac{c_\upsilon}{\lambda} = \frac{f}{1 - \beta_\upsilon} \ . \tag{8.91}$$

In this expression the speed of light c_υ is unknown, which enters in the normalised speed β_υ. From electrodynamics equations we have received a cyclical frequency of oscillations perceived by the receiver as (8.84), whence the frequency of oscillations at approach of the source and the receiver at $\varphi = 0$ will be written

$$f_\upsilon = f\frac{1 + \beta}{\sqrt{1 - \beta^2}} \ . \tag{8.92}$$

The expressions (8.91) and (8.92) determine the frequency of oscillations perceived by the receiver, which is calculated by different methods. Excluding them from value f_υ, we obtain the following expression for a normalised speed of propagation of electromagnetic waves between the approaching source and receiver with normalised speed β:

$$\bar{c}_\upsilon = \frac{\beta(1 + \beta)}{1 + \beta - \sqrt{1 - \beta^2}} \ , \tag{8.93}$$

where

$$\bar{c}_\upsilon = c_\upsilon / c_1 . \qquad (8.94)$$

In case of removing receiver and source $\beta < 0$. At $\beta \to 0$ the limits of expression (8.93) give $\bar{c}_\upsilon = 1$, i.e. for want of motion of a source relatively the receiver, the light is spread with usual the speed c_1. At $\beta = 1$ speed $c_\upsilon = 1$, i.e. in case of motion of the receiver or the source with the light speed the front of the oscillations moves together with a moving object and the receiver will perceive these oscillations, when the source completes with it. At superluminal velocity $\beta > 1$, value $\bar{c}_\upsilon$ is imaginary, i.e. the light oscillations do not reach the receiver. The relation (8.93) is submitted in Fig. 8.5. At $\beta = 0.6$ and 0.8 $\bar{c}_\upsilon = 1.2$. The maximum value is at $\beta = 1/\sqrt{2}$. The greatest speed of propagation of light between moving objects is $\bar{c}_{\upsilon\max} = 0{,}5(\sqrt{2}+1) = 1{,}207$.

At small speeds the value β^2 in a denominator (8.93) can be neglected, then we obtain

$$\bar{c}_\upsilon = 1 + \beta . \qquad (8.95)$$

This result coincides with the cases of motion of the source or the receiver of oscillations in media. The expression (8.95) testifies, that at small velocities the light speed is added with a relative speed υ of approach of the source and the receiver and is deducted, if they remove from each other. As follows from the graphic in Fig. 8.5, this property is fair at $\beta < 0.3$, and it can be used approximately up to $b <$ 0.6. At large speeds β it is necessary to take into account a precise relation (8.93) for the velocity of propagation of electromagnetic oscillations.

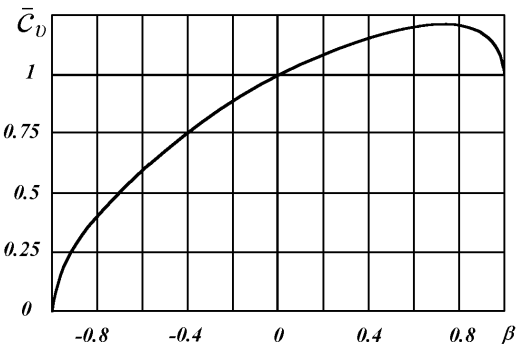


Fig. 8.5. Normalised speed of light $\bar{c}_\upsilon = c_\upsilon / c_1$ between bodies, which move from each other with normalised speed β.

At removing a source from the receiver ($\beta < 0$), speed of light between them is less than c_1. At removing with by light speed the velocity of propagation of light between the source and the receiver becomes equal to zero. This result has the important value for cosmology. According to the hypothesis "of large explosion" the far galaxies are removed from us with speed, close to the speed of light. The speed of light propagation from them of c_υ can be very small, therefore, the time necessary for propagation of light is $t_R = L/c_\upsilon$, where L is the distance up to galaxy, appears, to be considerably more than the time, past from the moment of "explosion" $t_S = L/c$. That is we can see a galaxy, which has not exist yet. This inconsistency is a serious test of the hypothesis "of large explosion".

It is necessary to note, that the c_υ value, entered by us, is not a usual speed and it cannot be combined with velocities of bodies motion. It links the distance between a relatively moving source and the receiver, and the period between a creation of oscillations and their arrival (see Fig. 8.4). Thus the distance l it is considered at the moment of radiation of oscillations. This value characterizes the time of propagation of action between two bodies. It is impossible to draw an analogy between value c_υ and speed of a sound a, or speed of propagation of waves in the water. In the last cases there is a material media, which motion is possible to observe and measure, and it is possible to fix the motion of bodies relatively it. In case of electromagnetic interaction, such media does not exist (it is not detected). Therefore, it is impossible to perceive value c_υ as speed of any material objects.

At small speeds of a source motion relatively the receiver according to (8.95) their speed geometrically is added to the propagation velocity electromagnetic oscillations. When the source approaches the receiver with velocity υ, the light between them is spread with speed $c_1 + \upsilon$, but during removing the light reaches the receiver with speed $c_1 - \upsilon$. This is confirmed by the observation of by the Jupiter shading of a satellite Io from different points of the Earth orbit, i.e. when the Earth approaches the Jupiter or removes from it. Due to the difference of observable times of shading Io the astronomer Olaf Remer defined the speed of light [47,48] in a 1676. In 1969 B. Wallace [122] noticed that at probing of Venus, the speed of radio waves is added to the speed of Venus relatively the Earth.

In Michelson's experiments [4] and other investigator's works the receiver and source did not move relatively one another, therefore the difference in speed of light was not found. The principle of the light speed constancy, at a relative motion of the receiver and source put in the basis of a Theory of relativity, is inconsistent.

8.7. THE PRINCIPLE OF THE RELATIVITY AND ITS MEANING

This principle is one of the bases of the relativity theory. It has different determinations and different understanding [71]. For example, we will put the formulations, used by the scientists of different orientations, in discussing the relativity theory. V.I. Sekerin cited the determination [48] of the book by Landsberg "Optics": "All processes of a nature flow past uniformly in any inertial system of a reference". The inertial system of a reference is a system of bodies, which move under inertia, i.e. without acceleration. A.M. Shalagin [73] considers, that the most valid determination of the principle of relativity is given in a textbook [26]: "... All laws of nature are identical in all inertial systems of a reference ". The second determination supposes a dissimilarity of processes in different systems, but it superimposes to use the identical the laws, describing these processes. As we have shown, just this position is used in the theory of relativity. The distinguished ac-

tions between the relatively moving bodies and the resting ones are described by identical expressions. Francly speaking, there is a deviation from the principle of relativity here, as the interactions not inside an inertial system, but between different systems are considered.

Despite of the different formulations of a principle of relativity it means that the linear motion mustn't change anything inside a system. A classical example of a principle is the cabin of a vessel or compartment of a train, which moves with a constant speed. If their windows are closed, if they are airtight, if they are supplied with an impenetrable screen for electrical action, it is possible to agree with such formulation of a relativity principle which is impossible to define: whether the cabin moves or rests. But when the surrounding subjects are seen from windows, when the air flows through the compartment, when the light of any source or the radio waves are received in a cabin, when in the compartment there are charges or currents able to interact with charges or currents, located on ground, then it is possible to define motion of the observer, the value of speed and it a direction by any of these phenomena.

So, the normalised formulation of the relativity principle is fair for a system of material bodies, which does not interact with the bodies of another moves relatively system. Automatically, a principle of relativity follows from a condition of the interaction absence, for the behaviour of bodies of a moving system will be the same, as in a resting system, because they are not acted by the other ones. Thus, it is necessary to notice, that the principle of relativity will be realized for an accelerated system of material bodies, isolated from the external actions. For example, in the freely dropping lift or in a satellite, on a planet etc. all motions happen as if they rest, and gravitational actions do not appear. In this case, the insulation from a gravitational field of bodies of a system consists in the fact that the speed brought from it by all bodies is identical and it does not change a picture of the objects motion in a system relatively one another.

The principle of relativity has only historical interest. It has appeared when the interactions by the forces were described. Then there were problems in different singularities of the motion and the interaction of bodies. For example, the law of inertia is closely connected with it, i.e. the first law of mechanics was entered. By it a body, on which the forces do not act, save rest or linear and uniform motion. In other words, the behaviour of bodies will not have differences, by which it is possible to define their motion.

Apparently, E. Mach for the first time has noticed that many our laws and principles exist in that way only by virtue of our determinations. So, relatively the first and the second laws of mechanics, given by Newton, in his work "Mathematical Beginnings of Natural Philosophy", E. Mach has told the following [36]: "It is not difficult to notice, that the first and second laws are already given in the previous determinations of force. According to these determinations, without force there is no also acceleration, and, therefore, there is only rest or linear and uniform motion. Further represents completely unnecessary tautology after acceleration is determined as a measure of force, once again to tell, that change of movement is proportional to force".

Let's return to a principle of relativity. After defining the action (see chapter 2) as the acceleration of a body, and the force, as one of the action measure no problems arise, which would require the applications of a principle of relativity. It is clear without it: if the bodies move is uniform and rectilinear, and there is no action on them, they will behave as if they are not forced, i.e. by only behaviour of bodies, without communication with the other bodies, we can not tell, if they move or rest. Our problem is to detect the actions on a considered body and their registration, instead of following a principle of relativity or other principles.

CHAPTER 9

NEARLUMINAL MOVEMENTS INSIDE OF «BLACK HOLES»

9.1. CONCEPT OF A «BLACK HOLE» AS RATIO OF PARAMETERS

Let’s consider the action of attracting centre on a particle, removing from it along the radius particle with velocity υ_0 on the radius R_0. Let's define the size of radius, at which the particle will be come off from the attracting centre. For the interaction by the classical law according to (4.83) when $R\to\infty$ and velocity $\upsilon\to 0$, we have

$$R_0 = -\frac{2\mu_1}{\upsilon_0^2}. \qquad (9.1)$$

Such interaction is possible, if the radius of attracting centre $R_c < R_0$. If the velocity of a particle υ_0 is equal to the light speed, a radius (9.1) in CTR is called gravitational:

$$R_g = -\frac{2\mu_1}{c_1^2}. \qquad (9.2)$$

The particles with light speed at radiuses equal to R_b and larger than R_g will be come off from the attracting centre, and the particles, located on $R_0 < R_g$ must have superluminal velocity to be come off it. According to (4.83) the maximum radius, on which the particles with light speed can be removed from attracting centre at υ_r (R_{max}) = 0, will be

$$R_{max} = \frac{R_g}{R_g / R_0 - 1}. \qquad (9.3)$$

The particles reach R_{max} at zero velocity, then return to attracting centre and drop on it.

In the frameworks of CTR it was represented, that the light consists of particles, on which the gravitation actions the same as on usual ponderable bodies. Therefore, with a radius of a star $R_c < R_g$ the particle of light removed from its surface, can not leave the orbit with a radius R_{max}, and such star at distances larger than R_{max} from it will be unobservable. That is, in the firmament it will look like "a black hole".

We have considered purely radial movement. In case of orbital movement of a particle with a kinematic moment h the gravitational radius R_g represents a radius of a pericentre of parabolic trajectory $\alpha_1 = -0.5$ (see definition α_1). That is, the light speed is the second space velocity at this radius. Therefore, at any movement of a particle with light speed at radiuses $R < R_g$ it can not be come off from the attracting centre. After a substitution of value μ_1 in (9.2) the gravitational radiuses according to (4.64) during a gravitational action receives a sight

$$R_{gg} = -\frac{2G(m_1 + m_2)}{c_1^2}, \tag{9.4}$$

and according to (107) at electrical action:

$$R_{ge} = -\frac{2q_1 q_2 (m_1 + m_2)}{c_1^2 m_1 m_2}. \tag{9.5}$$

We considered a question about "a black hole" on the basis of a classical interaction. At the gravity action in view of final speed of its propagation, by analogy with (4.58), the force of gravitation will determine by the law

$$\vec{F} = -\frac{G m_1 m_2 (1-\beta^2)\vec{R}}{\left\{R_{12}^2 - [\vec{\beta}\times\vec{R}_{12}]^2\right\}^{3/2}}. \tag{9.6}$$

In this case the particle having a velocity of light, as we have already repeatedly showed, moves without a change of velocity. Therefore, it will leave the attracting centre on infinity with the same velocity of light. At this interaction, according to (4.80), with the initial velocity v_0, the particle will leave infinity, if an initial radius

$$R_0 = -\frac{2\mu_1}{c_1^2 \ln(1-\beta_0^2)} = -\frac{R_g}{\ln(1-\beta_0^2)}. \tag{9.7}$$

At small β_0 formula (9.6) coincides with (9.1). However, with approach of initial velocity to velocity of light ($\beta_0 \to 1$), the radius is $R_0 \to 0$, i.e. the light particles from a surface of a body with any radius will leave the attracting centre on infinity. Therefore such body will not be represented by a «black hole».

For a conclusion about existence of «black holes» it is necessary to have, as minimum, two conditions. At first, the particles of light must represent ponderable particles. As we have shown in a chapter 8, light is the action, but it is not a body. Therefore, it cannot be shown as particles of light. Secondly, it is necessary to know the velocity of gravitational action. Now there are no experimental data

about light speed of its propagation, so, there are no bases for the statement about the existence of «black holes». Nevertheless, we will consider, in what situations at different hypothetical statements the «black holes» (BH) are possible:

Nature of light	Velocity of propagation of gravitational action	
	Infinite	Light
heavy particles	BH	–
Electromagnetic action	–	–

From the given table it is seen, that the «black holes» are possible only at the classical law of gravitation. As in the General Theory of Relativity the light speed of gravitation is accepted then the introduction of «black holes» in it frameworks is an unconditional error [117] at any hypothetical statements.

In this connection we do not consider the concept a «black hole» as some object of nature, but as a certain ratio of parameters as a radius R_g. Let's call it a light radius, as the particle, attracted from infinity, by classical law of interaction, by reaching this radius, gains the velocity of light c_1.

9.2. OBJECTS MICRO AND MACROCOSM AS "BLACK HOLES"

If the radius of an attracting centre is $R_c < R_g$, the movement of an attracted particle can pass inside of a «black hole», i.e. the radius of a pericentre of the orbit can be $R_p < R_g$. It corresponds to the parameter $\alpha < -1$. One of such trajectories at $\alpha = -1.12$ (see Fig. 5.5) was shown in a chapter 5. It is considering interaction of elementary particles, it is possible to see, that the parameter α can be less (-1). For example, if the radius of a pericentre will be equal to a radius of a proton $R_p = R_{pr} = 2.817 \cdot 10^{-13}$ cm, then, according to (9.5) the parameter of interaction of a proton with an electron is $\alpha = -1.956$. There are different representations of the sizes of elementary particles, for example, the authors [83] give the following size of a radius of a proton $R_{pr} = 1.023 \cdot 10^{-16}$ cm. In this case, the parameter of interaction can reach value $\alpha = -2754$.

In far space the set of objects is observed, which properties explain by a large mass and small sizes. Now, they are interpreted as «black holes» or neutron stars. Widely, it is known that there are stars of a different density. The value of a density varies in large ranges and its limits of limitation are unknown. The astronomical objects with a radius, smaller than R_g in many times, i.e. with the parameter $\alpha << -1$ are possible. If the gravitational action is spread with final velocity, the action of such objects will be subjected to the law (9.6). Therefore, the parameters movement of bodies round them, if they are known, will allow defining the velocity of gravity propagation.

The shown examples from micro and macrocosm show, that the research of movements at the values of interaction parameter $\alpha < -1$ represents a valuable interest, therefore, it will be made below. In chapter 5 we have considered its possi-

ble paths at variations of a trajectory parameter $-1 \le \alpha_1 \le 0$ and normalised the velocities in pericentres $0 \le \beta_0 \le 1$. There were established three kinds of trajectories, depending on value β_p in relation to parameter β_{pc}, defined by expression (5.41): 1) sub-lightspeed trajectories at $\beta_p < \beta_{pc}$; 2) trajectories of acquisition of a particle into a circular orbit at $\beta_p = \beta_{pc}$; 3) trajectories with light velocity at the pericentres $\beta_p \to 1$. As at $\beta_p \to 1$ the radial velocity (5.39) has a singularity for calculation of trajectories of the third type, we used the radial velocity (5.46), normalized to parameters at an intermediate point: R_0 , υ_{t0}. As well as in chapter 5, at $\alpha < -1$ equations (5.2) and (5.46), and also (5.16), are integrated on two segments: $\overline{R} > 1$ and $\overline{R} < 1$. The values of parameter of trajectory a varied as follows: α_1^0 = -2; -4; -10. The parameter of trajectory α_1^0, normalised to the velocity at the pericentres, i.e. to light, is determined according to (5.47), and the parameter of interaction, according to (5.45) is equal to

$$\alpha = 2\,\alpha_1 . \qquad (9.8)$$

The values of normalised tangential velocity β_{t0} varied so: β_{t0} = 0.9; 0.7; 0.5; 0.3; 0.1. The lower value β_{t0} was set so, that $\alpha < -1$. Therefore, all trajectories shown here differ from trajectories in chapter 5 and in works [59, 60], where they were calculated at $\alpha \ge -1$.

The values of normalised radial velocity varied so: β_{r0} = 0; 0.1; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9. The value β_{r0} = 0 corresponds to a premise of an initial point at the apocentre, as the integration is conducted from $\overline{R}$ = 1 up to $\overline{R} = \beta_{t0}$. It will be final trajectory with the least radius of an apocentre. With further increase of β_{r0} the radius of the apocentre will increase. Except a segment of integration $1 \ge \overline{R} \ge \beta_{t0}$ there is a segment $1 \le \overline{R} \le \overline{R}_a$, where the radius of the apocentre $\overline{R}_a$ is determined from a condition $\upsilon_r^0 = 0$. With further increase of β_{t0} the radius of the apocentre tends to infinity. From a condition $\lim_{R\to\infty} \overline{\upsilon}_r^0 = 0$, according to (5.46), the value β_{r0} will be written

$$\beta_{rp} = 1 - \beta_{to}^2 - exp\frac{2\alpha_1^0 \beta_{t0}^2}{\sqrt{1-\beta_{t0}^2}} . \qquad (9.9)$$

With radial velocity $\beta_{r0} = \beta_{rp}$ the trajectory becomes parabola-like. It is necessary to note that the expression (9.9) is identical to expression (5.50). With further increase of β_{r0}, the trajectories become hyperbola-like and when $\beta_{r0} = \sqrt{1-\beta_{t0}^2}$, they degenerate in a direct line, on which the particle moves with light speed.

After the integration the results were conduced to the parameters at the pericentres. The calculations were executed by assistance of the packet MATHCAD. The example of the program is given in Appendix 3, and calculated trajectories are given in Appendix 4.

9.3. TRAJECTORY AT VARIATIONS OF RADIAL VELOCITY

As we havealready noted, at the integration on a segment $\overline{R} < 1$ during decreasing $\overline{R}$, the radial velocity (5.46) decreases and becomes equal to zero at $\overline{R} = \beta_{t0}$, i.e. this point is a pericentre, which radius

$$R_p = R_0\beta_{t0} = R_0\upsilon_{t0} / c_1 . \qquad (9.10)$$

As all these trajectories are subjected to a conservation law of kinetic moment, (4.66) $h = R_0\upsilon_{t0} = R_p\upsilon_p$, the velocity at the pericentres can be written

$$\upsilon_p = \upsilon_{t0}R_0/R_p \qquad (9.11)$$

After a substitution of a pericentre radius from (9.10) we obtain the light velocity of a particle at the pericentres $\upsilon_p = c_1$.

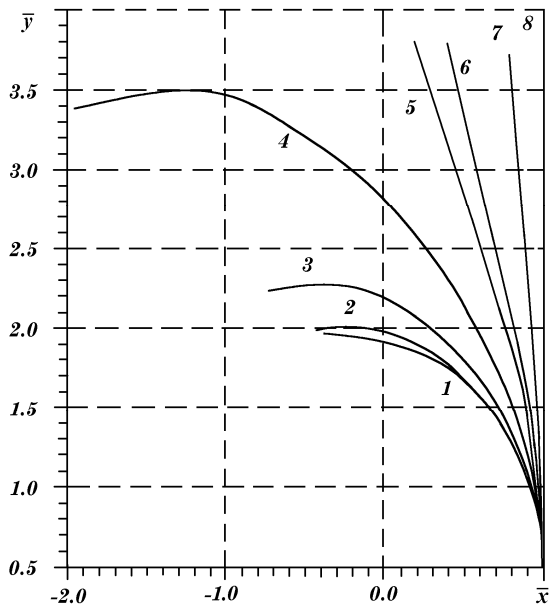


Fig. 9.1. Trajectories inside the «black hole» with the following parameters:

α_1^0 = - 2; β_{t0}= 0.5; α = - 2:

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
β_{r0}	0	0.1	0.3	0.5	0.659	0.7	0.8	0.866
$\overline{R}_a / \beta_{r\infty *}$	2	2.031	2.355	3.909	0*	0.419*	0.807*	1*
φ_a°	100.8	101.6	108.0	119.9	129.7	110.5	95.9	90
$\overline{T}_{0.5}$	3.879	3.987	5.053	10.473	---	---	---	---

The changes of parameters α_{10} , β_{t0} in the considered ranges have given the changes of interaction parameter in the area $-18 \le \alpha < -2$. Let's consider the trajectories at α = - 2 and β_{t0} = 0.5 (Fig. 9.1) when we change radial velocity β_{r0}. In this Figure and the other ones with the purpose of improving the image of trajectories the different scale on axes $\overline{x}$ and $\overline{y}$ is used.

When $\beta_{r0} = 0$ particles, moving from the pericentre ($\bar{x} = 1$, $\bar{y} = 0$) round attracting centre ($\bar{x} = 0$, $\bar{y} = 0$) during $\bar{T}_{0,5} = 3.879$ come in the apocentre $\bar{r}_a = 2$. It is necessary to note, that all considered results, including in Figures and in Appendix 4, are normalized to parameters in pericentres c_1, R_p. From here a halfcycle of orbit

$$T_{0,5} = \bar{T}_{0,5} R_p / c_1 . \qquad (9.12)$$

For this halfcycle polar coordinates φ varies at 100.8^0, i.e. the angular period of movement along the orbit makes 201.6^0, that is rather less than 2π. With increase of the radial velocity β_{r0} the radius of the apocentre $\bar{R}_a$, the angular period and the time of movement along the orbit is increased. When $\beta_{r0} = \beta_{rp} = 0.659$ radiuses $\bar{R}_a \to 8$ (see trajectory 5 in Fig. 9.1), i.e. the trajectory becomes parabola-like. Unlike a parabola, it moves to the infinity at $\varphi_a = 129.7$, not at 180. With further increase of β_{r0} the angle of deviation of trajectories decreases, and at $\beta_{r0} = 0.866$ we obtain a direct trajectory 8, on which the particle moves with light speed.

When $\alpha = -2$ the radius of a «black hole» is twice bigger than the radius of the pericentre. Therefore, the first trajectory ($\bar{R}_a = 2$) is completely inside a «black hole» and only at the apocentres concerns its surface, and the particle on trajectories *2*, *3*, and *4* periodically is pulled out of it. The particle on hyperbola-like trajectories *6÷8*, moving from infinity, enters inside a «black hole», moves there and again goes to the infinity. The particle on parabola-like trajectory *5* can be pulled out from the area of a «black hole». But in all cases the particle at the attracting centre moves with the velocity, approaching to light speed.

As it is seen from Fig. 9.1, and also from expression (5.46) the trajectories are determined in three parameters: $\alpha_1{}^0$, β_{t0}, β_{r0}. However, at all trajectories at the pericentres, the velocity is identical and is equal to c_1. Therefore, at the pericentres the trajectories are determined by two parameters. So, the parameter of interaction α, according to (9.8) and (5.47), is equal to $\alpha = 2\alpha_{10}\beta_{t0}$. If the parameter of interaction μ_1 is known, the parameter α has a radius of a pericentre

$$R_p = \frac{2\mu_1}{\alpha c_1^2} . \qquad (9.13)$$

The radius of a pericentre R_p and the velocity c_1 in it completely determine the movement of a particle. So, the value of radial velocity β_{r0} does not influence on the formation of trajectory at the pericentres. It gives the idea, that the particle which has come nearer to attracting centre with the velocity c_1, can then be removed on any trajectory, including on light one 8 in Fig. 9.1. Therefore, the point of the pericentre is unstable and the small disturbances in it can conduce to essential changes of a particle trajectory. With reference to a microcosm it can be the reason of the radioactivity of elements.

9.4. TRAJECTORY IN A CASE OF DIFFERENT TRANSVERSAL VELOCITIES

In Fig. 9.2 the trajectories at the same parameter α_{10} but with greater transversal velocity, $\beta_{r0} = 0.9$ are presented. The parameter α in this case has alsom increased, and the radius of a «black hole» in 2.8 times exceeds a radius of a pericentre of trajectory. At $\beta_{r0} = 0$ trajectories, in the comparison with similar ones in Fig. 9.1, has the radius of an apocentre and the angular halfcycle φ_α in 1.5

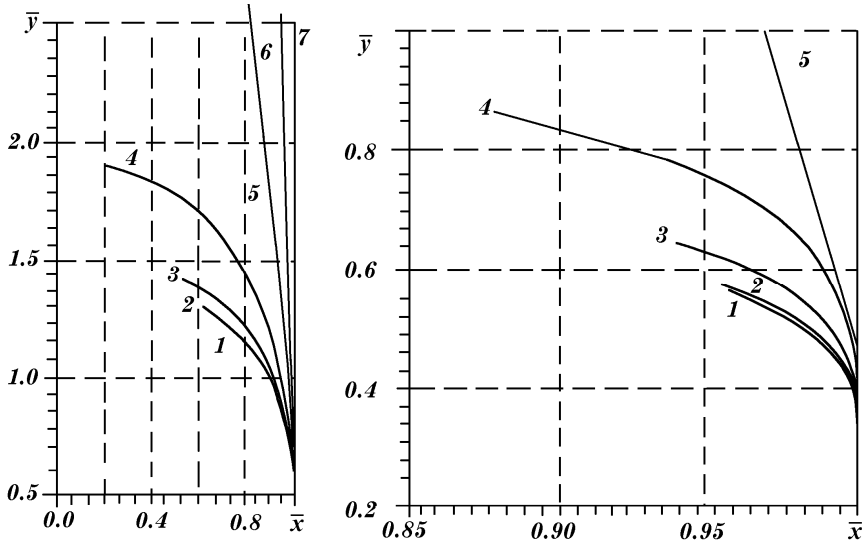


Fig. 9.2 (at the left). Trajectories inside a «black hole» with following parameters:

$\alpha_1^0 = -2$; $\beta_{t0} = 0.7$; $\alpha = -2.8$:

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*
β_{r0}	0	0.1	0.3	0.5	0.668	0.7	0.714
$\bar{R}_a/\beta_{r\infty *}$	1.429	1.437	1.521	1.913	0*	0.83*	1*
φ_a°	63.7	64.2	61.1	83.8	117.2	93.6	90
$\bar{T}_{0.5}$	1.6	1.625	1.864	2.934	---	---	---

Fig. 9.3 (on the right). Trajectories inside the "black hole" with following parameters:

$\alpha_1^0 = -2$; $\beta_{t0} = 0.9$; $\alpha = -3.6$:

N°	*1*	*2*	*3*	*4*	*5*
β_{r0}	0	0.1	0.3	0.4	0.435
$\bar{R}_a/\beta_{r\infty *}$	1.111	1.113	1.139	1.232	0*
φ_a°	30.5	30.9	34.4	44.5	110.5
$\bar{T}_{0.5}$	0.58	0.552	0.673	0.942	---

times smaller, and the temporary period is less in 3 times. The behaviour of remaining trajectories is similar. As is seen, trajectories *1, 2, 3, 4* in Fig. 9.2 are inside a «black hole», as $\overline{R}_a < \overline{R}_g = -\alpha$.

In Fig. 9.3 the trajectories are given at greater transversal velocity are. When $\beta_{r0} = 0$ radiuses $\overline{R}_a = 1.111$, i.e. it does not differ from the radius of a pericentre, and the angular halfcycle is equal to 30.5^0. If the particle moved in such orbit, per one turn it would make six hops on the altitude 0.111 radiuses. As follows from Fig. 9.3, all closed orbits are deeply inside a «black hole»; the greatest radius of the apocentre is in 3 times less $\overline{R}_g = -\alpha$. So, the increase of transversal velocity β_{r0} conduces to decreasing a hop size of both angular and time periods.

9.5. TRAJECTORY AT CHANGE OF INTERACTION PARAMETER

The two times increasing of a module parameter α_1^0 (see Fig. 9.4) conduces to the same increasing of the interaction parameter α in comparison with a situation shown in Fig. 9.1. At the same parameter of interaction μ_1 the radius of the pericentre, as follows from (9.13), will be 2 times less. However, as it is seen from Fig. 9.4, the relative radius of the apocentre $\overline{R}_a$ at $\beta_{r0} = 0$ has not varied (trajectory *1*), though the halfcycles, depending on the angle and time, have decreased. The parameter α_1^0 similarly influences on the other trajectories. Here the trajectories *1÷5* are also inside a «black hole».

In Fig. 9.5 the trajectories are presented with much greater parameter α_1^0. In a comparison with Fig. 9.1 the parameter α has increased in 5 times, but the radius of a pericentre R_p has decreased in 5 times. However, the size of the apocentre radius $\overline{R}_a$ has not varied for first orbit, and for remaining the small decreasing $\overline{R}_a$ with growth of radial velocity β_{r0} is observed. The halfcycles depending on the angle φ_a and time $\overline{T}_{0,5}$ have a smaller size in a comparison with trajectories at α_1^0 = - 4 and (-2). Thus, the increase of the module α_1^0 does not influence much on a relative radius of the apocentre, but reduces the halfcycles φ_a and $\overline{T}_{0,5}$. At the same time the absolute radius of the orbit decreases. As it is seen, the variations of the parameters α_1^0 and β_{t0} result approximately in identical variations of trajectories, but it the type of trajectories does not change. With small radial velocities, the trajectories are final, with a halfcycle smaller than π, and they are hyperbola-like at large radial velocities. The final trajectories can be inside the «black hole», and can go out of it.

At the pericentres the trajectories, , are factually characterized by one parameter α, which is possible to consider as a "depth" of the pericentre immersing

in the «black hole», i.e. parameter $(-\alpha) = R_g = R_g/R_p$ shows, how many times the radius of the «black hole» is more than the radius of the pericentre. In calculations it varied from $\overline{R}_g = 2$ (see Fig. 9.1) up to $\overline{R}_g = -18$ at $\alpha_1^0 = -10$ and $\beta_{t0} = 0.9$. In accordance with the increase R_g the final trajectories come nearer to the attracting centre, i.e. become less prolonged, and immerse stronger in the «black hole». Their halfcycle decreases on the angle and time. And the hyperbola-like trajectories become less curved and come nearer to a light trajectory.

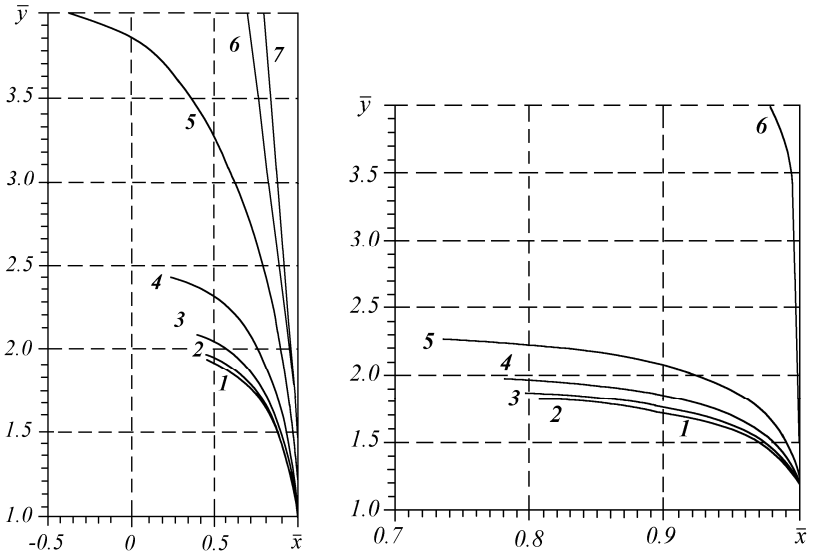


Fig. 9.4 (at the left). Trajectories " inside a black hole " with the following parameters: $\alpha_1^0 = -4$; $\beta_{t0} = 0.5$; $\alpha = -4$:

N°	*1*	*2*	*3*	*4*	*5*	*6*	*7*
β_{r0}	0	0.1	0.3	0.5	0.7	0.8	0.807
$\overline{R}_a/\beta_{r\infty *}$	2	2.011	2.114	2.436	4.025	38.93	0^*
φ_a°	76.8	77.2	79.2	84.2	95.6	108.6	108.8
$\overline{T}_{0.5}$	2.708	2.74	2.975	3.714	7.681	196.5	$2.99\ 10^4$

Fig. 9.5 (on the right). Trajectories inside a “black hole” in case of following parameters: $\alpha_1^0 = -10$; $\beta_{t0} = 0.5$; $\alpha = -10$:

N°	1	2	3	4	5	6
β_{r0}	0	0.1	0.3	0.5	0.7	0.864
$\overline{R}_a/\beta_{r\infty *}$	2	2.004	2.037	2.126	2.384	0^*
φ_a°	66.1	66.2	66.9	68.4	72.0	97.1

$\overline{T}_{0.5}$	2.121	2.133	2.188	2.334	2.76	---

On all trajectories the particle, approaching the pericentre, tends to the light speed. After reaching the pericentre, there is uncertainty in movement. As during the equations integration the radius of the pericentre according to (9.10) tends to such value, which is equivalent to the light speed in the pericentres, the movement of a particle after the pericentre along the unbent light trajectory (see trajectory *8* in a Fig. 9.1) is most probable.

9.6. MULTANGULAR ORBITS

The movement of a particle after reaching a pericentre is unstable. The minor deviation of velocity from light into direction of decreasing will conduce to a curvature of movement of a particle. Represents significant interest how limiting transition of speed of a particle with its approach pericentre is actually carried out. If the velocity of a particle will differ from c_1 ng parameter, which represents trajectory on a small δ, this value can be that missi of a particle after passing a pericentre. That is, to each trajectory there will correspond parameter δ. Therefore semiperiodes segments of trajectory can be to be connected and receive continuous trajectory of a particle. In case of $\varphi_a = \pi/k$, where k - integer, the particle during a full revolution will make k number of periods and it the trajectory will be constant in space. Such situations can be much. We have found them for case α_1^0 = -2 and β_{t0} = 0 in case of k = 2, 3 and 4.

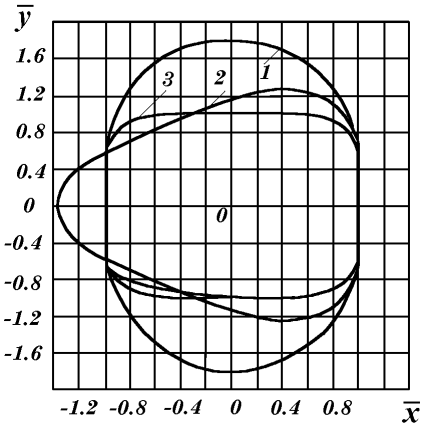


Fig. 9.6. Stables multangular trajectories inside of a «black hole» in case of following parameters: α_1^0 = - 2; β_{r0}= 0.

N°	β_{t0}	α	$\overline{R}_a$	φ_a°	$\overline{T}_{0.5}$	$\overline{T}_c$
1	0.552	-2.206	1.815	90	3.051	12.2
2	0.722	-2.888	1.385	60	1.454	8.72
3	0.815	-3.26	1.227	45	0.952	7.62

In a Fig. 9.6 such are submitted stables in space of orbit. On orbit *1*, have the ellipse-like form, the particle is gone round attracting centre, which position coincides centre of orbit, instead of with a focal point, biased relatively centre. Relative period of cyclical movement of a particle on such orbit $\overline{T}_c$ = 12.2. On direct segments the particle is gone almost with light velocity, and on curved - with least. Orbits *2* have three direct segments, and at orbit *3* - four. The movement of a particle happens on a triangle and square with rounded off tops. In case of large values of parameter α the tops of such polygonal orbits will be more acute.

The small increase of velocity of a particle at value δ will cause to reaching by it velocities of light, and it will abandon attracting centre. This process can be observed in a microcosm and, probably, such orbits stipulate the radioactivity of elements. Multangular orbits can be observed in a macrocosm, if the velocity of propagation of gravitation is final. The star on such orbit will give irregular radiation: on direct segments because of smaller time of presence it will be perceived less bright, and on angles - brighter. Therefore perceived light should be modulated definitely.

CHAPTER 10

SUPERLUMINAL MOTIONS

10.1. EXAMPLES OF SUPERLUMINAL MOTIONS

The position about impossibility of superluminal motions in modern physics was formed, mainly, in the interpretation of two experimental facts. Unsuccessful attempts to measure velocity of the Earth relatively luminiferous ether led to the idea, that the addition of the light speed and the motion velocity of the Earth in the exotic way, when the light speed remains constant, is possible. The second group of facts is connected with the diminution of an electron acceleration approaching its velocity to the speed of light, which was explained as the increase a particle mass and its rushing to the infinity with approaching to the light speed. As a particle mass must not be infinite, speed of light will be unattainable. If the charged particle gains speed of light in another way, its mass becomes infinite. Therefore, the exclusion of superluminal motions must be universal.

This logic and interpretation of the facts were formed under the influence of Lorentz's transformations. After they were given a transformation, sense of space, time and other parameters at transition between reference systems, the interpretation became logically closed.

Actually, the ether as media, in which light is spreaded, does not exist. The diminution of electron acceleration with its speed growth is explained by the fact that the force of action on it depends on velocity. We have defined the value of

force and with its help have calculated the interaction between the charged bodies of both magnets. Space and time do not vary at the transition from a motionless body to a moving one. Thus, the exclusion of superluminal motions is not necessary, as it has no a basis.

The motions with superluminal speed exist in the world, surrounding us. For example, if two accelerators, located at points *A* and *C* at the distance $2l$ = 598 ь (Fig. 10.1.), will emit one another towards the particles with velocity u = 299000 km/s, then in one microsecond each from particles will pass a distance l = 299 m, and they will meet at the point *B*. That is, the relative distance $2l$ is passed by particles during $t = 1 \cdot 10^{-6}$ seconds. Velocity of one particle relatively another is $\upsilon = 2l/t$ = 598000 km/s, that makes almost double speed of light. Such situations are observed in the experimental installations with the counter bundles of particles. In these installations the interactions of particles happen at relative velocity of the particles, reaching double speed of light. Many physicists agree with this situation [29, 75].

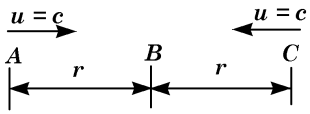


Fig. 10.1. Double speed of light in relative motion.

In 1963 G.D. Lomakin [30] advanced an interesting proof of superluminal particles existence. Determinating the lifetime of μ - mesons, the passed way by them is divided into speed of light. Such time of life is more, than the lifetime of slow μ - mesons. As the result of experimental data analysis Lomakin has come to a conclusion, that a longer way, passed by these particles, proves, that their velocity is more than the speed of light.

It is conventional, that the space particles enter in the atmosphere of the Earth with the light speed. As the Earth moves together with the Sun along its orbit with velocity 300 km/s, the excess above speed of light will make only 0,001 part. However, in space there are objects, which velocity is compared to speed of light. If such objects generate the flows of particles, their relative velocity can much exceed the speed of light.

A characteristic indication of superluminal motion of a particle is the its light radiation, known under a title of Cerenkov's radiation (Fig. 10.2). Here flashing particle, moving with superluminal velocity $\upsilon > c$, at the moment $t = 0$ is at the top of a cone. Earlier in a moment t_1 it was at a distance $b_1 = \upsilon t_1$, and light, radiated by it, reached an orb by a radius $a_1 = ut_1$. At a consequent moment $t_2 < t_1$ the particle was at the distance $b_2 = \upsilon t_2$ and light, radiated by it, reaches the sphere with a radius $a_2 = ut_2$. It is fair for any moment. Therefore, the signals, radiated by a particle, along the own way of the motion are focused on a conic surface, which apex angle is determined so:

$$\sin\alpha = \frac{a_1}{b_1} = \frac{a_2}{b_2} = u/\upsilon. \qquad (10.1)$$

The surface of a cone, thus, represents the concentrated front of action. In case of sound oscillations this cone is known as Mach's cone, and the front of action is called a shock wave. For light speed $u = c_1$, the angle between velocity of a particle and normal to a cone $\hat{\beta} = \arccos(c_1/\upsilon)$ is known as Cerenkov's angle radiation. Such radiation is observed at passing by the particles with sub-lightspeed $\upsilon \approx c$ in fluid,

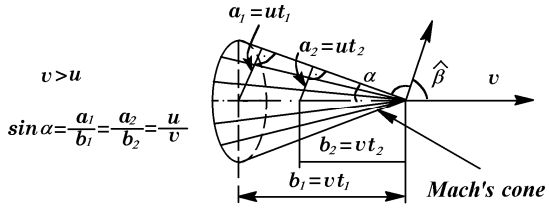


Fig. 10.2. Cerenkov radiation with the angle $\hat{\beta}$.

where speed of light $c_1 = c/n$, where n - index of refraction of fluid. At large n the sub-lightspeed of a particle becomes tens per cents more than the speed of light in media c_1. Such radiation is widely used in Cerenkov counters of elementary particles.

The existence of motions with superluminal velocity $\upsilon > c_1$ in media, but with smaller speed of light in vacuum $\upsilon < c$, conficts with STR. We have shown that all electrodynamics equations, including Lorentz transformations, depend on speed c_1. The existence of velocities at $\upsilon > c_1$, is determined for different medias various c_1 at, is the general rule. As the speed of light in vacuum is a particular case of media with $\varepsilon = \mu = 1$, then a special rule follows from the general one: the velocities of motion, which are more than the speed of light in vacuum, are possible.

Many investigators affirm [10, 15, 38, 72, 90], that the space particles, coming in the Earth atmosphere, create Cerenkov radiation. Such radiation can be observed at moonless night by the unaided eye. It is confirmed by the numerous experiments, that the observable meteors as an instantaneous striking in the sky coincide with the particles gushes of high-energy. So, according to Dobrotin. [15]: "Galbraith and Jellies have shown, that the flashes in the night sky coincide with the pulses in the counters and consequently are stipulated by Cerenkov radiation".

For explanation of this fact within the framework of the Theory of Relativity it is affirmed that the velocity of particles in this case exceeds the speed of light in the air, but they are less than the speed of light in vacuum. The stretched character of the coordination of observable superluminal motion with the Theory of Relativity is obvious. The speed of light in the air $c_1 = c/n$, where an index of refraction of an air according to [15] depends on its density

$$n = 1 + 2.9 \cdot 10^{-4} \cdot \rho/\rho_0,$$

i.e. the speed of light in an air on the sea level ($\rho = \rho_0$) differs from the speed of light in vacuum on

$$\delta = (c - c_1)\cdot 100\%/c = (n-1)\cdot 100\%/n = 0.029\%.$$

As with altitude density of the air ρ decreases, δ decreases, too. If to admit, that the velocity of particles υ does not exceed speed of light in vacuum, it must be in rigid limits $c > \upsilon > c(1 - \delta)$. This corresponds to the cone apex angle of superluminal radiation, distinguished from 90° on the value of the order δ, i.e. the cone degenerates in a plane. A characteristic property of Cerenkov radiation has a place only at an obviously expressed light cone. Such radiation is given in the references. Therefore, the observable Cerenkov glow testifies, that the space particles enter in the atmosphere of the Earth with velocity, greater than the speed of light in vacuum.

So high velocities of space particles can be stipulated by the different reasons. One of them is the gravitational attraction. Let's consider what velocities can give astronomical objects to a body, which, with zero initial velocity forced by gravitation, will come nearer to a surface of object. Calculated on (5.27), the parabolic velocities υ_{pa} are given in the table.

Object	m_1, kg	R, m	ρ, kg/m^3	υ_{pa}, m/s	Source
The Earth	$6.6\cdot10^{24}$	$6.37\cdot10^{6}$	$5.52\cdot10^{3}$	$11.75\cdot10^{3}$	[16]
The Sun	$1.98\cdot10^{30}$	$1.4\cdot10^{9}$	$1.41\cdot10^{3}$	$43.5\cdot10^{4}$	[16]
White dwarf such as a star of the Leyton	$2.5\cdot10^{30}$	$2\cdot10^{6}$	$6\cdot10^{10}$	$1.3\cdot10^{7}$	[16, 21]
Neutron star	$4.5\cdot10^{30}$	$8\cdot10^{3}$	$2\cdot10^{18}$	$2.7\cdot10^{8}$	[21]

As it is seen, they vary from 12 km/s for the Earth up to 270000 km/s for a neutron star. The velocities of bodies can be larger, which are forced by the quasars, kernel of galaxies or so-called candidates in "black holes". That is these velocities can exceed the speed of light. With velocities of such order, the bodies can be rejected from attracting centres or to exchange among themselves.

By new methods in astronomy - the radio- and x-ray telescopes in the last two decades many objects moving with velocities, large speed of light [35] are revealed. In the article [120] 30 strong extragalactic radiosources, which have compact components with visible tangential velocities large speed of light. One of the authors of the article, R.C. Vermaulen, in his other work considers statistical properties of superluminal components, which are noted in 66 galaxies, quasars and lacertides. The motions of jets, extensions of diffuse objects, relative motion of parts of breaking object, etc, are observed. For example, the observation of the radiokernel of quasar 3C395 on frequencies 5 and 1.6 GHz (centimetric range of waves) from 1979 to 1985 showed [111], that it consists of three components. The distance between the kernel and the slowly moving component makes 5 m/sec (arc milliseconds). The third component moves into direction of last one with superluminal velocity. Its angular rate is 0.64 msec/years.

The work consideres [106] the motion of two components of a source GRS1915 + 105 rigid X-rays from March 27 till April 30 1993. The unaccelerated deleting of components from each other with velocity, perpendicular to the line of observation and equal for each 1.25 and 0.65 c, is observed. Their relative velocity makes 1.9 c, i.e. reaches almost double speed of light.

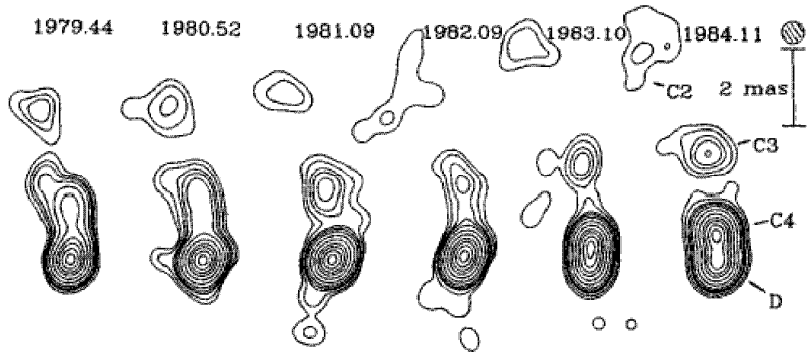


Fig. 10.3. Radioimages of quasar (Internet, according to John Biretta / Space Telecope Science Institute).

The radioimages of quasar 3C345 (Fig. 10.3) are fixed on the waves 10.7 GHz from 1979 to 1984 y. In it the velocities of jets exceed the speed of light in 7 times, if it is really exta-galactic and is removed on 1700 megaparsecs, as it is considered by the majority of astrophysicists. There are messages about velocities of motion, reaching 40 speeds of light. As in STR frameworks such motions are impossible, the attempts of explanation of observable appearances without violation of a principle of limiting speed are undertaken. The most widespread explanations [5, 106] of superluminal transversal velocity there are the selection of such a source direction of motion, at which its the sub-lightspeed will be considered as superluminal. The given approach, naturally, cannot refute the superluminal motions: the angle between the velocity of object and the direction of observation should be determined by the results of measurements. It shouldn't be calculated from the supposition, which they want to prove. The analysis of this approach, which is shown in the work [110], brings the author to a conclusion, that the observable superluminal motions represent serious test of the relativity theory.

So, the existing objects in a deep space can give bodies superluminal velocities and such velocities are noted. Moving bodies and jets of substance with similar velocities create flows of elementary particles, which velocity is also superluminal. They enter in the atmosphere of the Earth, create Cerenkov radiation and generate showers of secondary particles, which testify to huge energy of space particles. Millenium Twain [119] has paid attention, that such properties of elementary particles, as their mass, charge and radius can be coordinated with a magnetic moment and a pulse moment, if they represent a structure, rotated with superluminal velocity. He has come to a conclusion, that our entire world, both

macro and micro is superluminal, i.e. there are objects in it, which move with velocities larger than the speed of light.

It is necessary to note that in modern theoretical physics the concept "tachyons" [3, 19, and 39] is entered. There are hypothetical particles, which must satisfy STR. Their imaginary mass is postulated. The scientists investigate, how to break a principle of causality with the help of tachyons, i.e. the effect must precede the cause. A title "tachyons" means overtaking time. Unlike such fancy tachyons, we consider actual superluminal motions.

10.2. OBSERVATION AND INTERACTION AT SUPERLUMINAL VELOCITIES

Obtained as the result of calculation the superluminal velocities of far astronomical objects cause many problems. To decide them it is impossible without understanding, how, as the superluminal objects should be observed. The analysis of activities on explanation of superluminal motions [5, 35, 106, 110] shows, that there is no conventional method of their consideration. We think, that this problem contains many vague problems. Let's analyze the observation of such motions on an example of supersonic ones. Let source of sound *S* (Fig. 10.4) at a moment $t = 0$ moving in media with velocity $\upsilon > a$ is in a point *S*. In this media at the distance p there is a motionless receiver *R*. The sound oscillations propagated in an air with the velocity *a*, were imposed on the front as Mach cone with the apex angle α, equal,

$$\sin\alpha = a / \upsilon. \qquad (10.2)$$

Motionless relatively the air the observer *R* will hear at this moment a source at the point *S'*, located on a perpendicular to the front *SR*. The source has passed a distance *S'S* during time

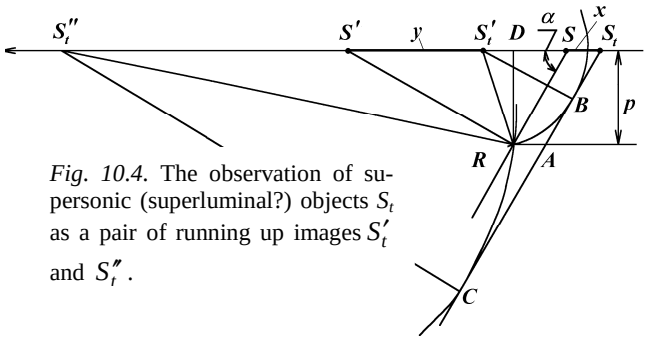


Fig. 10.4. The observation of supersonic (superluminal?) objects S_t as a pair of running up images S_t' and S_t''.

$$T = S'S/\upsilon, \qquad (10.3)$$

and the sound for this time was spread at the distance

$$S'R = T\,a = S'S/\sin\alpha. \qquad (10.4)$$

A bit later *t* the source has moved exactly to the point S_t, and Mach cone will take a position S_tA, where

$$SS_t = RA = \upsilon t = x. \qquad (10.5)$$

Here $x = \upsilon\, t$ is a symbol of the passed distance.

After passing front the receiver *R* will perceive a sound, which has reached it from a source found, as it is seen from Fig. 10.4, at once in two points S_t' and S_t''. At a moment t the sound will also reach points *B* and *C* accordingly, located on Mach cone S_tA. Thus, the receiver, after passing the front, will register a sound from two sources S_t' and S_t''. In case of light the observer of a superluminal source (if the analogy to a sound here is allowed!) will see it at once in two positions. Thus, after passing the front, the seen source at a point *S'* will bifurcate and further will be observed as two sources S_t' and S_t''. Let's define the motion velocities of two images, observed supersonic (superluminal?) source.

From a triangle $\Delta S_t'SR$ it follows:

$$S_t'R^2 = S_t'S^2 + RS^2 - 2S_t'S \cdot RS \cdot \cos\alpha. \qquad (10.6)$$

Let's express $S_t'S_t$ through $S_t'S$ and *x*:

$$S_t'S_t = S_t'S + x. \qquad (10.7)$$

The distance between the observable sources *S'* also S_t' can be written:

$$y = S'S - S_t'S. \qquad (10.8)$$

Let's express *RS* and *S'S* through the aim distance *p*:

$$RS = p/\sin\alpha; \qquad (10.9)$$

$$S'S = \frac{RS}{\cos\alpha} = \frac{p}{\sin\alpha \cdot \cos\alpha}. \qquad (10.10)$$

Substituting in (10.8) *S'S* and $S_t'S$ from (10.7), we obtain

$$y = \frac{p}{\sin\alpha \cdot \cos\alpha} - S_t'S_t + x. \qquad (10.11)$$

From a triangle $\Delta S_t'S_tB$, with allowance for $S_t'B = S_t'R$, follows

$$S_t'S_t = S_t'B/\sin\alpha = S_t'R/\sin\alpha. \qquad (10.12)$$

Whence, with allowance for (10.7), we discover

$$S_t'R = S_t'S_t \cdot \sin\alpha = (S_t'S + x)\sin\alpha. \qquad (10.13)$$

We substitute in (10.6) $S_t'R$ and *RS* from (10.13) and (10.9) accordingly. After transformation we have:

$$S_t'S^2 - 2S_t'S\left(\frac{p}{\sin\alpha\cdot\cos\alpha} + x\cdot\mathrm{tg}^2\alpha\right) + \frac{p^2}{\sin^2\alpha\cdot\cos^2\alpha} - x^2\cdot\mathrm{tg}^2\alpha = 0\,. \quad (10.14)$$

The solution of a quadratic equation (10.14) will be

$$S_t'S = x\ \mathrm{tg}^2\alpha + \frac{p}{\sin\alpha\cos\alpha} \pm \frac{1}{\sin\alpha}\sqrt{x^2\ \mathrm{tg}^4\ \alpha + 2xp\ \mathrm{tg}^3\ \alpha}\,. \quad (10.15)$$

By (10.8) and (10.10) we will express $S_t'S$ through *y*:

$$S_t'S = \frac{p}{\sin\alpha\cdot\cos\alpha} - y\,. \quad (10.16)$$

Then the distance of observable object y, depending on the objects distance *x*, aim distance *p* up to the observer and angle of Mach cone α, according to (10.15), will be written

$$y = -x\,\mathrm{tg}^2\alpha \pm \frac{1}{\sin\alpha}\sqrt{x^2\mathrm{tg}^4\alpha + 2xp\,\mathrm{tg}^3\alpha}\,. \quad (10.17)$$

Two signs $\pm$ determine two positions S_t'' and S_t' of the observable object, which at the moment *t* is at the point S_t (see Fig. 10.4).

So, the supersonic object, located at the point *S*, at the moment of overfilling front on the receiver *R* is observed at the point *S'*. Then it bifurcates: one object S_t' moves in the direction of the object *S*, and other S_t'' moves in the converse direction. Let's define velocities of motion of observable objects:

$$v_s = \frac{\mathbf{d}y}{\mathbf{d}x}\frac{\mathbf{d}x}{\mathbf{d}t} = v\frac{\mathbf{d}y}{\mathbf{d}x} = v\mathrm{tg}^2\alpha\left[-1 \pm \frac{xtg\,\alpha + p}{\sin\alpha\sqrt{x^2\mathrm{tg}^2\alpha + 2xp\mathrm{tg}\alpha}}\right]. \quad (10.18)$$

Here *p* is the aim distance, and $x = vt$, where *t* is the time counted from the beginning of the front arrival to the observer *R*. As it is seen from (10.18), at the initial moment ($x \to 0$) velocities of motion of observable objects will be infinite. With the course of time ($x \to \infty$) the velocity of an image in the direction of the object motion, with allowance for (10.2), will be

$$v_s' = v\cdot\mathrm{tg}^2\alpha\,(1/\sin\alpha - 1) = \frac{va}{v+a} \le a, \quad (10.19)$$

and in return

$$v_s'' = -v\cdot\mathrm{tg}^2\alpha\,(1/\sin\alpha + 1) = -\frac{va}{v-a} \le -a \quad (10.20)$$

It follows from here, that the absolute velocity of an image motion in the opposite direction is more, than in direct one. In the direct direction the image moves with subsonic speed. At the motion of an object with the sound speed $v = a$, (10.2) the angle of cone Mach is $\alpha = 90°$. The velocity in the opposite direction $v_s'' \to \infty$, and velocity in direct, as follows from (10.19), will be

$$v_s' = 0{,}5a. \quad (10.21)$$

In this case the observer will see object, moving from infinity after the front overfilling on him. Originally, the velocity of motion of an image will be infinite. Afterwards, it will come nearer to $0{,}5a$. With such velocity the image of the object will be removed from the observer in a direction of the object motion.

At the motion with velocity $v > a$ the object first, is observed (see Fig. 10.4) at the point S', then its image will move to the left and to the right from the point *S'*. To the motion to the left happens with higher velocity, than to the right, and soon disappears.

The motion of a direct image S_t' happens at the smaller velocity and is observed longer. The image will be removed from the observer with decreasing velocity, and at a longer distance its velocity will become less than the sound or light for the cases of superluminal motion. This result is relatively interesting. If the observer has missed a moment of the front overfilling, that is relatively probable during observing the astronomical objects, the return image S_t'' will disappear, for him and he will see a direct S_t' moving with sub-lightspeed.

The last example shows, what exotic situations are possible at observation of superluminal objects. Therefore it is necessary to analyse the results of observing as macro-objects, so as micro (for example, space particles or elementary particles in nuclear physics and during acceleration) with allowance for the observable velocities (10.18). It is seen, that if we do not estimate this phenomenom in physics of elementary particles it can lead to the interpretation of superluminal particles as the birth of a pair of two particles.

The author could observe these actions of the superluminal motions in the childhood. We have seen supersonic military planes in the sky long before the information about them appeared in the press. Suddenly in the pure sky, where there were no outside objects, the explosion or shot was heard. The place of a sound source was not determined, because the sound filled in the whole space. Then from a certain point in the heaven the sounds of two removed from each other planes were perceived. Later we learned to discover plane, creating a sound. It was far ahead, closer to the horizon of following him a sound image. Then the plane left for limits of visibility, but its sound image still created the illusion of a flying plane.

We have considered the problems of superluminal motions observation. The other group of problems is connected with the interaction of bodies at such motions. Let us research a singularity of interaction of charged bodies at superluminal motion. At $\beta > 1$, as it is seen from the law for force (4.92), at certain angles φ

between $\vec{R}$ and the vector of velocity $\vec{v}$, the denominator is a real number and the expression for force exists. If we consider a moving charge q_1, at $\beta > 1$ it the act on a located ahead motionless charge q_2 won't have time to be spreaded. Therefore, the interaction between bodies will begin, when the charge q_2 is behind a charge q_1, i.e. at $\varphi > \pi/2$. It follows from (4.92), that the denominator is equal to zero at

$$\sin\varphi_M = 1/\beta. \qquad (10.22)$$

As $\varphi_M > \pi/2$, (10.22) is identical (10.2), thus

$$\varphi_M = \pi - \alpha. \qquad (10.23)$$

At value φ_M, according to (10.22), the denominator (4.92) is equal to zero, and the force tends to infinity. Thus, a moving charge q_1 will begin to act on a motionless charge q_2, when last gets on the front of Mach cone. Inside Mach cone the force of a charge action q_1 on a charge q_2 will be final, but opposite at a sign. It is stipulated by a negative sign a numerator $(1-\beta^2)$ in the law of force (4.92). Thus, the charged body q_2, located inside Mach cone, will be attracted to a charge q_1, and at the opposite sign it will be repelled.

We will consider the isolines of force at a superluminal motion. Similarly (4.93) we will record an equation of constant value of force line:

$$R = \sqrt{B\frac{\beta^2 - 1}{\left(1-\beta^2\sin^2\varphi\right)^{3/2}}}\quad , \qquad (10.24)$$

where

$$B = -\frac{q_1\, q_2}{\varepsilon\, F} = \text{const}. \qquad (10.25)$$

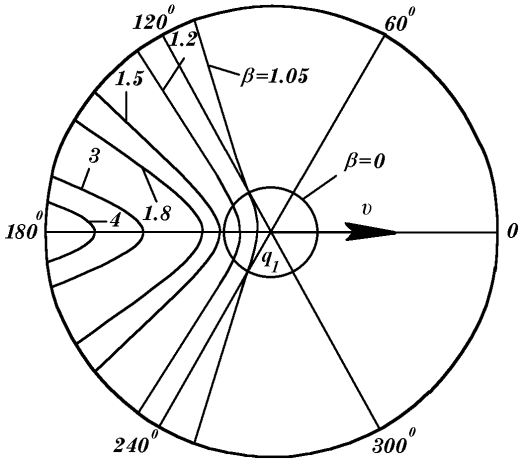


Fig. 10.5. The isolines of forces of action of a charged body, moving with superluminal velocity υ to the motionless charged body q_2 located at on different angular distances from it, having different values of normalised velocity β.

The expression (10.24) at B = 1 is represented in Fig. 10.5 in polar coordinates. The isoline of force (4.93) of motionless charged body q_1 is shown by the circle with radius R = 1. From a Fig. 10.5 it is seen, what at β < 1,5 the isolines in

area $\varphi = \pi$ are situated inside a circle R = 1, i.e. in this area the action forces of a moving superluminal charge are less motionless than the ones of a fixed charge.

With approach φ to φ_M the isoline tends to infinity, i.e. the force beyond all bounds grows. At $\beta \geq 1.5$ the isolines are outside of a circle R = 1, i.e. the force of action of a superluminal charge is more force of motionless one.

The tops Mach cones at different β are placed in the centre of a charge q_1, and the formings are parallel to the isolines. For example, a cone with a polar angle $\varphi_M = 120^\circ$ is Mach cone at $\beta \approx 1.2$. From Fig. 10.5 it is seen, that with increase β the apex angle of Mach cones decreases. At light speed of motion, (β = 1), Mach cone is degenerated in a plane, perpendicular to velocities υ.

10.3. ACCELERATION OF PARTICLES BUNCHES UP TO SUPERLUMINAL VELOCITIES

As we mentioned earlier, it is possible to reach light speed by electromagnetic action. And how to surpass it? For this it enough to create the installations, so that the particle was accelerated by the object, which moves relatively the installation with velocity u in a direction of a particle acceleration [55, 115, 118]. Then the particle can be sped up by this object up to speed c_1 relatively it, and the absolute velocity of a particle, i.e. its velocity, relatively the installation, will be equal to a sum of speed c_1 and velocity u of the object:

$$\upsilon_a = c_1 + u.$$

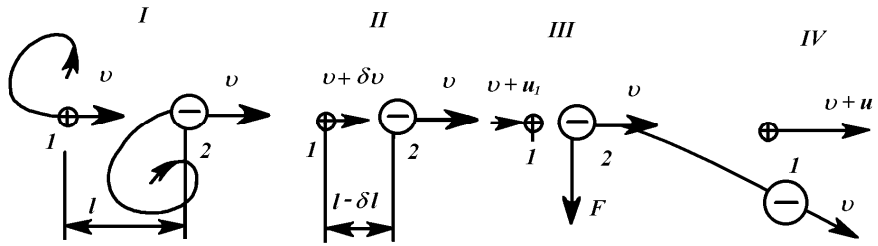


Fig. 10.6 is applied. Superluminal acceleration at attraction opposite of charged bunches: 1, 2 - number of bunches; I, II, III, IV - number of positions.

Such objects can be bunches of charged particles. At the beginning the both bunch are accelerated by independent accelerators (Fig. 10.6, position I) up to identical velocity υ. Then, bunches begin to be attracted to each other. As the mass of a bunch *2* is more significant than mass of a bunch *1*, only the velocity of a bunch *1* varies, it accelerates (position II). As shown in position II, the initial distance l between bunches decreases on δl, and the velocity of an accelerated bunch will increase on $\delta\upsilon$. After approaching bunches (position III) the accelerated bunch *1* increases velocity u_1. In this position to an accelerating bunch *2* is acted by the

external action F, due to which it is withdrawn from a path of an accelerated bunch *1*. During the interaction at a divergence of bunches the initial increase of velocity u_1 will decrease up to u. Thus, after the process of acceleration, the bunch *1* will have full velocity $\upsilon + u$.

It is possible to take a bunch of charged particles, which are created, for example, in modern accelerators, as a fast moving acting object. Such bunch with N particles, each charge q, will have a full charge $Q_a = qN$. If the bunch with a charge Q_{a1} (see Fig. 10.6) will move after an acting bunch with a charge Q_{a2} of an opposite sign, it will gain a relative velocity u, which can be defined by expression (7.46). For this purpose we we consider bunches as points. We take an acting bunch with mass M_2, which is significantly greater than mass M_1 of an accelerated bunch. Then the diminution of an acting bunch can be neglected. The bunch will be accelerated until it comes nearer to an acting bunch. Considering a distance between as their centres at the given moment equal to the diameter d of the greatest of them and considering the bunches of protons and electrons with a charge of particles q, a relative velocity of an accelerated bunch at $u_0 = 0$ will be the following

$$u^2 = c^2 - c^2 \exp\left[-\frac{2q^2 N_2}{m_1 c^2 d}\right]. \qquad (10.26)$$

Here N_2 is a number of particles of an acting bunch, m_1 is a mass of one particle from an accelerated bunch; the acceleration is made in media with $\varepsilon = \mu = 1$.

It follows from expression (10.26), that the bunch of electrons can be sped up by a bunch of protons with the values of the order d = 2 sm and with number of particles $N_2 \geq 4 \cdot 10^{12}$ up to velocity $u_r = 0.3c$. Up to the same velocity the bunch of protons can be sped up by a bunch of electrons with number of particles $N_2 \geq 6.2 \cdot 10^{14}$. Thus mass of a bunch of protons must be $M_1 = m_p N_1 < N_2 m_e$, i.e. the number of particles in a bunch of protons $N_1 < N_2 \, m_e/m_p$.

As in modern accelerators it is possible to receive the bunches of particles with velocity, very close to speed of light, after accelerating a moving bunch, the accelerated particles will receive velocity relatively the installation

$$\upsilon_a = c + 0.3c = 1.3c,$$

which considerably exceeds the speed of light. Therefore, even if the vacuum is not very high in the installation, braking particles at the expense of superluminal radiation (so-called Cerenkov radiation) will not reduce this velocity up to sublightspeed.

After the accelerated bunch reaches an acting bunch, it will begin to anticipate it and decelerate. The further interaction between bunches must be prevented. It can be executed in several ways. If to act on bunches by a magnet, the acting bunch will deviate and to leave a path of motion of accelerated bunch.

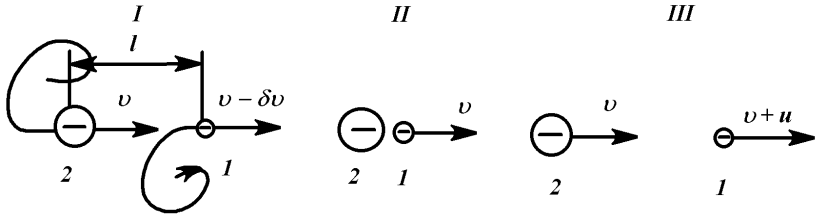


Fig. 10.7. The superluminal acceleration at the repulsion of equally charged bunches: 1, 2 - number of bunches; I, II, III - number of positions

Thus the vector of magnetic intensity $\vec{H}$ must be perpendicular to the velocity of bunches. The calculations show that the delay of an accelerated bunch will be the less, than more value of the magnetic intensity. In case of acceleration of a proton bunch electron, the small strength must be, as the mass of electrons of an interacting bunch, injected from acceleration, is very small. At acceleration by a proton bunch, it is necessary to apply much stronger magnetic action to stop the acceleration. The calculations show, the strength 15 ÷ 20 kG, widely used in accelerators, is quite sufficient for it.

It is possible to dilute the bunches after approach by electrical action, perpendicular velocity of bunches. It is possible to apply both magnetic, and electrical action simultaneously.

In considered example the bunches had the opposite charges. At equal charges the process of acceleration will happen as follows (Fig. 10.7.). In basis I the accelerated bunches *1* has the velocity smaller than the velocities of a bunch *2* on value $\delta\upsilon$. Value $\delta\upsilon$ and the initial distance l between bunches is selected so that at approach to a minimum distance, their relative velocity was equalled to zero (position II). Further at the expense of repulsive forces the acceleration of a bunch *1* and, after going out of a bunch *2* boundaries, the accelerated bunch will have velocity $\upsilon + u$.

As well as in the previous case, at technically feasible density of bunches the relative velocity $u = 0{,}3c$ can be reached. At initial velocity $\upsilon = c$ and $\delta\upsilon = u$ the accelerated bunch in a position II will have velocity $\upsilon_1 = c$, and its final velocity will make $\upsilon_1 = 1{,}3c$, i.e. will be superluminal. In this way, at acceleration of electrons the accelerating bunch of electrons must have $N_2 \geq 4 \cdot 10^{12}$ of particles, and at acceleration of protons $N \geq 6{,}2 \cdot 10^{14}$ of protons in a bunch.

So, the bunches with a number of particles order $10^{12} \div 10^{13}$ are necessary for obtaining superluminal electrons, and for acceleration of protons must be the bunches with a number of particles order $10^{14} \div 10^{15}$. At this number of particles the bunches must have the values 1÷10 sm.

Such proton bunches are obtained in many large modern accelerators. It is affirmed in [1], that the number of particles of a proton synchrotron reaches $2 \cdot 10^{12}$ in a pulse. Moreover, it is planned to increase the number of particles in a pulse up

to $5\cdot10^{13}$. Still, lots of protons have bunches, obtained in accumulative rings. In [2] it was planned to receive up to $4\cdot10^{14}$ of protons in a bunch. Therefore, in any such accelerator it is possible to accelerate electrons up to the superluminal velocity, and it is possible to reach even greater velocity, than 1.3*c*.

Acceleration of protons up to superluminal velocities needs a number of particles in a bunch on two orders more, than at acceleration of electrons. As the mass of electrons is in 2000 times less than the mass of protons, they have stronger repulse; therefore is difficult to reach a large density of electrons in a bunch. However, a necessary bunches of electrons can be derivated as rings. Such rings have received some contributors. According to [22], the electron rings with the number of particles $6\cdot10^{12}$ are obtained.

Sarantsev [46] has received the rings with number of electrons 10^{13}. The values of rings can be less than 1 mm; therefore it is possible to accelerate protons up to superluminal velocity by them. According to Sarantsev statement, there are a number of capabilities to obtain the rings with the sizes $10^{-3} \div 10^{-4}$ sm. By such bunches it is possible to velocity up the protons up to velocity greater than 1.3*c*.

The activities on obtaining electron bunches with even large number of electrons were conducted. Hodataev [68] anticipated a capability of obtaining the small bunches with $4\cdot10^{16}$ of electrons. As it was given in [28], in our country the bunches with $2.5\cdot10^{15}$ of electrons were obtained. Thus, at teleological creation of electron bunches it is possible to reach a necessary density of particles in them to have a capability to accelerate protons up to velocity 1.3*c* and more.

10.4. MULTISTAGE ACCELERATORS OF SUPERLUMINAL PARTICLES

With the help of the ways, considered above one bunch can give the other, relatively itself, a velocity not larger than c_1. To receive these bunches in accelerators it is possible with velocity not greater than c_1. Therefore, a double speed of light is the greatest velocity, up to which it is possible to velocity up the particles by an indicated way. By an explained below multistage way it is possible to surpass and double the speed of light.

For obtaining protons with light speed the large accelerators are required. Some more large costs will require acceleration of heavy ions up to light speed. But if to accelerate the bunches protons or ions up to this velocity by electron, the costs will considerably decrease. By electrons bunch in $10^{14} \div 10^{15}$ particles and speed *c* it is possible to speed up to light using the scheme, shown in Fig. 10.6 velocity a proton bunch with initial velocity 0.7c. In position I in this case, bunches have different velocities and are located as much closer to each other, i.e. *l* is minimum.

As the relative velocity of a proton bunch at the beginning is negative $u = 0{,}7c - c = -0{,}3c$, it will remove, until it reaches a relative velocity $u = 0$, and in relation to the installation, its velocity will be *c*. Then it will begin to come nearer

to an electron bunch and it velocity relatively the installation will become equal to 1.3*c*. Such acceleration is possible, if we can reduce the bunches at the minimum distance $l \approx d$ at the beginning.

The superluminal proton bunch can be accelerated, using the scheme of Fig. 10.5 further in the same way, but other electron bunch with velocity $\upsilon = c$. In this case at the beginning of acceleration by the protons will have velocity relatively electrons $u_0 = 1.3c - c = 0.3c$. After a proton bunch approaching the electron, the relative velocity of the first one, as it follows from (7.46), will be $u_0 = 0.412c$. That is, after the second stage of acceleration, the proton bunch will have velocity 1.412c. The given bunch can be accelerated by electron with light speed and further, but each time the incremental velocity will decrease.

At all stages of the acceleration it is possible to use electron bunches from the same accelerator, transmitted in different instants. At a multistage way it is possible to use electron bunches and with smaller in the order by number of particles, for example $10^{13} \div 10^{14}$, but then the greater number of stages is required. To prefer the intensity of a bunch to the number of stages or on the contrary, it is possible, comparing the costs of creation of these installations.

Such multistage way of acceleration allows accelerating the particles up to velocity 2*c* and more. With this purpose it is necessary to receive a superluminal bunch of electrons and superluminal bunch of protons, and then again to use them for accelerating each other. And if the velocity of bunches was 1.5*c*, then at the message of accelerated relative velocity 0.3*c*, its velocity relatively the installation will be 1.8 *c*. After the second stage of such acceleration it will receive velocity 1.912 *c* etc.

10.5. ACCELERATION OF SINGLE PARTICLES UP TO SUPERLUMINAL VELOCITIES

We will consider the collision acceleration of equally of charged particles. Let proton with velocity υ_0 in infinity (Fig. 10.8) direct on a motionless positron. Then, relatively the proton the positron has velocity $\upsilon_{rel} = \upsilon$ at the direction of a proton, i.e. it comes nearer to the proton and brakes. Its velocity, according to (7.46), will be written so:

$$\upsilon_{\text{rel}} = c_1 \sqrt{1 - (1 - \beta_0^2) \exp\left[\frac{2\mu_1}{c_1^2}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right]}. \qquad (10.27)$$

Approaching on the least distance $R_{\min}$ (Fig. 10.9) relative velocities will become equal to zero. From (10.27) we will define $R_{\min}$, substituting $\upsilon_{\text{rel}} = 0$ и $R_0 \to \infty$:

$$R_{\min} = -\frac{2\mu_1}{c_1^2 \ln\left(1 - \beta_0^2\right)}. \qquad (10.28)$$

Thus, the mass of a proton m_{pr} is significantly more than the mass of a positron m_{po} and its velocity will not vary, i.e. $v_{pr} = v_0$. The same velocity also will have positron at the point of approach.

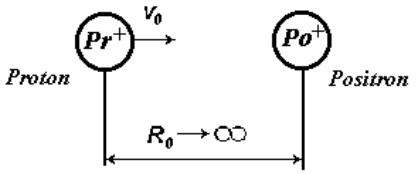


Fig. 10.8. A beginning of acceleration stage.

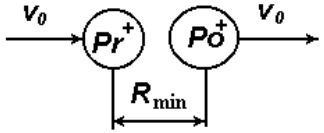


Fig. 10.9. A mean acceleration stage.

After approaching a minimum distance the positron begins removing from a proton (Fig. 10.10). Its velocity at removing in infinity will be defined from (10.27) at $v_0 = 0$ $R \to \infty$ and $R_0 = R_{\min}$ in the following kind:

$$R_0\, v_{rel} = v_0. \qquad (10.29)$$

Therefore a full velocity of a positron will be

$$v_{po} = v_0 + v_{rel} = 2v_0.$$

Let's calculate the initial velocity of a proton, which allows particles to be pulled together at a distance, equal to a sum of radiuses of a proton and the positron $R_{\min} = R_{pr} + R_{po}$. From (10.28) it is followed

$$\beta_0 = \sqrt{1 - \exp\frac{-2\mu_1}{c_1^2\left(R_{pr} + R_{po}\right)}} = \sqrt{1 - \exp\alpha_{pr}}\,, \qquad (10.30)$$

where

$$\alpha_{pr} = -\frac{2\mu_1}{c_1^2\left(R_{pr} + R_{po}\right)} = -\frac{2e^2}{c_1^2 m_{po}\left(R_{pr} + R_{po}\right)}. \qquad (10.31)$$

Fig. 10.10. The end of acceleration stage.

After a substitution of parameters we obtain $\alpha_{pr} = -1.34$, and the initial velocity of a proton $\beta_0 = 0{,}859$. At this velocity of proton action on a positron, the last will be sped up to $\beta_{po} = 2\,\beta_0 = 1.72$c, i.e. we have superluminal positrons. If the velocity of a proton will be more than $0.859c$, the particles will be pulled together at a distance, smaller than the sums of their radiuses, therefore, their further interaction will not be determined by normalised ratios. Let's note, that by this way it is possible also to accelerate electrons by antiprotons.

Considered acceleration can be executed, bombarding by a bundle of heavy particles a slowly moving cloud of easy particles. The probability of direct collision will depend on a density of ensembles of these particles. Further it is necessary to separate the superluminal particles from the heavy sub-lightspeed particles. The last must be rejected by a transversal electromagnetic action. After rejecting devices along an axis of the installation, only the superluminal particles will move. They can be detected with Cerenkov radiation. Under the analysis collision of results it is possible to define a probable density of particles angular distribution, which will characterize the availability of superluminal particles. Thus, the parameters of trajectories normalised in Appendix [59], and also in the work [60] can be used.

We have considered the collision acceleration of equally of charged particles. By the same way it is possible to accelerate the opposite charged particles, for example bombarding by protons a cloud of electrons, it is possible to accelerate the last one up to the superluminal velocity. However, it is possible in particular conditions, which emerge at the analysis of trajectories, shown in chapter 5. A proton with velocity $v_0 = 0.67c$, directed to an electron with an aim distance $a = 1.2 \cdot 10^{-12}$ sm (the Fig. 10.11), will seize it into hyperbolic-like trajectory round itself, on which the electron will be wrapped up round a proton for a period at 360^o (see trajectory 1.1.3.7 on one seconds 193 of [59] or trajectory *7* in a Fig. 5.1).

At stage I a proton with velocity v_0 (see Fig. 10.11) fly on an electron with an aim distance a. At stage II the motion of an electron relatively a proton is shown. Having initial velocity in infinity ($-v_0$), it goes round a proton along a hyperbola-like trajectory, approaching at the pericentres at the distance $R_p = 0.9\ 10^{-12}$ sm. Then along the lower branch moves into infinity with velocity v_0 relatively a proton. At stage III the velocities of particles relatively the installation after interaction are presented. The electron moves at the direction of a proton motion and has the superluminal velocity $2v_0 = 1.34c$.

The process of acceleration happens within the limits of a distance 10^{-9} sm, i.e. practically at a point, therefore, the length of the installation will not be large. However, the electrons must be free, as the electrons, connected by other interaction, will not create necessary situations for superluminal acceleration: at bombardment by protons of atoms their electrons can not be sped up to the superluminal velocity.

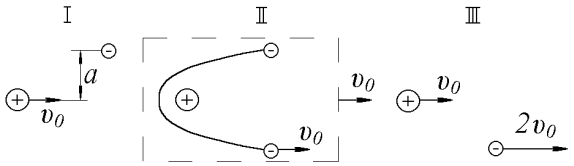


Fig. 10.11. Stages of acceleration at collision are the oppositely charged bodies.

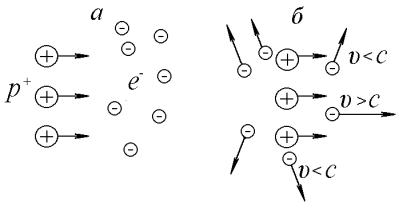


Fig. 10.12. Acceleration of electrons at bombardment by protons of an electron ensemble: *a* - before collision; *b* - during collision.

For a realization of this way by a

bundle of monofast-track protons it is possible to bombard a cloud of motionless free electrons (Fig. 10.12, *a*). Because of different aim distances, the different trajectories of electrons (see Fig. 10.12, *b*) will be received, from which the light speed has only electrons, moving in the direction of protons bundle motion. All static characteristics of process of acceleration can be calculated, if the density of particles ensembles is known.

It is necessary to note, that the trajectory of acceleration will slightly differ from trajectory *7* in Fig. 5.1. Let's consider the results of calculations on searching trajectory, which will lead to the greatest superluminal velocity:

α_1	β_p	β_{ox}	φ_a
-0.25	0.96	-0.617	2.45
-0.28	0.958	-0.743	3.09
-0.3	0.951	-0.704	3.11

Here β_{ox} is a component of normalised velocity in the direction of the axis *x* in Fig. 5.1, and φ_a is an angular halfcycle in radians. How it is seen, after the acceleration, shown in the scheme shown in Fig. 10.11 velocities of a particle can reach $\upsilon = 2\beta_{ox} c = 1.5c$ of the light speed. However, the parameters of these trajectories are in narrow limits. The minor change of parameters calls deviation of electrons in a perpendicular direction to a line of motion, or acquisition by their proton, or deviation in the necessary direction, but with small velocity.

10.6. PERSPECTIVE OF APPLICATION SUPERLUMINAL MOTIONS

The space particles generate broad showers of elementary particles, which simultaneously cover large territories on a surface of the Earth. Maybe, this abundance of particles is created by one superluminal particle. This property can be used for further space communication. The small quantity of superluminal particles can be directed to the other planet of a solar system or even to a planet of the other star. Such particles will not be dispersed similarly to radio waves or light and to lose the intensity. At the entrance in the atmosphere of a planet they will create showers of particles, which will be fixed on large distances from an aiming point. Probability of a gross error of a transmitted signal therefore decreases. The information, thus, will be contained in temporary intervals between transmitted superluminal particles. Apparently, the communication by of superluminal particles is the most energetically expedient.

The debris of the kernels and a variety of elementary particles observed in broad atmospheric showers, testify, that the superluminal particles can effectively destroy substance. Therefore, the bundle of superluminal particles of certain intensity can be a more effective weapon, than a laser ray. Such bundle of superluminal

particles can be used for dividing into parts of the asteroid, approaching the Earth. After falling on Jupiter in 1995 of Levi-Shumekher comet fragments, the problem of antiasteroidal protection of the Earth was realized by the mankind. And the superluminal particles here can be used in different ways. Except effective "knife" the bundle of superluminal particles can be applied to a carving of asteroids into parts for a distant location of asteroids. The superluminal particles, modulated on time, after collision with asteroid will beat out a part of particles with opposite directional velocity, which, entering the atmosphere of the Earth, will create the showers of atmospheric particles. As the result of their filing, the distance up to asteroid, and also its other parameters will be determined.

The other way of antiasteroidal protection is the change of asteroid trajectory by jet . As it is seen from Tsiolkovsky's formula (7.10), the more the speed of jet, the less fuel is required and the less the energetic and material inputs. At superluminal velocity of jet the efficiency of a jet engine is the greatest.

Jet superluminal drivers are the unique means, which will allow the person to visit the planets of other stars. August 12-14, 1997 the American astronautical associations conducted working conference (Breakthrough Propulsion Phisics Workshop) on searching the strategy of researches, which will allow the person to create the drivers for intersidereal travellings. The analysis of materials of this conference has shown, that there is no an alternative to jet superluminal driver.

We will consider the flight of the spacecraft with such driver up to the nearest star situating at the distance l = 3 l y. (of light years). Let's make the flight with earth acceleration g. For a year of flight, t_1 = 1 year, the spacecraft will set up light speed

$$\upsilon = gt_1 \approx c \qquad (10.32)$$

also will pass a distance

$$l_1 = gt_1^2/2 = 0.5 \cdot ct_1 = 0.5 \text{ c.r.} \qquad (10.33)$$

According to Tsiolkovsky's formula (7.10), the mass of the spacecraft will decrease at the expense of a jet with light speed $u = c$:

$$m_1 = m_0 \exp(-\upsilon / u) = m_0 / e, \qquad (10.34)$$

where e = 2.73 is the basis of a natural logarithm. Then the spacecraft during t_2 = to 2 years moves with velocity $\upsilon = c$ at inertia, also passes the distance l_2 = 2 l y., then during t_3 = 0.5 years turns an engine against motion and throws the velocity up to zero. For this time it passes, according to (10.33), the stayed distance l_3 = 0.5 y. and its mass decreases up to

$$m_2 = m_1 \exp(-\upsilon/u) = m_0 \exp(-2\upsilon/u) = m_0 / e^2. \qquad (10.35)$$

Thus, the spacecraft for 4 years can reach a star, removed from the Earth at 3 light years. Making a return path in the same three stages: the acceleration 0.5 years, flight at inertia 2 years, braking 0.5 years - spacecraft returning to the Earth, will have, according to (10.35) the mass

$$m_B = m_2 \exp(-2\upsilon/u) = m_0 \exp(-4\upsilon/u) = m_0 / e^4, \qquad (10.36)$$

i.e. a useful mass from the initial mass of the spacecraft will make $m_B = 0.018m_0$. It is quite real ratio, which allows agreeing in a design of the spacecraft energetic needs and technical feasibilities. For a comparison we will consider a returned mass of the spacecraft at usual propulsion jet with velocity $u \approx$ km / s, i.e. $u = 10^{-5}$ c. According to (10.36), $m_0 = m_B \exp(4 \cdot 10^5)$. This astronomical number for initial mass of the spacecraft m_0 testifies, that it is impossible to reach the nearest stars in another way.

We have considered an example of travelling to the nearest star with acceleration g. The whole "road" has taken 8 years. At $v > c$ the flight time will be reduced, i.e. for this time it is possible to make a flight up to the farther stars. The normalised ways of acceleration of particles up to the superluminal velocity can be used for a superluminal jet driver, schematically shown in Fig. 10.13. It consists of two accelerating devices for heavy particles: *1* is for protons (*Pr*); *2* is for antiprotons (*aPr*) and two rejecting devices: *3* is for antiprotons and *4* is for protons. These devices provide circulation of heavy particles in a plane *xy* on two circuts: in circut I - protons; in circut II - antiprotons. In devices *1* and *2* heavy particles are introduced in turn with velocity v_0 on an axial line of *x* jet drivers. In rejecting devices these particles are injected from an axial line: in an circut I - protons are injected and in circut II antiprotons are injected.

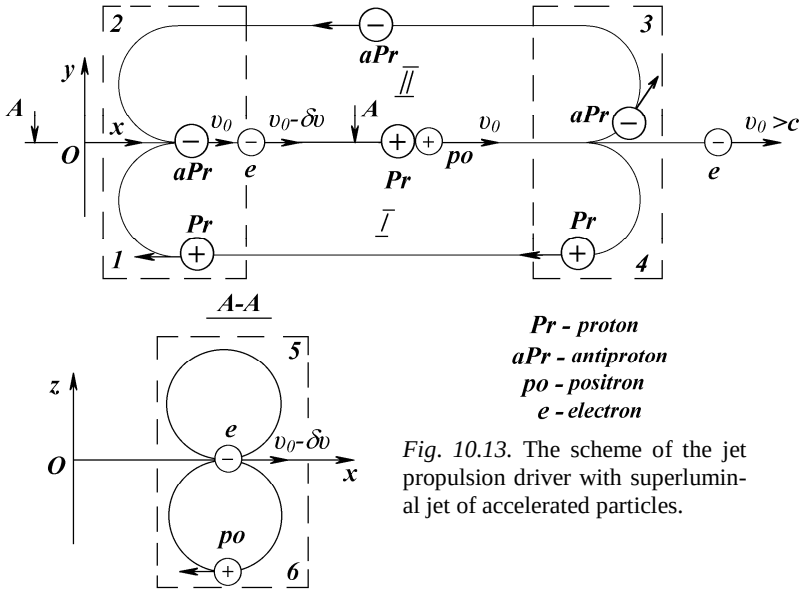


Fig. 10.13. The scheme of the jet propulsion driver with superluminal jet of accelerated particles.

In perpendicular to a delineation of a plane (see section *A-A*, plane *xz*) two circuts of an ejection and acceleration of easy particles are located: *5* - for electrons (*e*), *6* - for positrons (*po*). These devices in turn with velocity $v_0 - \delta v$ introduce easy particles on an axial line of *x* jet drivers.

On the axial line *x* the collision acceleration equally charged particles happens, it is considered in item 10.3 and 10.5. In a left-hand part (Fig. 10.13) the initial stage of acceleration of an electron by an antiproton is shown. As the speed of injection of an electron is less than the velocity of an antiproton, the antiproton catches up with it. Thus the velocity of an electron grows.

At the moment of approach the velocity of an electron will become equal to v_0, then the electron will begin removing from an antiproton, and its velocity will increase. In a mean part of a driver the moment of approach is shown at the interaction of a proton and positron, and in a right member we show a completing phase of interaction of an antiproton with an electron. The antiproton by the rejecting device *3* returns in a circulating circut II, and the superluminal electron continues motion along the axis *x*. Thus happens, therefore the driver remains electrically of the neutral. Due to the symmetry of devices the torques are not created in it. At $v_0 = 0.859c$ the speed of jet, as shown in item 10.5, will be equal to 1.72 speed of light.

We have considered the acceleration of single particles, however, this scheme is applicable for bunches of particles, too. Thus, the condition should be executed, than the accelerating bunches had mass larger than the mass of accelerated bunches.

The perspectives of using the superluminal particles are not limited by mentioned examples. As any new phenomenon, they will have exotic properties, which will present new technological capabilities to the people.

CHAPTER 11

GRAVITATIONAL INTERACTIONS

11.1. VELOCITY OF GRAVITY PROPAGATION

All bodies in the world around are attracted to each other. All subjects drop on the surface of the Earth, being attracted to its centre. The Moon, due to the Earth attraction, twins round it and is not fly away from the Earth. The sun attracts the Earth, and it moves round it. In the circum-solar space due to the attraction of the Sun the planets, asteroids and other bodies move; in total they represent a lens-shaped structure, which exists due to the gravitation. The stars with their planetary systems are attracted to each other and form starry associations and accumulations, which due to mutual attraction form galaxies, which also due to attraction are integrated in metagalaxies. So, the gravitation acts on huge distances, therefore, a title "the world law of gravitation" is quite justified for expression of force interactions of two bodies

$$\vec{F} = -G\frac{m_1 m_2 \vec{R}}{R^3}, \qquad (11.1)$$

which masses are m_1 and m_2. I. Newton has introduced the law (11.1) on the base of analysis and generalization of observable gravitational interactions. The gravitational constant G was determined due to G. Cavendish experiment in 1798. The becoming of the law (11.1) was stipulated by a large activity of many scientists at calculation of celestial bodies motions.

The acceleration is a direct expression of action, therefore, the latter, as it has already been noted earlier, is possible to describe without application of force. Probably, such description will be easier and will be occasions for erroneous conclusions less. One of them, following from a force-method of gravity description, consists in searching the reason of gravity. It seemed, if there is a force of action on a body, there must be a subject, which applies it. It must be directly adjoined to the body and influence on it. There were suggested various kinds of such subjects, for example, the particles of ether, which are involved by a gravitating body, and this flow tightens the attracted body. Notwithstanding what, mechanisms of action are thought out till now, any of them is not capable to give consistent treatment of gravitation.

This is one direction of mechanisms searching a short-range action of gravitation. The other one consists in a field representation of gravitation. By analogy to an electromagnetic field the gravitational field is supposed. Here the illusion of a short-range action is created by the mathematical images, for example, the gradient of a potential creates a force. But the physical mechanisms at a level of abstraction have already become customary, so that the field direction freely developed, until a geometric explanation of gravitation as a curvature of space-time was introduced in the GTR.

In a field explanation of gravitation by analogy to an electromagnetic field there appeared a question about the velocity of gravity propagation. The motion of the Moon round the Earth, the motion of the Earth and planets at the close analysis has appeared a little bit distinguished from the results of solving the problem of two bodies interaction. To explain the differences a lot of hypothesises were put forward, including the new laws of world gravitation. One of them is connected to speed of gravitation propagation, equal to the speed of light. However, the scientists, taking into account the action of other bodies of a solar system and attracting singularities of the form of a main body, have explained these differences. Thus, the law (11.1) became more and more justified with each new explanation.

It is necessary to note here, that in 1787 Laplace in the work "an Account of a system of the world", and also in "Treatise on a celestial mechanics" [99] from the analysis of the Moon motion came to a conclusion, that if the speed of gravitation is final, it must exceed the speed of light in 100 million times. Nevertheless, the hypothesis about light speed of gravitation afterwards was put forward many times. It had got the greatest actuality at consideration the precession of planets perihelions.

According to the law (11.1), a closed orbit is an ellipse (see formula (5.3)), and there is no pericentre in the precession. As the precessions were observed, the scientists, including I. Newton, selected the new laws of gravitation, which would

allow taking into account the precession. U. Le Verrier [45] used one of the particular cases of the law, offered by a I. Newton, in the following way:

$$\vec{F} = - G \frac{m_s m \vec{R}}{R^3} + \vec{P}, \qquad (11.2)$$

where m_s is a mass of the Sun; m is a mass of Mercury, and $\vec{P}$ is a revolting force, which expresses the action of remaining planets.

In 1859 U. Le Verrier, according to N.T. Rousver [45], presented in his work [101], calculated Mercury perihelion advance according to (11.2), and planets contribution in it:

Planet	The contribution on [45]	The contribution on [50]
Venus	280.6″	277.856″
The Earth	83.6″	90.038″
Mars	2.6″	2.536″
The Jove	152.6″	153.584″
Saturn	7.2″	7.302″
Uranus	0.1″	0.141″
Neptune	---	0.042″
Oblateness of the Sun	---	0.010″
All for one century	526.7″	531.509″

It is affirmed in the references from GTR, that the calculated $\sim 530''$ per one century differs on $\sim 40''$ from a really observable perihelion advance, which value is not given. It is necessary to note, that the dissonance of calculation in $43''$, in view of an extremely small precession of a perihelion (it is considered for 100 earth years, i.e. for 415 of Mercurial years), is a rather good prognosis. Secondly, this result is obtained at the solution of an approximate problem (11.2), instead of exact one, when to each celestial body the force not only from the Sun, but also from each of the planets is applied. That is, the problem of many bodies is not decided in full volume. Thirdly, the parameters of all acting bodies such as a mass, a position on trajectories and the parameters of trajectories are known with insufficient accuracy, to hope for the best coincidence even at the solution of a problem of many bodies. In fourth, there are many other factors: the action of huge quantity of unobservable small bodies, the form of the Sun, circum-solar substance etc., which can essentially affect on the rotation of Mercury perihelion. Despite of it, many scientists put forward the various hypothetical laws of gravitation for explanation of anomality (i.e. misalignment $43''$) of Mercury perihelion advance. One of them is the light speed propagation of gravitation. Different authors offered a number of ratios for gravity, which depends on velocity of motion and acceleration that is stipulated by a final speed of propagation of gravitation. It is Weber's force (2.11) and the onessimilar to it. Apparently, the greatest value of them has the force [45]

$$\vec{F} = -G\frac{m_1 m_2}{R}\left[1 - \frac{3}{c^2}\left[\frac{\mathbf{d}R}{\mathbf{d}t}\right]^2 + \frac{6R}{c^2}\frac{\mathbf{d}^2 R}{\mathbf{d}t^2}\right], \qquad (11.3)$$

deduced by Gerber and published in 1898 [91]. As the result of solving the problem of two bodies with force (11.3) he received the following expression for a perihelion advance:

$$\delta\varphi_T = \frac{24\pi^3 a^2}{T^2 c^2 (1-\varepsilon_t^2)}, \qquad (11.4)$$

which for Mercury is equal to 41″ per one century. The expression (11.4) was known to Mach [36] and A. Einstein [88] and was widely discussed in 1916 - 1917 [45]. In 1915 A. Einstein introduced the expression (11.4), taking into account a general theory of relativity [76].

Both Gerber formula (11.3), and GTR method are based on a hypothesis about speed of gravitation, equal to the speed of light. Notwithstanding that the obtained displacement (11.4) is close to a calculated anomaly of Mercury perihelion in 39″, it cannot be considered as a proof, and, furthermore, as a proof of light speed of gravitation. At first, the existence of the anomaly, as we have already mentioned, is stipulated by approximate character of a problem solution. Secondly, the registration of the gravitation light speed has given a perihelion advance 41", and the value 527" + 39" = 566", which is on the order larger, is observed. There are no proofs that, the light speed of gravitation improves the result. The value 41" is calculated for a problem of two bodies. Let's imagine, that the problem of many bodies was decided because of the law of gravitation (11.1), and it gave the result of a perihelion advance 527" per one century. Then the problem of many bodies was decided because of the law of gravitation with light speed of propagation, and the result of displacement 527" + 41" = 568" was obtained. Then it would be possible to assert, that the light speed of gravitation considerably improves the results. As these problems were not decided, the supposition about light speed of gravitation is deprived of any basis.

As we have already noted, in a General Theory of Relativity the light speed of gravitation was entered with the purpose of creation of a unified field theory. The expression (11.4) was one of the main proofs of GTR validity. The second proof has become a curvature of a ray of light passing near the Sun, and the third one was decreasing of frequency of light at its removing from the attracting centre. As to deviation of a ray, how we repeatedly showed in a chapter 5 and 9, at light speed of body the action propagated with the same speed, leaves trajectory of a body unbent. Here the situation is identical to a situation with a «black hole» and is expressed in the table in item 9.1. The curvature of a ray of light is only possible in the case, when the light consists of energetic particles and the speed of gravitation is infinite.

During Venus passing along the disk of the Sun in 1761 Lomonosov observed a swelling of the Sun edging at Venus departure from the disk: "When Venus came forward from the Sun, its front edge became approaching the Sun edge

and… it was about the tenth part of Venus's diameter, a knob appeared on the Sun edge. This shows a refraction of solar rays in Venus's atmosphere" [31]. Visible for an eye the refraction must have value about a degree. Not smaller refraction of rays must be in the Sun atmosphere. In this connection how to estimate the deviation, confirmed by the observations of a ray of light ($\Delta\varphi = 1.75$" [26] due to a gravitational field of the Sun?

The reddening of light at its removing from a star is possible, if the light is not an action, but there is some substation, which has mass, by means of which the star will decelerate the motion of light and change it frequency. Such properties of light are not revealed. There is no also reddening of light. Light spread between the points with the gravitational potentials Φ_1 and Φ_2 according to [26], changes the frequency on value

$$\Delta f = f(\Phi_1 - \Phi_2)/c^2 \ . \qquad (11.5)$$

Gravitational potential on a surface of a star $\Phi_1 = -Gm_s/R_s$, and at a viewpoint $\Phi_2 \to 0$. Then the light which is radiated by a star with the frequency f, will be observed with frequency, reduced by a value

$$\Delta f = -f\frac{R_g}{2R_s}. \qquad (11.6)$$

For a neutron star its radius R_s comes nearer to a gravitational radius R_g and $\Delta f = f/2$. As the expression (11.5) is approximate and is fair for a weak field (for strong fields (11.6) should be $\Delta f = -f R_g/R_s$), there must be an even more essential change of frequency. In a limit at $R_s = R_g$ the frequency of light tends to zero. However opened in XX v. white dwarfs, neutron stars, candidates in the «black holes» do not give such a change of frequency. Therefore, "the experimental confirmations" of three effects in GTR remain on conscience of the scientists, declared them. For every scientist they must be a visual example of the responsibility for the results, presented to a society.

The fourth GTR effect deals with gravitational waves. They also are a corollary of light speed of gravitation. Begun by many scientists in the second half of 20-th centuries, the activity on searching gravitational waves were finished without any result. The gravitational waves are not detected.

We will note, that the observable perihelion advance of Mercury in different sources is represented by different values. For example, in the review of magazine " News AS of Byelorussia " on D.V.Tal'kovsky's paper it is informed: "The observable precession of Mercury perihelion makes 5599.74" per one century; due to the influence of no-sphericity of the Sun, disturbances from planets and other factors the astronomers managed to explain 5557.18" from this value within the framework of Newton's theory of gravitation. The divergence on 43" per one century made a large problem for a theoretical astronomy, and its explanation by Einstein in GTR frameworks become the triumph of the latter". Here observable value is in 10 times more than the former. The offset value of the same order on the basis of comparing the orbits of Mercury in 1850 and 1950 is given in the work [21]: " This effect is so small, that for hundred years the perihelion of Mercury turns

only on 1°33'20" (5600"), as shown in Fig. 5.3 (see [21]). From this observable full turn the theory of Newton can explain only the turn on 1º33'37" (5557") for one century. There is an excessive motion of a perihelion 43 seconds of an arc for one century... which Einstein received for the velocity of precession exactly 43" for one century that is just that value, which was not explained by Newton theory. "

So, the observable offset value is equal to 5600" per one century, instead of 566". The value of observable displacement 5600" is confirmed also in the third source [64]. Then it follows, that the displacement, calculated in Newton theory, makes 5557" per one century, instead of 526.7" or 531.5", as shown by two sources in the above-stated table. Nevertheless, explanation of a divergence between observation and calculation in 41" or 43" has become a triumph of a relativity theory. As modern Physics is constructed on the relativity theory, above-mentioned "triumph" is a testimony of a steep physics crisis in the XX century.

The shown analysis testifies, that there are no proofs of light speed of gravitation. Therefore, the problem on speed of propagation of gravitational action remains opened. How we have shown in ch.5, for interactions spread with limited speed an angular halfcycle of final trajectories is $\varphi \neq \pi$, i.e. the pericentre of the orbit will displace in space. We solved this problem at a large velocity of bodies. Further we will consider it at small velocities of bodies motion.

A small displacement of the orbit parameters of celestial bodies that is rotation of the pericentre, the change of its lengths, declination of the orbital plane and its rotation in space, represents a special interest from a position of existence and development of a solar system, the history of the Earth and a change of its climate. Put forward in 1864 by J. Croll and developed by M. Melankovitch the astronomical theory of approach of ice ages finds broad support of many scientists [18]. If the gravitation has a final speed of propagation, at large temporary periods this property can affect on the rotation of a perihelion of our planet. The change of an eccentricity and declination of a plane of its orbit is stipulated by the action of other planets. Actually there is a problem of many bodies, which will be further considered.

11.2. PRECESSION OF MERCURY PERIHELION

For determination of real precession of Mercury perihelion the analyses of many centuries observations of a planet is necessary in system of coordinates, which, in turn, is subject to changes due to secular perturbations of an orbit of the Earth and movement of Solar system. Therefore, the result depends on the method of data processing observation. The laws of changing the planets perihelion (see, for example, Astronomical year-book in 1949), are based on S. Newcomb's theory (1835 - 1909) which for first three ones look like the following:

Mercury - $75^\circ 53' 58.91'' + 5599.76'' T + 1.061'' T^2$;

Venus - $130^\circ 09' 49.8'' + 5068.99'' T - 3.515'' T^2$;

Earth - $101^\circ 13' 15'' + 6189.03'' T + 1.63'' T^2 + 0.012'' T^3$,

where T is time in centuries, counted from epoch 1900 y. As we can see from here, a linear component of precessions Mercury is equal to $5599.76''$ per one century.

Now we will consider what contribution to precession can give the registration of gravitation propagation speed at interaction of two bodies. As it was shown in chapter 4, it follows from an electrodynamics, that at interactions dependent on velocity of relative motion of two objects, the force is determined by expression (4.58). From d'Alembert's solution (3.28) it follows, that such an action is spread with a final speed. The conjectured earlier laws of interaction, for example (2.11), (2.12), (11.3) etc., were based on different hypothesises. In special and General Theory of Relativity the description of interactions, notwithstanding that it is expressed as a change of spatially - temporary parameters, is constructed on relations of electrodynamics, and, as we have already shown in item 4.6, the relativistic problem of two bodies (4.90) is reduced to our equations of trajectory (4.80), (4.82). Therefore, the latter represents a precise solution of a problem of two bodies, which interaction is spread with final speed.

The results of integrating equations (4.80), (4.82), as it was already noted, almost for 100 trajectories are submitted in the book [59], and the periods of orbits and radiuses of their apocentres for final trajectories are given in Appendix 5. In Fig. 5.5 these trajectories are represented at $\alpha_1 = -0.7$. At velocity at the pericentres, making 0.1 from the speed of light ($\beta_p = 0.1$), for one period the pericentre of orbit displaces on $\delta\varphi_T{}^\circ = 2(\varphi_a - \pi) = 0.8°$. With increase of velocity β_p the offset value of the pericentre for trajectories *2*, *3*, *4* receives values $\delta\varphi_T{}^\circ = 9^\circ$; 35°; 296.2° accordingly. At a limiting velocity $\beta_{pc} = 0.714$ the displacement of the pericentre aims to infinity. As it was mentioned earlier, in this case the attracting centre into a circular orbit seizes a particle. Alongside with growth $\delta\varphi_T$ at the increase of velocity the radius of apocentre $\overline{R}_a = 2.482$ decreases to $\overline{R}_a = 1$.

In Fig. 11.1 we represent the change of relative period of orbit $\overline{\varphi}_T$, where

$$\overline{\varphi}_T = \varphi_T / 2\pi, \tag{11.7}$$

$$\varphi_T = 2\varphi_a \tag{11.8}$$

depending on β_p at different parameters of trajectories α_1. The verticals designate the asymptotes, to which the curves rush reaching the limiting velocity β_{pc}. At a fixed α_1 the relative radius of apocentre with growth of velocity β_p decreases from maximum value at $\beta_p = 0$ up to $\overline{R}_a = 1$ at $\beta_p = \beta_{pc}$. Therefore, a displacement of pericentre tends to infinity at an asymptote, but in this case the orbit is a circle, which has no a pericentre.

How it is seen from Fig. 11.1, with increase of an absolute value α_1 the displacement of pericentre is increased at the same velocity at the pericentres β_p. But with increase of $|\alpha_1|$ the limiting value of velocity decreases β_{pc}, therefore there is an optimum of displacement in area α_1 = - 0.7. For example, at α_1 = - 0.7, the orbit with a relative period $\bar{\varphi}_T$ = 1.5 is possible, i.e. for two periods the particle will make three whole turnovers, and its two-period trajectory will be stabile in space. At relative period

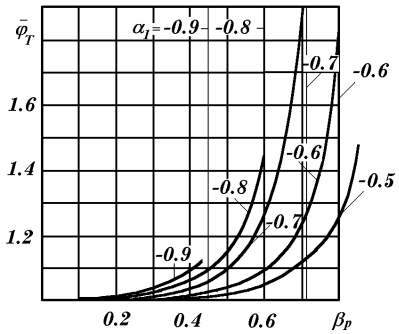


Fig. 11.1. Relative period of orbits ($\bar{\varphi}_T = \varphi_T / 2\pi$), depending on velocity at the pericentres β_p at different parameters of trajectories α_1. The vertical asymptotes have abscissas $\beta_p = \beta_{pc}$:

α_1	-0.5	-0.6	-0.7	-0.8	-0.9
β_{pc}	0.866	0.8	0.714	0.6	0.435

$$\bar{\varphi}_T = 1 + 1/k \tag{11.9}$$

in space the *k*-periodic trajectories will be stabile. In them the particle for *k* periods will make *k* + 1 turnover, after which it again continues motion on the same trajectory. If $\bar{\varphi}_T$ = 2, the particle for one period will make two turnovers. Its trajectory will represent two close to a circle curves, enclosed one into the other and adjoining on a horizontal axis.

With increase of $|\alpha_1|$ the radius of apocentre, as it is seen from Appendix 5, decreases from $\bar{R}_a \to \infty$ at α_1 = - 0.5 up to $\bar{R}_a$ = 1 at α_1 = -1. With increase of β_p the radius of apocentre also decreases and comes nearer to $\bar{R}_a$ = 1 at $\beta_p = \beta_{pc}$.

The motion of planets at the pericentres happens at small velocities. The observable parameters of planets are enumerated in the parameters of their trajectories. At this stage the various algorithms of calculation and various parameters of trajectories are possible. We will base calculations on the following data about Mercury: the length of a semi-axis *a*, the eccentricity ε_t, the period T, the distance 1 AU, the mass of the Sun m_s. The parameter of trajectory according to (5.14), α_1 through an eccentricity ε is determined so:

$$\alpha_1 = -1/(\varepsilon_t + 1), \tag{11.10}$$

and the radius of the apocentre pursuant to (5.15) will be

$$R_p = a\,\frac{2\alpha_1 + 1}{\alpha_1}\ . \tag{11.11}$$

As the velocity at the pericentres υ_p enters in α_1, it can be expressed through the parameter of trajectory

$$\upsilon_p = \sqrt{\frac{G\,m_s}{R_p\,(-\alpha_1)}}\ . \tag{11.12}$$

After a substitution in (11.12) R_p and α_1 we obtain a normalised velocity at the pericentres

$$\beta_p = \sqrt{\frac{G\,m_s(1+\varepsilon_t)}{(1-\varepsilon)\,a_1\,c^2}}\ . \tag{11.13}$$

Substituting numerical values of parameters, we discover the following values for Mercury orbit: α_1 = – 0.829, $\beta_p = 1.96 \cdot 10^{-4}$.

How it is visible from Fig. 11.1, at small β_p < 0.3 the periods of trajectory $\bar{\varphi}_T$ come nearer to unit asymptotically, so it is impossible the to evaluate value $\bar{\varphi}_T$ at such small β_p, which are inherent to Mercury. The numerical integration of equations (4.80), (4.82) at such parameters gives an error much more exceeding an expected perihelion $\delta\bar{\varphi}_T$ advance. In this connection we consider the approximate solution of the trajectory equation at small β_p. Let's record an exponential multiplicand, included in radial velocity,

$$\exp\frac{2\,\mu_1/(c_1^2\,R)}{\left[1 - h^2/(c_1^2\,R^2)\right]^{0.5}} = \exp\frac{-\,\bar{r}_g}{\sqrt{1-\beta_t^2}} = \mathrm{e}^{-A}\ , \tag{11.14}$$

where $\bar{r}_g = R_g/R$; $\quad \beta_t = \upsilon_t^2/c_1^2 = h^2/(c_1^2\,R^2)$; $\quad A = \dfrac{\bar{r}_g}{\sqrt{1-\beta_t^2}}$.

As we consider the Sun, for it $\bar{r}_g$ << 1, and at small β_t value A << 1. Therefore, the exponential curve can be decomposed in Taylor's number:

$$e^{-A} = 1 - A + \frac{A^2}{2} - \frac{A^3}{6} = 1 - \frac{\bar{r}_g}{\sqrt{1-\beta_t^2}} + \frac{\bar{r}_g^2}{2\left(1-\beta_t^2\right)} - \frac{\bar{r}_g^3}{6\left(1-\beta_t^2\right)^{1.5}} + \ldots$$

Decomposing denominators of addends in a right member and limiting by the terms of the second order, we obtain

$$e^{-A} = 1 - \frac{R_g}{R} + \frac{R_g^2}{2R^2} - \frac{BR_g^2 + R_g^3/6}{R^3} + \frac{BR_g^3}{2R^4} + \frac{3B^2R_g^2}{8R^5} + \ldots \tag{11.15}$$

where

$$B = \frac{h^2}{(-2\mu_1)}. \tag{11.16}$$

In (11.15) we limited by addends with exponents *r* less than 6. Substituting (11.15) in (4.80), with allowance for the addends, which are not higher than R^3 we have

$$v_r = c_1 \sqrt{1 - D + \frac{DR_g}{R} - \frac{1}{R^2}\left(\frac{DR_g^2}{2} + \frac{h^2}{c_1^2}\right) + \frac{DE}{R^3}}, \qquad (11.17)$$

where

$$D = \left(1 - \beta_p^2\right) \exp \frac{2\mu_1}{c_1^2 \sqrt{R_p^2 - h^2/c_1^2}}; \qquad (11.18)$$

$$E = \frac{R_g^2}{6} - \frac{h^2 R_g^2}{2\mu_1}. \qquad (11.19)$$

Substituting v_r in (4.82), the equation of trajectory will be the following:

$$\varphi = -\frac{h}{c_1} \int \frac{\mathbf{d}y}{\sqrt{F + My - Ny^2 + DEy^3}}, \qquad (11.20)$$

where

$$F = 1 - \left(1 - \beta_p^2\right) exp \frac{2\mu_1}{c_1^2 \sqrt{R_p^2 - h^2/c_1^2}}; \qquad (11.21)$$

$$M = DR_g; \qquad (11.22)$$

$$N = \frac{DR_g^2}{2} + \frac{h^2}{c_1^2}; \qquad (11.23)$$

$$y = 1/R. \qquad (11.24)$$

At $\beta_0 \to 0$ factors have the following order: $F \approx 1$; $D \approx 1$; $R_g \approx 1/c_1^2$; $E \approx 1/c_1^4$; $M \approx 1/c_1^2$; $N \approx h/c_1^2$; $DE \approx 1/c_1^4$. Therefore, neglecting an addend with y^3, we can integrate the expression (11.20) in a boundary condition $\varphi(R_p) = 0$, i.e. the angle is counted from the pericentre:

$$\varphi = \frac{1}{\sqrt{Nc_1^2/h^2}} \arcsin \frac{M - 2N/R}{\sqrt{4FN + b^2}}\Bigg|_{R_p}^{R}. \qquad (11.25)$$

Expression (11.25) with allowance for the boundary condition can be copied as

$$R = \frac{2N}{M + \sqrt{4FN + b^2}\cos\left(\sqrt{N\,c_1^2/h^2}\,\varphi\right)}. \qquad (11.26)$$

According to (11.26), at $\varphi = 0$, the radius receives a minimum value $R = R_p$, and the maximum value $R = R_a$ will be at

$$\sqrt{Nc_1^2/h^2}\;\varphi_a = \pi. \qquad (11.27)$$

As the full angular period is equal to $\varphi_T = 2\varphi_a$, the displacement of the pericentre for one period with allowance for (11.27) will be

$$\delta\varphi_T = \varphi_T - 2\pi = 2\pi\left[\frac{1}{\sqrt{Nc_1^2/h^2}} - 1\right]. \qquad (11.28)$$

Using expansion (11.15), we will transform a radicand

$$Nc_1^2/h^2 = 1 + \frac{2\mu_1^2\left(1 - \beta_p^2\right)}{c_1^2 h^2}\left[1 - \frac{R_g}{R_p} + \frac{R_g^2}{2R_p^2}\right]. \qquad (11.29)$$

With allowance for (11.29) the displacement of the pericentre for one turnover (11.28) will be written so:

$$\delta\varphi_T = -2\pi\alpha_1^2\beta_p^2\left(1 - \beta_p^2\right)\left[1 - \frac{R_g}{R_p} + \frac{R_g^2}{2R_p^2}\right]. \qquad (11.30)$$

As a gravitational radius of planets is $R_g << R_p$, the influence of addends in the second bracket is not essential, therefore, finally at $\beta_p << 1$ a rotation of the pericentre is obtained as

$$\delta\varphi_T = -2\pi\alpha_1^2\beta_p^2. \qquad (11.31)$$

As is seen, the displacement of the pericentre happens in the opposite direction, i.e. for a period the particle passes along the orbit an angular distance, smaller than 2π. It contradicts all solutions, obtained by us, including the one shown in Fig. 11.1. On an absolute value the displacement (11.31) exceeds the values, calculated numerically:

Parameters		$\delta\varphi_T$	$\lvert\delta\varphi_T\rvert$
α_1	β_p	numerical calculation	under the formula (11.31).
– 0.7	0.1	0.0128	0.029
– 0.7	0.3	0.15	0.27
– 0.7	0.5	0.60	0.769

Thus, the obtained approximate analytical solution (11.31) is mistaken.

In Fig. 11.2 a relative period of calculated trajectories is constructed, depending on $\beta_p^{2.5}$. In such processing the relation at $\beta_p \to 0$ become linear:

$$\overline{\varphi}_T = 1 + A\beta_p^{2.5}. \qquad (11.32)$$

Then the rotation of the pericentre will be

$$\delta\varphi_T = 2\pi\left(\overline{\varphi}_T - 1\right) = 2\pi A\beta_p^{2.5} \quad . \qquad (11.33)$$

For Mercury at α_1 = - 0.829 from Fig. 11.2 we define A = 0.8 is defined. Then according to (11.33) a displacement of the pericentre at $\beta_p = 1.96 \cdot 10^{-4}$ will make $\delta\varphi_T = 2.7 \cdot 10^{-9}$. For 100 earth years Mercury does 415 revolutions, therefore, the displacement for this period will make $\delta\varphi_T = 0.23''$.

An extremely small size of displacement is problematic not only for observation, but also for calculation. We have shown, that the approximate solution of an equation gives a large error. For Mercury the equations (4.80) and (4.82) were also numerically integrated. It appeared, that the results hardly depend on conditions of integration. The reason of errors in the integration of a trajectory equation (4.80) consists in the fact that on the boundaries of integration $R = R_p$ and $R = R_a$ a radial velocity reduces in zero.

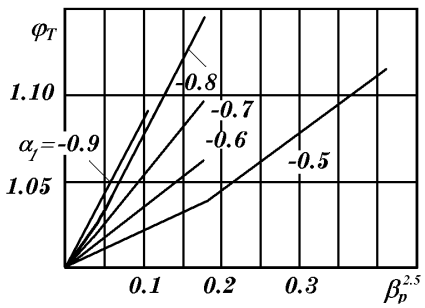


Fig. 11.2. Asymptotic relations $\overline{\varphi}_T$ at small β_p.

To receive the approached solutions with error smaller $2.7 \cdot 10^{-9}$ is the big problem. Apparently, the method, which we used to define a correct offset value of Mercury perihelion, is the only possible in this case.

Perihelion advance (11.4), obtained by Gerber and confirmed by A. Einstein after a substitution ε, T and a, according to (5.14), (5.36) and (5.15), receives a kind

$$\delta\varphi_T = 6\pi\alpha_1^2\beta_p^2 \quad . \qquad (11.34)$$

As we can see, the expression, obtained by us, (11.31) gives in 3 times smaller on an absolute value result. G.I. Suhorukov and others [64] have received in 3 times a smaller perihelion advance of Mercury on a comparison with (11.34), too the numerical value (11.34) for Mercury makes $\delta\varphi_T = 5\ 10^{-7}$, which, though in 200 times more than the value, obtained by us, $2.7\ 10^{-9}$, nevertheless, is rather represents enough small. In A. Einstein work [76] the problems of two bodies are reduced to an integral

$$\varphi = \int \frac{\mathbf{d}y}{\sqrt{\frac{2A}{B^2} + \frac{\alpha}{B^2}y - y^2 + \alpha y^3}}, \qquad (11.35)$$

which is identical to an integral, obtained by us, (11.20). The latter was solved limiting by the addends with y^2. A. Einstein integrated (11.35) also approximately, but by the other method. This explains the difference of an result (11.34) from (11.31). And both they are incorrect approximate solution of a precise equation of trajectory (4.80) - (4.82).

In summary we will bring some totals.

1. The real value of planets perihelions precession now remains uncertain.
2. The relation of force of interaction of two bodies to speed results in the precession of the pericentre. At very small speeds the displacement of pericentre for a turnover is determined by the expression (11.33), and for Mercury it makes $2.7 \cdot 10^{-9}$ radians or 0.23″ per one century. This effect represents such a small, size that it can never be confirmed by the observations.
3. The offset value of Mercury perihelion determined by U. Le Verrier on the basis of Newton's gravitation law, is doubt full and for its finding the problem of many bodies must be decided.

11.3. INTERACTION OF MANY BODIES

According to the law of world gravitation (11.1), the body with mass m_2 gives body with mass m_1 acceleration

$$\vec{w}_{21} = \frac{\vec{F}}{m_1} = -\frac{Gm_2}{R_{21}^3}\vec{R}_{21} \quad , \qquad (11.36)$$

where $\vec{R}_{21}$ is a position vector from the second body to the first one. The acceleration of the first body is determined by the mass of an acting body and the distance between bodies.

If there is a third body, it according to (11.1) will give the first body the acceleration

$$\vec{w}_{31} = \frac{\vec{F}}{m_1} = -\frac{Gm_3}{R_{31}^3}\vec{R}_{31} \quad . \qquad (11.37)$$

Having N of bodies each of N - 1 bodies gives to object m_1 the acceleration, defined by the mass of an acting body and distance up to it. Therefore, the full acceleration of body m_1 will be written so:

$$\vec{w}_1 = \sum_{i=2}^{N}\vec{w}_{i1} = -G\sum_{i=2}^{N}\frac{m_i\,\vec{R}_{i1}}{R_{i1}^3}. \qquad (11.38)$$

Under the action of acceleration $\vec{w}_1$ the speed $\vec{v}_1$ of body m_1 and its arrangement $\vec{R}_{i1}$ in relation to the other bodies varies. It results in change of value of acceleration $\vec{w}_1$. Similar acceleration has each of N - 1 bodies. Thus, the acceleration of

each body varies with the passage of time, the velocities and position of bodies vary. The determination of these values makes a problem of many bodies.

We will consider the interaction of spherical bodies (particles) among them. The system of bodies is isolated, i.e. it is not acted on outside. The particles at the motion can adjoin and merge. At the collision of particles the kinetic energy of motion passes in a thermal energy of particles. The contact can happen from the tangent component of speed, therefore, the particle after confluence gains it's own rotation. The listed problems should be clarified as the result of the solution of many bodies problem.

We will decide this problem in Cartesian coordinate system *x, y, z* and the number of each body will be characterized by three numbers *i, j, k,* varying in limits,

$$i_1 \le i \le i_2 , \qquad j_1 \le j \le j_2 , \qquad k_1 \le k \le k_2 . \tag{11.39}$$

The total number of bodies is determined by expression

$$N = (i_2 - i_1 + 1)\,(j_2 - j_1 + 1)\,(k_2 - k_1 + 1). \tag{11.40}$$

In these symbols of particles mass will be m_{ijk}, and the coordinates and projections on them of velocities and accelerations will be written so:

$$x_{ijk}, y_{ijk}, z_{ijk}, u_{ijk}, \upsilon_{ijk}, w_{ijk}, \dot{u}_{ijk}, \dot{\upsilon}_{ijk}, \dot{w}_{ijk} \quad . \tag{11.41}$$

The position vector from the body with mass m_{ijk} up to the body m_{mnl} will be recorded with two indexes:

$$\vec{R}_{im} = \vec{i}\,(x_{mnl} - x_{ijk}) + \vec{j}\,(y_{mnl} - y_{ijk}) + \vec{k}\,(z_{mnl} - z_{ijk}) . \tag{11.42}$$

In such symbols a projection of a body acceleration m_{mnl} to the axis *x*, located under action remaining *N* - 1 bodies, according to (11.38) will accept a kind

$$\dot{u}_{mnl} = -G \sum_{\substack{i=i_1 \\ j=j_1 \\ k=k_1}}^{\substack{k=k_2 \\ j=j_2 \\ i=i_2}} \frac{m_{ijk}\,(x_{mnl} - x_{ijk})}{R_{im}^3} \text{ , при } R_{im} \ne 0. \tag{11.43}$$

Condition $R_{im} \ne 0$ means, that the summation is carried out on *N* - 1 particle, i.e. the particle m_{mnl} is eliminated from summation. The similar expressions will be written for projections of acceleration on an axis *y* and *z*.

As the result of integration of acceleration $\dot{u}_{mnl}$ we obtain a projection of a particle velocity to the axis *x*, and after an integration of speed we obtain the law of a particle motion m_{mnl} under the action of many bodies:

$$u_{mnl} = u_{mnl0} + \int_0^t \dot{u}_{mnl}\ \mathbf{d}t , \tag{11.44}$$

$$x_{mnl} = x_{mnl0} + \int_0^t u_{mnl}\ \mathbf{d}t , \tag{11.45}$$

where $x_{mnl0} = x_{mnl}(0)$, $u_{mnl0} = u_{mnl}(0)$, are the initial values of coordinates and the speed of particles at the moment instant $t = 0$. For projections of speed υ_{mnl}, w_{mnl} and coordinates y_{mnl}, z_{mnl} for the axes *y* and *z*, the expressions will be similar to ratios (11.44), (11.45). The equations (11.43) - (11.45), recorded for three projections, completely determine the motion of each from *N* bodies. Two groups of integral equations (11.44), (11.45) are included in this set of equations, which in other record will be two groups of differential equations. The total of these equations is $N_u = 3\ N\ 2 = 6N$. $6N$ beginning conditions should be known for the solution of N_u equations, i.e. initial positions and speeds of all *N* bodies in projections to 3 axes of coordinates. According to (11.42), in R_{im} the components of all projections are included, therefore, the equations depend from each other on coordinates and cannot be integrated separately.

To control the right solution of a problem it is necessary to watch the conservation of integral values. The mass of all bodies should be saved during the motion.

$$m = \sum_{ijk}^{N} m_{ijk} = \text{const} , \tag{11.46}$$

where the summation is conducted on indexes *ijk* for *N* particles.

As the projection of a momentum with mass m_{mnl} to the axis *x* is equal to $P_{mnl} = m_{mnl}\,u_{mnl}$, then *x* - projection of a momentum of the whole system of bodies with allowance for (11.44) will be

$$P_x = \sum_{mnl}^{N} m_{mnl}\,u_{mnl} = \sum_{mnl}^{N} m_{mnl}\,u_{mnl0} - G \int_0^t \sum_{mnl}^{N} \sum_{ijk}^{N-1} \frac{m_{mnl}\,m_{ijk}\,(x_{mnl} - x_{ijk})}{R_{im}^3}\mathbf{d}t.$$

Under an integral the double summation is made, at which the differences of final coordinates with the same masses are repeated twice, but with different signs. For example, at $N = 2$; $m = i = n = j = 1$; $l = k = 1, 2$ we have

$$\sum_{mnl}^{2} \sum_{ijk}^{1} m_{mnl}\,m_{ijk}\,(x_{mnl} - x_{ijk}) = m_{111}\,m_{112}\,(x_{111} - x_{112}) + m_{112}\,m_{111}\,(x_{112} - x_{111}) = 0 . \tag{11.47}$$

Therefore, the integrand expression is equal to zero, and momentum of all bodies

$$P_x = \sum_{mnl}^{N} m_{mnl}\,u_{mnl0} = P_{x0} = \text{const} , \tag{11.48}$$

i.e. is saved during the motion.

We will record the x-projection angular momentum of body:

$$M_{mnlx} = P_{mnlx}\, y_{mnl} - P_{mnlz}\, z_{mnl}. \qquad (11.49)$$

Then for the whole system with allowance for (11.43), (11.44) is

$$M_x = \sum_{mnl}^{N} m_{mnl}\left(w_{mnl0}\, y_{mnl} - \upsilon_{mnl0}\, z_{mnl}\right) - G\int_0^t \sum_{mnl}^{N}\sum_{ijk}^{N-1} \frac{m_{mnl}\, m_{ijk}\left(y_{ijk}\, z_{mnl} - z_{ijk}\, y_{mnl}\right)}{R_{im}^3}\mathbf{d}t\,.$$

It is easy to be convinced, that the double summation under an integral is similar to (11.47). It is signify that the *x*-projection of the angular momentum of system bodies, is the following

$$M_x = \sum_{mnl}^{N} m_{mnl}\,(w_{mnl0}\, y_{mnl} - \upsilon_{mnl0}\, z_{mnl}) = M_{x0} = \text{const}\,,$$

i.e. it is also saved during the motion. It is necessary to note, that at confluence of particles their quantity decreases, the angular momentums of bodies system will vary, and a part of it will pass in a angular momentum of bodies own rotation. The obtained results are fair for the components of momentums and angular momentum on the axes *y* and *z*:

$$M_y = \sum_{mnl}^{N} (P_{mnlx}\, z_{mnl} - P_{mnlz}\, x_{mnl}) = \text{const}\,; \qquad (11.50)$$

$$M_z = \sum_{mnl}^{N} (P_{mnly}\, x_{mnl} - P_{mnlx}\, y_{mnl}) = \text{const}\,.$$

By dividing a pulse (11.48) of bodies systems on the mass *m*, we will receive the speed it of motion along the axis *x*:

$$u = P_{x0}/m = \text{const}. \qquad (11.51)$$

As we can see, the system of bodies will move with constant speed, despite of the interactions, happening inside it. The speed of motion *u* is connect ed with centre of masses of a system, which *x*-coordinate is determined by the expression

$$x = \sum_{mnl}^{N} m_{mnl}\, x_{mnl}/m\,. \qquad (11.52)$$

The important characteristic of interacting bodies is the kinetic energy

$$E_c = 0.5\sum_{mnl}^{N} m_{mnl}\left(u_{mnl}^2 + \upsilon_{mnl}^2 + w_{mnl}^2\right). \qquad (11.53)$$

At the interaction of two bodies the trajectory (5.3) can be an ellipse, parabola or hyperbola. At elliptical orbit ($\alpha_1 < -0.5$) energy (11.53) will vary from minimum at the apocentres up to maximum at the pericentres. At hyperbolic orbit ($\alpha_1 > -0.5$) the energy of approach-correcting bodies will be increased, will reach a maximum at the pericentres, and then will decrease and be aimed to a constant value along the infinity. Such behaviour of a kinetic energy will be observed at the interaction of many bodies: at their retraction it will decrease, at retraction of bodies at one point it will to be increased, and at stabile existence of a system of bodies the kinetic energy must vary in constants limits. Therefore, the character of behaviour E_c in time will testify to stability of a system, and the inexplicable change of a kinetic energy can testify to the mistakes of a problem solutions.

If R_{mnl} and R_{ijk} - radiuses of interacting particles, when they approach at the distance $d < (R_{mnl} + R_{ijk})$ such particles we consider merged in one. As at collision the particles have relative closing velocities, the kinetic energy of particles will pass in a thermal energy of the merged particle. Let's calculate it. Let's consider the motion of a particle m_{ijk} relatively a particle m_{mnl}. The projections of it's relative attitude and speed (see Fig. 11.3) on an axis *x* will be written so:

$$x_r = x_{ijk} - x_{mnl}; \qquad (11.54)$$

$$u_r = u_{ijk} - u_{mnl}. \qquad (11.55)$$

From the equality of a static moment relatively the point C_x

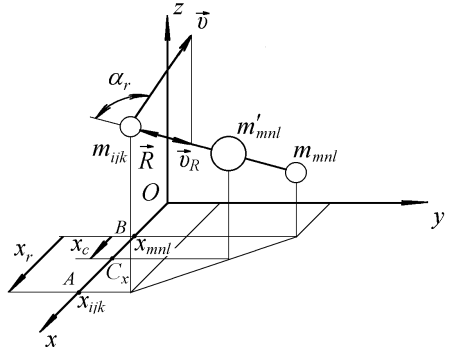


Fig. 11.3. Relative velocities at confluence of two particles.

$$m_{mnl}\, x_c = (x_r - x_c) m_{ijk}. \qquad (11.56)$$

x-coordinate of the centre of two masses is determined

$$x_c = \frac{m_{ijk}\, x_r}{m_{mnl} + m_{ijk}}\,. \qquad (11.57)$$

Then new *x*-coordinate of the merged particle will be

$$x'_{mnl} = x_{mnl} + \frac{m_{ijk}\, x_r}{m_{mnl} + m_{ijk}}\,, \qquad (11.58)$$

and its mass is $m'_{mnl} = m_{mnl} + m_{ijk}$. If the mass $m_{mnl} < m_{ijk}$, to a particle is given a number of a greater particle m_{ijk} is, i.e. it mass is designated m'_{ijk}.

As *x*-component of a pulse of the merged particle is equal to a sum of momentums of making particles

$$P'_{mnlx} = P_{mnlx} + P_{ijkx}, \quad (11.59)$$

that after a substitution of their values we determine the expression for its speed:

$$u'_{mnl} = \frac{m_{mnl}}{m_{mnl} + m_{ijk}} u_{mnl} + \frac{m_{ijk}}{m_{mnl} + m_{ijk}} u_{ijk}. \quad (11.60)$$

The relative radius of a particle m_{ijk}

$$\vec{R} = \vec{i}\, x_r + \vec{j}\, y_r + \vec{k}\, z_r \quad (11.61)$$

and it relative speed (see Fig. 11.3)

$$\vec{v} = \vec{i}\, u_r + \vec{j}\, v_r + \vec{k}\, w_r \quad (11.62)$$

are directed to each other at the angle α_r, which can be expressed from determination of a scalar product $\vec{R}\vec{v} = Rv\cos(\alpha_r)$ as follows:

$$\cos(\alpha_r) = \frac{x_r u_r + y_r v_r + z_r w_r}{Rv}. \quad (11.63)$$

Then the radial speed of a particle m_{ijk} relatively a particle m_{mnl}, to which the approach happens, will be:

$$v_R = v\cos(\alpha_r) = \frac{x_r u_r + y_r v_r + z_r w_r}{R}. \quad (11.64)$$

At the collision of the particles their kinetic energy will turn to a thermal energy according to radial speeds of particles relatively the centre of masses. As relatively the centre of masses the radial pulses of particles are equal, their radial speeds will be

$$v_{Rmnl} = v_R \frac{m_{ijk}}{m_{mnl} + m_{ijk}}; \quad v_{Rijk} = v_R \frac{m_{mnl}}{m_{mnl} + m_{ijk}}. \quad (11.65)$$

The kinetic energy of two merged particles will pass in the increment of a thermal energy of a new particle. Then its thermal energy with allowance for thermal energy of the particles components E_{mnl} also E_{ijk} will be written:

$$E'_{mnl} = E_{ijk} + E_{mnl} + \frac{m_{mnl} v^2_{Rmnl}}{2} + \frac{m_{ijk} v^2_{Rijk}}{2} = E_{ijk} + E_{mnl} + \frac{m_{ijk} m_{mnl}}{m_{mnl} + m_{ijk}} \frac{v^2_R}{2}, \quad (11.66)$$

where a relative closing velocity v_R should be determined at a distance between particles equal to a sum of their radiuses:

$$R = d = R_{mnl} + R_{ijk}. \quad (11.67)$$

Approach-correcting particles, as it is seen from Fig. 11.3, except radial making speed have transversal, which at the formation of the merged particle will give it the rotation. Let's calculate the own moments, i.e. the spins S of new-formed particles. Let's consider the projections of speeds and the position of a particle on

a plane yz (see Fig. 11.4). Let's define the angular particles momentums relatively the centre of masses C:

$$\Delta S'_{mnlx} = m_{ijk} (-w_{ijk} MN - v_{ijk} GF) + m_{mnl} (w_{mnl} ND + v_{mnl} FE), \quad (11.68)$$

where distances up to centre of masses C are determined similarly to (11.57):

$$MN = -\frac{m_{mnl}}{m'_{mnl}} y_r, ND = -\frac{m_{ijk}}{m'_{mnl}} y_r; \quad GF = \frac{m_{mnl}}{m'_{mnl}} z_r, FE = \frac{m_{ijk}}{m'_{ijk}} z_r. \quad (11.69)$$

Here y_r and z_r are determined similarly (11.54).

After a substitution of distances (11.69) in (11.68) we obtained the x-projection of a spin increment of the merged particle

$$\Delta S'_{mnlx} = m_{re} (w_r y_r - v_r z_r), \quad (11.70)$$

where

$$m_{re} = \frac{m_{mnl} m_{ijk}}{m_{mnl} + m_{ijk}}; \quad (11.71)$$

m_{re} is a normalised (reduced) mass at the interaction of two particles.

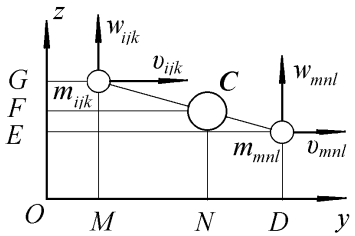


Fig. 11.4. Formation of a projection of a spin on an axis x at confluence of particles.

The projections of a spin increment of the merged particle on the axes y and z will be similarly recorded. And with allowance for the spins of initial particles the spin of the merged particle on these axes will be:

$$S'_{mnly} = S_{ijky} + S_{mnly} + m_{re} (u_r z_r - w_r x_r), \quad (11.72)$$

$$S'_{mnlz} = S_{ijkz} + S_{mnly} + m_{re} (v_r x_r - u_r y_r). \quad (11.73)$$

In expressions (11.70), (11.72), (11.73) a relative speeds v_r and w_r are similarly (11.55). The relative distances and speeds at determination of increment of spins should be calculated at the moment of particles contact. However, as the interaction of two merging particles at the moment of confluence much more exceeds of action of the remaining, ones the value of a angular momentum will be saved on small distances before the confluence. Therefore, the increment of a spin can be determined at the point close to the point of confluence.

The summation of spins on all particles represents total systems angular momentum made in the own rotation of bodies:

$$S_x = \sum_N S_{mnlx}, \; S_y = \sum_N S_{mnly}, \; S_z = \sum_N S_{mnlz}. \quad (11.74)$$

The spins of a system S_x, S_y, S_z and the moment of pulses M_x, M_y, M_z make a total system angular momentum of the of interacting bodies, which components

$$M_{sx} = M_x + S_x, \qquad M_{sy} = M_y + S_y, \qquad M_{sz} = M_z + S_z, \qquad (11.75)$$

are saved during the existence of an isolated system of interacting bodies at every possible motions and transmutations.

Alongside with a kinetic energy E_c of a system it is necessary to consider a thermal energy

$$E_t = \sum_N E_{mnl} \qquad (11.76)$$

If the problem is decided disregarding the heat transfers, the thermal energy will grow in a developing system of bodies, and in the stationary one ones it will remain constant. Especially strong increase of a thermal energy will happen at an accretion of a substance.

The problem of many bodies represents the greatest interest in the studying behaviour of large-scale arrays of substance in the universe. The solution of these problems will allow to understand the gear of formation of planets, stars, planetary systems, sidereal accumulations, galaxies and other objects, which are observed and can be observed in space. Originally, these arrays of substance can be considered as statically homogeneous, with initial mean density

$$\rho_0 = \frac{m}{abc}, \qquad (11.77)$$

where a, b, c are values of an area along the axis x, y, z, accordingly. The form of area can be not a parallelepiped, then the values a, b, c will be the values of a parallelepiped, enveloping the area. The coordinate axes are directed so that $a > b > c$. Let's pass to the dimensionless variables:

$$\tilde{x} = x/a, \quad \tilde{y} = y/a, \quad \tilde{z} = z/a, \quad \tilde{m}_{ijk} = m_{ijk}/m. \qquad (11.77,\text{a})$$

After a substitution them in (11.43), we obtain expressions for acceleration in the normalised kind

$$\tilde{\ddot{u}}_{mnl} = -\sum_{N-1} \frac{\tilde{m}_{ijk}\left(\tilde{x}_{mnl} - \tilde{x}_{ijk}\right)}{\tilde{R}_{im}^3}, \qquad (11.78)$$

where $\tilde{R}_{im} \neq 0$. Thus we determine the transformation for time, and also for a derivative from coordinates on time as follows:

$$\tilde{t} = t\sqrt{\frac{Gm}{a^3}} = t\sqrt{G\rho_0 \tilde{b}\tilde{c}}; \qquad \tilde{u} = u\sqrt{\frac{a}{Gm}}; \qquad \tilde{\dot{u}} = \dot{u}\frac{a^2}{Gm}, \qquad (11.79)$$

here

$$\tilde{b} = b/a, \qquad \tilde{c} = c/a. \qquad (11.80)$$

As we can see, the normalised acceleration (11.78) differs from (11.43) by absence of G. Therefore, it is possible to consider all consequent equations, since (11.44), in the normalised kind, substituting $G = 1$ and the absolute parameters on dimensionless ones, noted above ~.

The array of the substance in volume $V = abc$ can be divided into elementary volumes $V_{ijk} = \tilde{V}/N$, in which centre we locate the bodies with masses

$$m_{ijk} = m/N. \qquad (11.81)$$

To have the sides of elementary volumes close on value at different forms of areas, we will act in the following way. The quantity of steps, i.e. the intervals between bodies along axes x, y, z, we will designate M_S, N_S, K_S accordingly. Then the distance between bodies along the axis x will be $\Delta x = a/M_S$, and the quantity of steps along the axes y and z we take as the whole part of fractions:

$$N_S = \mathrm{Int}(b/\Delta x), \qquad K_S = \mathrm{Int}(c/\Delta x). \qquad (11.82)$$

If i_1, j_1, k_1 is a number of the first nodes on axes x, y, z accordingly, the numbers of the latter ones will be written

$$i_2 = M_S + 1; \qquad j_2 = N_S + 1, \qquad k_2 = K_S + 1, \qquad (11.83)$$

and the total number of nodes N will be defined by expression (11.40).

The masses of bodies m_{ijk} are put in nodes. If mean, on their diameter, density of bodies ρ_{ijk}, than, considering bodies as orb its, their, radiuses will be defined as

$$R_{ijk} = \left(3m_{ijk}/4\pi\rho_{ijk}\right)^{1/3}. \qquad (11.84)$$

The density of celestial bodies ρ_{ijk} vary over a wide range. To calculate them, it is necessary to set them, certain relations of radiuses of body from the other properties, for example the masses and temperatures. The radius of bodies is necessary for calculation of a distance d between the bodies during their confluence. At the first stage it is possible for all particles to set identical density, to a equal for example, mean density of the Earth:

$$\rho_{ijk} = \rho_p \approx 5 \cdot 10^3 \text{ кг/м}^3. \qquad (11.85)$$

Let's record in a normalised kind a distance between bodies during a contact with allowance for of density (11.77) and (11.85), when the masses of particles are determined (11.81)

$$\tilde{d} = \frac{R_{mnl}}{a} + \frac{R_{ijk}}{a} = \left(\frac{3\tilde{b}\tilde{c}\rho_0}{4\pi N\rho_{mnl}}\right)^{1/3} + \left(\frac{3\tilde{b}\tilde{c}\rho_0}{4\pi N\rho_{ijk}}\right)^{1/3}. \qquad (11.86)$$

At identical density of particles ρ_p the normalised distance between the particles depends on a ratio of density as follows:

$$\tilde{d} = 2\left(\frac{3\tilde{b}\tilde{c}\rho_0}{4\pi N\rho_p}\right)^{1/3}. \qquad (11.87)$$

The important conclusion follows from here. All equations, since (11.78), and then from an equation (11.44) on (11.53), recorded in a normalised kind (G = 1 and variables with ~), for want of collisions do not depend on the value of area. Therefore, the solution of a problem of many bodies for the formation of a planetary system can be widespread on the formation of a sidereal system, sidereal accumulations, galaxies etc. However, if there is confluence of bodies the normalised value of confluence depends on relative density ρ_0/ρ_p. If ρ_p for usual celestial bodies varies not very much, ρ_0 with the increase of the value of the area decreases considerably. It will result in decreasing $\tilde{d}$ with increase of the value of area, therefore the integration is necessary for conducting up to the smaller values. The thermal energy of bodies will be increased by consideration of a problem of many bodies for large areas. Thus, the problem becomes not self-similar.

Despite an apparent simplicity of many gravitating bodies problem: the simple equation for force (11.1) plus the equation of motion (2.4), in the literature [49] There are a lot of diverse statements of its solution. We have begun this problem with the purpose to confirm the mechanisms, of formation of circulation at formation of vortexes in artificial [58] and natural [62] conditions with the reference to formation of space systems. During work we have met the problems of mistaken solutions, which made us search for the other statements of a problem. Explained above is the last one. We consider it the most successful. This statement allows monitoring the parameters and destiny of each body. At the same time, it envelops a whole system and allows to understand as it develops, what its properties depend on, how they will vary. The given statement enables to consider also the interaction between the systems and influence of external systems to the absolute properties of a system.

The normalised statement allows simply setting an initial condition of a system, for example, its initial rotation, its dominant mass, and the degree of a discretization of a system. By giving number N of bodies in a system the same program can be adapted to different ones on power of the COMPUTER. By a variation of number of bodies N in a system it is possible to check up the relation of the solution to a degree of its discretization etc.

On the considered statement the program Galactica on the Turbo Basic language was developed. With its help it is possible to consider problems of any number of bodies, for example at M_S = 1; $\tilde{b}$ = 0,5; $\tilde{c}$ = 0,25 5 the problem of two bodies will be decided. Examples solving the problems of two, three and four bodies by this program are given below.

11.4. EXACT SOLUTION OF AXISYMMETRIC MANY BODIES PROBLEM

We will consider the interaction of n bodies, evenly located on a circle with the radius r_0 (Fig. 11.5) at the angular interval $\varphi_1 = 2\pi/n$. The body m_2 will give the body m_1 acceleration, which projection to an axis x is equal to

$$w_{21x} = -\frac{Gm_2}{AB^2}\cos\alpha . \qquad (11.88)$$

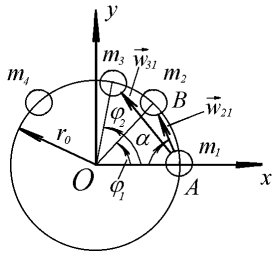
Triangle OBA is an isosceles, therefore $2\alpha + \varphi_1 = \pi$, whence we obtain:

$$\alpha = (\pi - \varphi_1)/2 = (\pi - 2\pi/n)/2 = \frac{\pi}{2} - \frac{\pi}{n} . \qquad (11.89)$$

From this triangle the distance $AB = 2\ r_0 \sin(0{,}5\varphi)$ is determined. Then the acceleration (11.88) will be written so:

$$w_{21x} = -\frac{Gm_2}{4r_0^2\sin(\pi/n)} . \qquad (11.90)$$

The angular distance between a particle m_3 and m_1 is equal to $\varphi_2 = 2\varphi_1 = \frac{4\pi}{n}$ and it will give a particle m_1 acceleration along the axis x



$$w_{31x} = -\frac{Gm_3}{4r_0^2\sin(2\pi/n)} . \qquad (11.91)$$

The particle m_4 will give the particle m_1 acceleration

$$w_{41x} = -\frac{Gm_4}{4r_0^2\sin(3\pi/n)} \qquad (11.92)$$

Fig. 11.5. Interaction of the axisymmetricly located bodies.

At an even number of particles $n = 2k_e$, where k_e is the integer, at the opposite m1 end of diameter will be a particle with number is $k_e + 1$, and the remaining particles with numbers $(k_e + 2) \div n$ will be located on the lower semicircle, symmetric ally to particles $i = 2 \div k_e$.

For each of the particles on the upper semicircle the multiplicand with sine can be written as $1/\sin[(i-1)\pi/n]$, which for a particle with number $(k_e + 1)$ will be equal to 1. Therefore, according to (11.90) - (11.92), the total acceleration given to the particle m_1 by remaining n - 1 particles with equal masses $m_1 = m_2 = m_3 = \ldots m_n$, will be written as

$$w_{1x} = -\frac{Gm_1}{4r_0^2}\left(1 + \sum_{i=2}^{k_e}\frac{2}{\sin[(i-1)\pi/n]}\right) . \qquad (11.93)$$

At odd number of particles $n = 2k_0 + 1$ on the opposite m_1 end of the diameter there will be no particle and the first addend in (11.93) will be absent. In this case the total acceleration of the first particle in a projection to the axis x will be the following:

$$w_{1x} = -\frac{Gm_1}{2r_0^2}\sum_{i=2}^{k_0+1}\frac{1}{\sin[(i-1)\pi/n]} . \qquad (11.94)$$

By virtue of a rotational symmetry the acceleration of a particle m_1 on the axis y is equal to zero, therefore, the acceleration w_{1x} determines the action of all particles completely. From Fig. 11.5 and expressions (11.93) and (11.94) it is seen, that each from n particles is given the acceleration, by remaining ones which is directed to their centre of masses and which in an inverse proportion to quadrate of a particle distance up to it, i.e.

$$\vec{w}_1 = -\frac{Gm_1\vec{r}_0}{r_0^3} f(n), \qquad (11.95)$$

where at an even number of particles

$$f(n) = 0{,}25\left(\sum_{i=2}^{k_e}\frac{2}{\sin[(i-1)\pi/n]}+1\right), \qquad (11.96)$$

and at the odd one

$$f(n) = 0{,}5\sum_{i=2}^{k_0+1}\frac{1}{\sin[(i-1)\pi/n]}. \qquad (11.97)$$

The acceleration (11.95) for a particle m_1 can also be written as the force of action on it

$$\vec{F}_1 = -\frac{Gm_1^2\vec{r}_0}{r_0^3} f(n). \qquad (11.98)$$

The multiplicand $f(n)$ depends on mutual arrangement of bodies. If during motion it will not vary, i.e. $f(n)$ = const, the acceleration (11.95) or force (11.98) will differ from the force of the world gravitation (11.1) only by constants. Therefore, the motion of each of n bodies will happen under the action of a central force (11.98) similarly to the motion of one body, being under the action of the other. Their trajectories will be conic sections, in which there is a centre of masses in a focal point.

In connection with an identical kind of forces (11.98) and (11.1) these interactions can be united node, i.e. in centre of masses (p. O in a Fig. 11.5) in addition to place n + 1 body with mass m_0, which will act on each from n bodies by force (11.1) or to give acceleration.

$$\vec{w}_0 = -\frac{Gm_0\vec{r}_0}{r_0^3}. \qquad (11.99)$$

Then the total acceleration of each from n bodies will be equal to a sum of accelerations (11.95) and (11.99):

$$\vec{w} = \frac{\mathbf{d}^2\vec{r}_0}{\mathbf{d}t^2} = -\frac{G\vec{r}_0}{r_0^3}\left[m_0 + m_1 f(n)\right]. \qquad (11.100)$$

As we have shown in chapter 5, at Coulomb interaction, fair for the given case, too the trajectory is described by expression (5.3), and the time of motion along it is (5.22), (5.25), (5.27) and (5.30). The parameters of motion are determined by the parameter of trajectory α_1, which depends on the parameter of the

interaction μ_1, included in the equation, of the relative motion (4.63). Comparing equations (4.63) and (11.100), we obtain the expression for the parameter of interaction in a problem of axisymmetricly located n bodies round a central body m_0 as follows:

$$\mu_1 = -G\left[m_0 + m_1 f(n)\right]. \qquad (11.101)$$

The expression (11.101) is the key in this problem. With allowance for (11.101) equations (11.100) can be written so:

$$\frac{\mathbf{d}^2\vec{r}_0}{\mathbf{d}t^2} = \frac{\mu_1\vec{r}_0}{r_0^3}.$$

It has the solution as an equation of trajectory (5.3), where the parameter of trajectory

$$\alpha_1 = \frac{\mu_1}{r_{0p}v_{0p}^2}, \qquad (11.101a)$$

where r_{0p} and v_{0p} are the parameters of pericentres of particles in relation to the centre of masses of a system of bodies.

The radial velocity is calculated according to (5.1), and the transversal is equal to $\bar{v} = 1/\bar{r}_0$. All values here are normalized to the parameters at the pericentres v_{0p} and r_{0p}. The time of motion of bodies on trajectories is determined according to (5.21), (5.25), (5.26) and (5.30), where the parameters are normalized to values at any point r_{00}, with speed v_{0t0} and v_{0r0} relatively the centre O (Fig. 11.5). The motion of each n body happens at the conservation of a kinematic moment h = const. If to accept, that in Fig. 11.5 the bodies are represented at the pericentres, the line of apsides of the body m_1 passes on the axis x. Its apocentres will be located on the negative values of the axis x. The apsides of body m_2 is inclined to the axis x at the angle $\varphi_1 = 2\pi/n$, body m_3 – at the angle $\varphi_2 = 2\bullet 2\pi/n$; m_4 – $\varphi_3 = 3\bullet 2\pi/n$ etc. The central body with mass m_0 will be motionless at p. O.

So, the axisymmetrical problem of n bodies is reduced to a problem of the interaction of two bodies. The motion of each from them happens along a constant in space trajectory, which belongs to a set of a curve conic section and is determined by the parameter α_1. The solution of this problem has the important theoretical value. For example, when $m_0 >> m_i$ the system represents a central star and moving round it n planets. In this case, despite of actions of the planets against each other, their orbits are constant in space and the pericentres do not preces. There are many other questions, which can be answered by solutions of this problem. The practical value of the given problem consists in the capability to determine an actual error of computing algorithms of many bodies problem.

11.5 TRAJECTORIES OF MOTION AT INTERACTION OF SEVERAL BODIES.

We will analyse the results of the program Galactica, solution which algorithm is circumscribed in item 11.3. At the beginning we will consider the interaction of two bodies with different mass: $m_1 = 2m_2$. At the initial moment t = 0 a

position of particles at a point of apsides and transversal speed v_{t0} are set. The trajectories of motion of bodies are considered at change v_{t0} = 0 up to some value, characteristic of hyperbolic trajectories. At zero transversal velocity the particles move radial up to a contact.

The motion happens relatively the centre of masses *O* (Fig. 11.6). The speed in pericentres of particles it is set by their rotation with angular rate ω relatively to p. *O*:

$$v_{p1} = \omega r_{01}, \quad v_{p2} = \omega r_{02}, \text{ where } r_{01} + r_{02} = a.$$

Then the velocity of one particle relatively other is $v_p = v_{p1} + v_{p2} = \omega a$. As the parameter of trajectory is in this case determined

$$\alpha_1 = -G(m_1 + m_2)/(av_p^2), \qquad (11.102)$$

then, passing to dimensionless parameters, according to (11.77a) and (11.79), we obtain

$$\alpha_1 = -\frac{1}{\tilde{\omega}^2}, \qquad (11.103)$$

where

$$\tilde{\omega} = \omega\sqrt{a^3/Gm}, \qquad m = m_1 + m_2. \qquad (11.104)$$

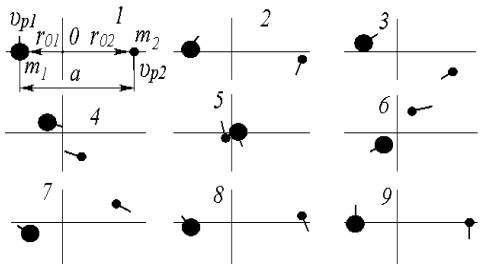


Fig. 11.6. Stages of motion of two bodies at α_1 = - 0.5597 at the instants 2.31 $\tilde{t}$:
1 - 0.001; *2* - 0.5; *3* - 1.5; *4* - 2.5; *5* - 3.0; *6* - 3.5; *7* - 4.5; *8* - 5.5; *9* -6.0.

At α_1 = -1, the particles move along the circles, at $\alpha_1 > -1$ particles move along the ellipses and the distance between them is increased. If $\alpha_1 < -1$, that happens at small angular rate $\tilde{\omega}$, the distance between particles will decrease. That is the particles begin motion not from the pericentre, but from the apocentre with parameters R_a and v_a. Therefore we will designate this value α_1 as α_1^a, according to determination, we have

$$\alpha_1^a = \frac{\mu_1}{R_a v_a^2} = \frac{\mu_1 R_a}{R_P^2 v_P^2} = \alpha_1 \frac{R_a}{R_P}. \qquad (11.105)$$

After a substitution in (11.105) values $\overline{R}_a$, according to (5.7), we obtain the expression for the parameter of trajectory

$$\alpha_1 = -\frac{\alpha_1^a}{2\alpha_1^a + 1}, \qquad (11.106)$$

where α_1^a - value of expression (11.103) at $\alpha_1 > -1$.

In Fig. 11.6 the moment of motion of two bodies are shown at initial angular rate $\tilde{\omega}$/2.31 = 0.2. The images are taken from a screen of the screen monitor. For each from nine positions the times are normalised with the factor 2.31 $\tilde{t}$. The introduction of a factor is stipulated by the fact, that this and all consequent illustrations are calculated according to the version of the program Galactica, in which the parameters were normalized not to total mass *m*, but to mean density ρ_0. In position *1* the bodies are at the apocentres. The primes at bodies show a direction of speeds of bodies. The bodies approach each other in a rotary motion relatively and in position *5* reach the pericentres. Then they are removed from each other and in position *9* come to the apocentres.

In Fig. 11.7 the trajectories of two bodies are submitted at change of initial speed ($\tilde{\omega}$/2.31) from zero up to 0,7. The initial distance between particles is equal to node. The beginning of coordinates is located in apsid by one of the particles. At $\tilde{\omega}$ = 0 (the trajectory *1*) happens radial approach of bodies. At $\tilde{\omega}$/2.31 = 0,1 particles move along prolate elliptical orbits, beginning the motion from the apocentre. With increase $\tilde{\omega}$ the eccentricity of orbits decreases and for case *6* the trajectories become the circles. With further increase of initial speed $\tilde{\omega}$, the particles begin to run up, i.e. the initial point for them is the pericentre. For situation *10* at ($\tilde{\omega}$/2,31) = 0,613, the trajectories of particles become parabolas, and with further increase $\tilde{\omega}$ they are transformed into hy-

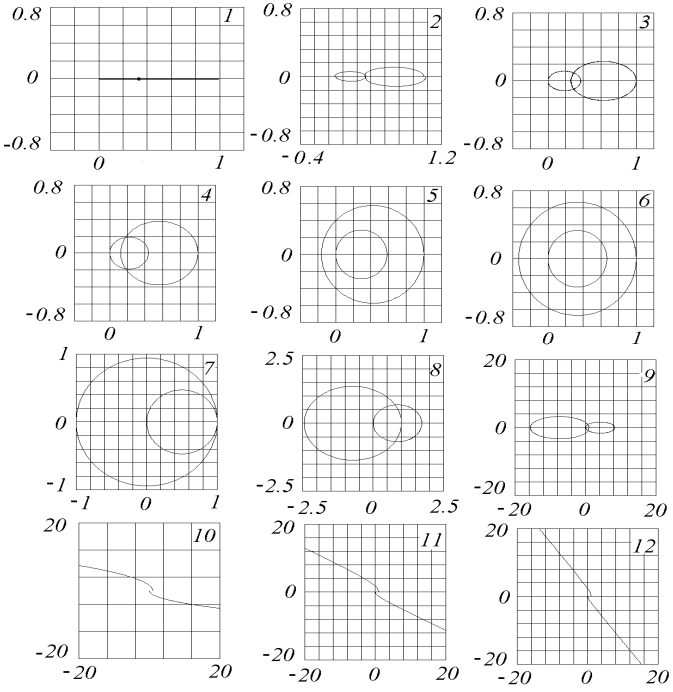


Fig. 11.7. Trajectories of motion of two particles at a variation of initial transversal velocity $\tilde{\omega}$/2.31 (in brackets parameter of trajectory α_1) is indicated:
1 - 0 (0);
2 - 0.1 (- 0.514);
3 - 0.2 (- 0.5597);
4 - 0.3 (- 0.658);
5 - 0.4 (-0.872);
6 - 0.433 (- 1);
7 - 0.5 (- 0.75);
8 - 0.55 (- 0.62);
9 - 0.6 (- 0.521);
10 - 0.613 (- 0.5);
11 - 0.63 (- 0.473);
12 - 0.7 (- 0.3826).

perbolas. With increase $\tilde{\omega}$, hyperbolas are rectified and at $\tilde{\omega} \to \infty$ are transformed in verticals. Shown in Fig. 11.7 numerical solutions of a problem of two bodies with a high degree of accuracy agree with the analytical solutions. In Fig. 11.7 it is illustrated by the normalised values of parameter α_1, on which value $\tilde{\omega}$ assigned at the numerical score, was calculated, according to (11.103), to receive trajectories as a circle *6* and parabola *10*. As the result of the numerical solution the trajectories as a circle and parabola were received and their parameters were maintained with high accuracy.

We will record the equations of these trajectories in a system of the centre of masses (see Fig. 11.6). As the distances of particles from the centres of masses are in the equation

$$r_{01}m_1 = r_{02}m_2,$$

and sum of distances $r_{01} + r_{02} = R$, where R - distance between particles,

$$r_{01} = R\, m_2/m, \qquad r_{02} = R\, m_1/m. \tag{11.107}$$

The equation of trajectory (5.3) at a position of pericentre at the angle φ_0 will be written as

$$R = \frac{R_P}{(\alpha_1 + 1)\cos(\varphi - \varphi_0) - \alpha_1}. \tag{11.108}$$

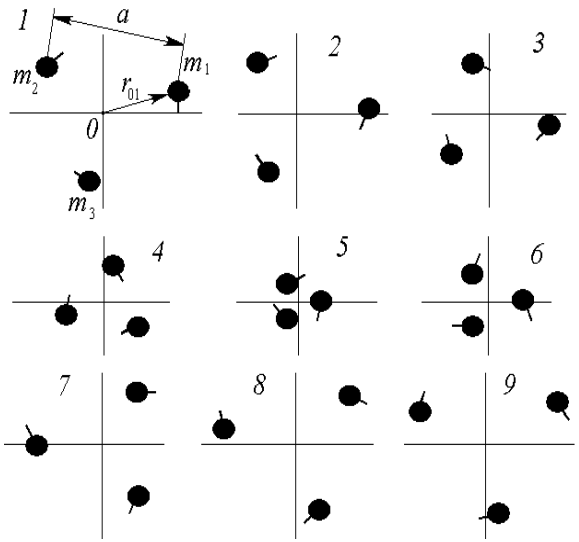


Fig. 11.8. Stages of motion of three bodies at α_1 = - 0.6356 at the instants 3.27 $\tilde{t}$:
1 - 1.0; *2* - 2.0; *3* - 3.0; *4* - 4.0; *5* - 5.0; *6* - 6.0; *7* - 8.0; *8* - 9.0; *9* - 10.0.

Receiving the arrangement of pericentre for the first body with the angle φ_0 = 0, and for second at $\varphi_0 = \pi$, according to (11.107) and (11.108) the equations of their trajectories relatively the centre of masses *O* are obtained as follows:

$$r_{01} = \frac{R_P m_2 / m}{(\alpha_1 + 1)\cos\varphi - \alpha_1}, \qquad r_{02} = \frac{R_P m_2 / m}{(\alpha_1 + 1)\cos(\varphi - \pi) - \alpha_1}. \tag{11.109}$$

It is necessary to note, that a direction of rotation in Fig. 11.6 and the consequent ones by virtue of the opposite direction, on a screen of the screen monitor happen clockwise. It is necessary to take into account, comparing the expressions for trajectories (11.109) with their images in Figures.

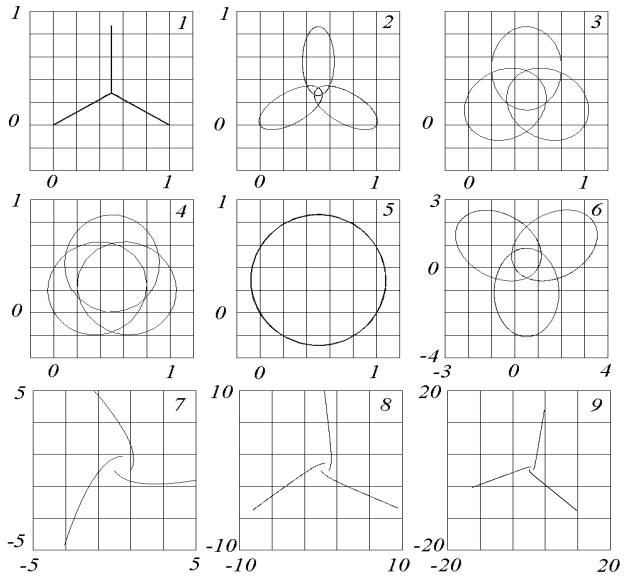
Now we will consider the axisymmetrical motion of three bodies. In Fig. 11.8 the motions of three bodies are shown at initial angular rate $\tilde{\omega}$/3,27 = 0,2. At such initial transversal velocity, the particles approach and in a position *5* reach the pericentres, then again begin to be removed from each other and in position *9* come nearer to the apocentre.

In Fig. 11.9 the influence of initial transversal velocity $\tilde{\omega}$/3,27 on trajectory of motion of three bodies is submitted. At $\tilde{\omega}$ = 0 the three-radial radial approach of three bodies (trajectory *1*) happens. At nonzero value $\tilde{\omega}$ the particle approach each other up to pericentres (trajectories *2, 3, 4*), and then return to the apocentre. At the initial transversal velocity, appropriate to α_1 = - 1, all particles move along circle *5*. At some greater value $\tilde{\omega}$, the particles are retreated from each other (trajectory *6*), reach apocentres and again return in pericentres. At $\tilde{\omega}$, appropriate to α_1 = - 0,5, the particles move along parabolas *7*. At some greater initial velocity $\tilde{\omega}$, the particles move along hyperbolas *8* and *9*.

We will define the analytical expressions for trajectories of three particles. For three bodies without central (m_0 = 0), according to (11.101) and (11.101a), the parameter of trajectory will be

$$\alpha_1 = -Gm_1 \big/ (\sqrt{3} r_{0p} v_{0p}^2). \tag{11.110}$$

Fig. 11.9. Trajectories of motion of three particles at a variation of initial transversal velocity $\tilde{\omega}$/3.27 (in brackets parameter of trajectory α_1) is indicated:
1 - 0 (0); *2* - 0.1 (- 0.528); *3* - 0.2 (-0.6356); *4* -. 25 (- 0.675); *5* - 0.306 (-1); *6* - 0.4 (- 586); *7* - 0.433 (- 0.5); *8* - 0.5 (- 0.375); *9* - 0.6 (- 0.26).



From Fig. 11.8 it is seen, that the distance of each particle up to centre with masses *O* and initial

polar angle can be written

$$r_{0p} = a/\sqrt{3} = R_p/\sqrt{3}, \quad \varphi_{0i} = \pi/6 + 2\pi(i-1)/3. \qquad (11.111)$$

Then the equation of trajectory of each particle, according to (11.108), relatively centre of masses *O* will be written so:

$$r_{0i} = \frac{r_{0p}}{(\alpha_1+1)\cos(\varphi - \varphi_{0i}) - \alpha_1}, \quad i = 1, 2, 3, \qquad (11.112)$$

where $r_{0p} = R_p/\sqrt{3}$ is the least distance of trajectories up to centre with masses +.

As well as in case of two bodies, at the solution of equations on the program Galactica, the initial velocity at the apsis point is set by the rotation of area round the centre of masses *O* with the angular rate $\omega = v_{0p}/r_{0p}$. After a substitution it in (11.110) and fulfilment of transformations the expression for parameter of trajectory is obtained

$$\alpha_1 = -1/\tilde{\omega}^2, \qquad (11.113)$$

which is identical to the expression (11.103) at the interaction of two bodies. Here, as well as in case of two bodies, at $\alpha_1 < -1$, the motions begin at the apocentre, therefore, according to (11.106), it is necessary to bring α_1 into accord with the parameters in apocentres.

In Fig. 11.10 the axisymmetric motion of four bodies is shown at $\tilde{\omega}$ = 0,2. The bodies begin motion at the apocentres, in a position *4* reach the pericentres, then begin to remove and in a position *8* come into the apocentre.

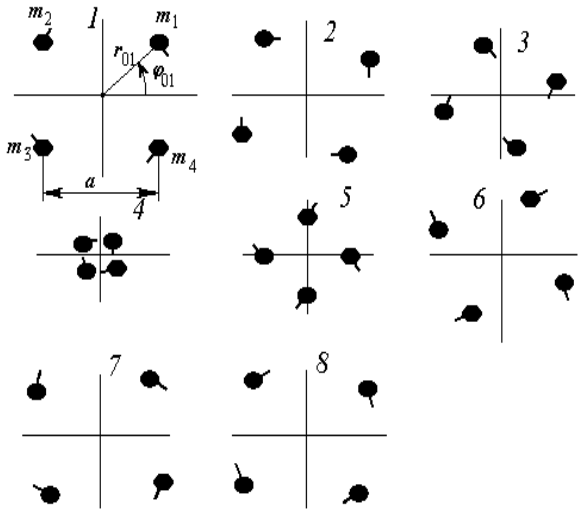


Fig. 11.10. Stages of motion 4-th bodies at α_1 = - 0.587 at the instants $2\tilde{t}$: *1* - 0.01; *2* - 1.0; *3* - 2.0; *4* - 3.0; *5* - 4.0; *6* - 5.0; *7* - 6.0; *8* - 7.0.

In Fig. 11.11 the influence of the initial transversal velocity $\tilde{\omega}/2$ on the trajectory of motion of four bodies is shown. At $\tilde{\omega}$ = 0 happens (see trajectory *1*) four-radial radial approach of bodies. With increase of $\tilde{\omega}$, the bodies come nearer to the centre of masses along the elliptical trajectories, which eccentricity with the growth of $\tilde{\omega}$ decreases, and at

$\tilde{\omega}$ = 0,822 the trajectory becomes a circle *6*. The further increase of $\tilde{\omega}$ results in the retraction of particles, at the beginning along the elliptical orbits *7*, then along parabolas *8* and hyperbolas *9*. In case of four particles, according to (11.101) and (11.101a), the parameter of trajectory will be written as

$$\alpha_1 = -\frac{G\left(0{,}25 + \sqrt{2}/2\right)m_1}{r_{0p}v_{0p}^2}. \qquad (11.114)$$

The initial angle of the particles pericentres is $\varphi_{0i} = \pi/4 + \pi(i-1)/2$. Then according to (11.108) the trajectories of each from four particles will be determined by the equation (11.112) as for case of three particles. At the numerical solution of equations the rotation of area round the centre of masses *O* (see Fig. 11.10) with velocity is set $\omega = v_{0p}/r_{0p}$, where $r_{0p} = \sqrt{2}a/2$. Expressing (11.114) through ω, we obtain

$$\alpha_1 = -\frac{1+\sqrt{2}/4}{2\tilde{\omega}^2}. \qquad (11.115)$$

At $\alpha_1 < -1$ the parameters of trajectory are enumerated under the formula (11.106).

The axisymmetrical motion of three and four bodies is considered without a central body. It leads it, according to (11.101), to increasing on an absolute value of values μ_1 and α_1. Therefore, for the same trajectories, according to (11.103) and (11.115), it is necessary to increase the initial transversal velocity by increase $\tilde{\omega}$. In all cases the central body will be motionlessly in the centre of a system masses.

So, we have compared the precisely solutions (11.112) of axisymmetrical interactions of many bodies with the numerical ones. The precisely solutions are stabile in space.

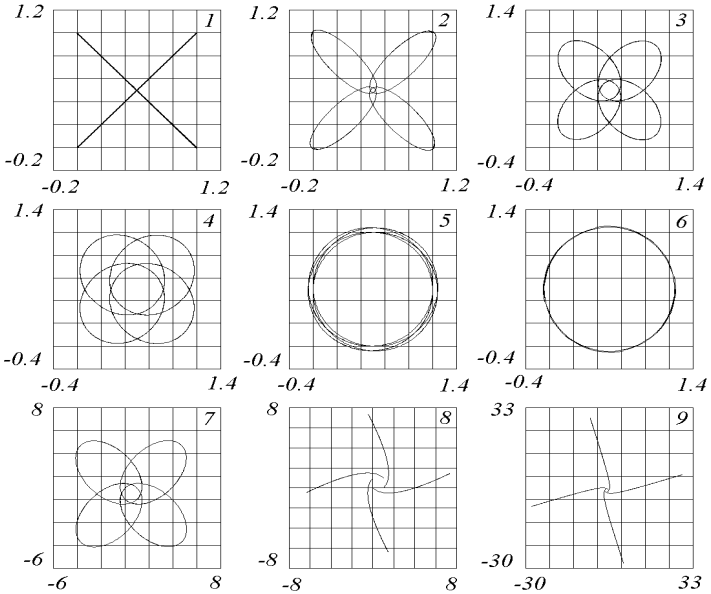


Fig. 11.11. Trajectories of motion of four particles at a variation of initial transversal velocity 0.5 $\tilde{\omega}$ (in brackets parameter of trajectory α_1) is indicated: *1* - 0 (0); *2* - 0.1 (- 0.515); *3* - 0.2 (- 0.587); *4* - 0.3 (- 0.681); *5* - 0.4 (-0.948); *6* - 0.411 (- 1); *7* - 0.5 (- 0.559); *8* - 0.582 (- 0.5); *9* - 0.6 (- 0.47).

However, numerical solutions give open-ended trajectories: the more their pericentre displaces, the bigger the orbit eccentricity is (see trajectory *2* in Fig. 11.7). Improving the numerical method we can essentially reduce the change of orbits during several periods, but it is impossible to liquidate it completely. It is exhibited early after a large number of periods. These changes are stipulated by the fact, that the integration happens not according to a precise acceleration, which is determined by the world law of gravitation (11.1), but it is executed with an error. Therefore, the approximate solutions of a problem of many bodies, for example as expression (11.2), used by U. Le Verrier, also will lead to the change of orbit, even if it is not present. In this connection, we consider, that the problem of secular changes of the orbits parameters of planets in a Solar system is not completed and requires further research.

11.6. CENTRAL-SYMMETRICAL ACCRETION

At the accretion of substance the impacts of attracted parts happen and the mechanical energy of motion passes in thermal. The numerical experiments with the program Galactica have shown, that this process calls large algorithmic difficulties. For example, it is necessary to fix a moment of a contact. Besides the impact of several bodies can happen simultaneously. In this connection there are problems, which can be solved precisely with the help of accretion.

We will consider the collision of two particles. Earlier we have the defined radial velocity (5.27) at radial motion and time (5.30). At collision the particles merge in one, which is in centre of their masses. Each particle velocity relatively the centre

$$\upsilon_{r1} = \upsilon_r\, m_2/m; \qquad \upsilon_{r2} = \upsilon_r\, m_1/m,$$

where υ_r is a relative radial closing velocity of particles at the moment of collision. Then the thermal energy selected at collision of two particles, will be

$$E_t = \frac{m_1\upsilon_{r1}^2}{2} + \frac{m_2\upsilon_{r2}^2}{2} = \frac{m\upsilon_r^2}{2}.$$

If at the distance R_0 between particles their closing velocity is υ_{r0}, at a point of collision $R = d$ their radial velocity will be defined by expression (5.27), and the thermal energy accepts a kind

$$E_t = \frac{m_{re}\upsilon_{r0}^2}{2} - \mu_1 m_{re}\left[\frac{1}{d} - \frac{1}{R_0}\right], \qquad (11.116)$$

where

$$m_{re} = \frac{m_1 m_2}{m_1 + m_2}. \qquad (11.117)$$

In the dimensionless variable (11.77) - (11.79) the expression for a thermal energy (11.116) will be written so:

$$\tilde{E}_t = \frac{\tilde{m}_{re}\tilde{\upsilon}_{r0}^2}{2} + \tilde{m}_1\tilde{m}_2\left[\frac{1}{\tilde{d}} - \frac{1}{\tilde{R}_0}\right], \qquad (11.118)$$

where

$$\tilde{E}_t = E_t\frac{a}{Gm^2}. \qquad (11.119)$$

Let's consider the distribution of bodies in space with density $\rho(r)$, radius r_0 and velocity $\upsilon_r(r,0) = 0$. Let in an initial moment $t = 0$ the density in all space is constant, i.e. $\rho = \rho_0$. The body with mass m_1 located at a distance $r_1 < r_0$from centre, is acted by bodies of an internal system ($r \le r_1$) and external with $r > r_1$. Let's calculate the force of action of an internal system (Fig. 11.12, *a*) in a projection to the axis *z*.

By virtue of a rotational symmetry, the elements orb ring of an

$$\mathbf{d}m = \rho r\mathbf{d}r 2\pi r\sin\theta\mathbf{d}\theta$$

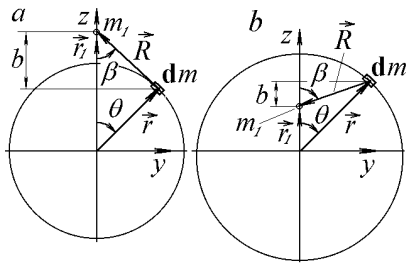


Fig. 11.12. The force of action of central-symmetric distributed substance in a spherical layer with a radius *r* on mass m_1, located: *a* - outside the substance; *b* - inside it.

act uniformly on body m_1, therefore, the force of action of a ring will be

$$\mathbf{dd}F_z = -\frac{2\pi G\rho m_1 r^2\sin(\theta)\mathbf{d}\theta\cos(\beta)\mathbf{d}r}{R^2} \qquad (11.120)$$

It is fair for all angles $0 \le \theta \le \pi$. It follows from geometric reasons

$$b = r_1 - r\cos(\theta), \quad R = \sqrt{b^2 + r^2\sin^2(\theta)}, \quad \cos(\beta) = b/R. \qquad (11.121)$$

Then it is possible to record (11.120) as an integral

$$\mathbf{d}F_z = 2\pi\rho G m_1\mathbf{d}r\int_1^{-1}\frac{(\bar{r}_1 - \gamma)\mathbf{d}\gamma}{\left(\bar{r}_1^2 + 1 - 2\bar{r}_1\gamma\right)^{3/2}}, \qquad (11.122)$$

where $\bar{r}_1 = r_1/r$, $\gamma = \cos(\theta)$.

Let's designate $x = \bar{r}_1^2 + 1 - 2\bar{r}_1\gamma$, then $\gamma = \left(\bar{r}_1^2 + 1 - x\right)/\left(2\bar{r}_1\right)$ and $\mathbf{d}\gamma = -\mathbf{d}x/2\,\bar{r}_1$. After a substitution in (11.122) we obtain

$$\mathbf{d}F_z = -\frac{\pi G\rho m_1\mathbf{d}r}{2r_1^2}\int\left[\frac{\bar{r}_1^2 - 1}{x^{3/2}} + \frac{1}{\sqrt{x}}\right]\mathbf{d}x. \qquad (11.123)$$

After integrating (11.123) and returning to a variable γ the force will be written as

$$\mathbf{d}F_z = -\frac{2\pi G\rho m_1 \mathbf{d}r}{r_1^2}\left.\frac{1-\bar{r}_1\gamma}{\sqrt{\bar{r}_1^2+1-2\bar{r}_1\gamma}}\right|_1^{-1}. \qquad (11.124)$$

The process of limits substitution we will consider more detailed, as it represents large interest:

$$\mathbf{d}F_z = -\frac{2\pi G\rho m_1 \mathbf{d}r}{r_1^2}\left[\frac{1+\bar{r}_1}{\sqrt{\bar{r}_1^2+1+2\bar{r}_1}} - \frac{1-\bar{r}_1}{\sqrt{\bar{r}_1^2+1-2\bar{r}_1}}\right]. \qquad (11.125)$$

The radicands represent quadrates of binomial, therefore they are positive, and the number, extracted from them, is also positive. The second addend at $\bar{r}_1 > 1$

$$-\frac{1-\bar{r}_1}{\sqrt{\bar{r}_1^2-2\bar{r}_1+1}} = -\frac{1-\bar{r}_1}{(\bar{r}_1-1)} = 1. \qquad (11.126)$$

Let's note, that at $\bar{r}_1 < 1$ this addend will be equal to (-1). After a substitution (11.126) in (11.125) the force of action of a spherical surface with thickness **d***r* on the body m_1 will be written so:

$$\mathbf{d}F_z = -\frac{Gm_1}{r_1^2}\,4\pi\rho r^2\mathbf{d}r,$$

and the force of action of all substance contained in a full-sphere with the radius *r*, will be

$$F_z = -\frac{Gm_1}{r_1^2}\int_0^r 4\pi\rho r^2\mathbf{d}r. \qquad (11.127)$$

This force is fair at any position of the axis *z* relatively spherical area of substance, i.e. it is directed on the radius inside an orb. As the mass of substance contained in a full-sphere with the radius *r*, is equal to

$$m_2 = \int_0^r 4\pi\rho r^2\,\mathbf{d}r, \qquad (11.128)$$

then the force of action of a spherical accumulation of substance with centrally symmetric distribution of density $\rho(r)$ on mass m_1, according to (11.127) and (11.128) will be written:

$$F_z = -\frac{Gm_1m_2}{r_1^3}\vec{r}_1, \qquad (11.129)$$

where $r_1 \geq r$. That is without dependence from distribution of substance density in a full-sphere the expression for force coincides the law of world gravitation (11.1).

We will consider now the force of action of a spherical layer with a radius *r* on body with mass *m1* located inside sphere (see Fig. 11.12, *b*). The projection of force to the axis *z* will be written similarly to (11.120), but with a converse:

$$\mathbf{dd}F_z = \frac{2\pi G\rho m_1 r^2\sin(\theta)\mathbf{d}\theta\cos(\beta)\mathbf{d}r}{R^2}. \qquad (11.130)$$

The same expressions (11.121), as in the first case follow from geometric ratios. With their registration expression (11.130) will be copied

$$\mathbf{d}F_z = 2\pi G\rho m_1\mathbf{d}r\int_1^{-1}\frac{(\bar{r}_1-\gamma)\mathbf{d}\gamma}{\left(\bar{r}_1^2+1-2\bar{r}_1\gamma\right)^{3/2}}. \qquad (11.131)$$

The expression (11.122) is identical to (11.131), however here $\bar{r}_1 < 1$. The integral (11.131) has the solution (11.125), but at a substitution of boundary value $\gamma = 1$ in the second addend at $\bar{r}_1 < 1$, according to (11.126), it will be equal to (-1), and the integral (11.125) and, therefore, (11.131) will be

$$\mathbf{d}F_z = 0. \qquad (11.132)$$

Thus, the spherical shell by a thickness **d***r* does not act on a mass, located inside it. As this conclusion is fair for each spherical layer, the composition of forces on all elements **d***r* of a full-sphere with a cavity with the radius *r* gives force, equal to zero, on a particle, located inside a cavity. Let's note, that this outcome is known in electrostatics: inside a charged orb there are no forces of action on a charged particle.

Now we will consider the motion, of a substance in a centrally symmetric array stipulated by own attraction. Only a substance of internal layers r ≤ r1 will act any body located on the radius r1 ≤ r0. The value of force is determined by expression (11.129), where $m = m(r_1)$ is the mass of substance in a full-sphere with the radius r_1. Then the acceleration of body at $r = r_1$ will be written

$$w = \frac{\mathbf{d}^2 r_1}{\mathbf{d}t^2} = -\frac{Gm(r_1)}{r_1^2}. \qquad (11.133)$$

In an initial moment the density of substance everywhere is identical, therefore

$$m(r_1) = \frac{4\pi}{3}\rho_0 r_{10}^3, \qquad (11.134)$$

where $r_{10} = r_1(0)$ is a position of body m_1 at the initial instant ($t = 0$). Let's consider that during the motion of a substance some parts do not overtake the others, therefore the mass of substance inside an orb with a radius $r_1(t)$ remains all time of a constant, i.e. $m(r_1) = \text{const}$. Then the equation (11.133) is identical to an equation of radial interaction of two bodies, as the result of which solution we have received a radial velocity of motion (5.27) and the time of motion (5.30). At zero initial velocity the radial velocity and time will be written so:

$$v_r(r_1) = \sqrt{2Gm(r_1)\left(\frac{1}{r_1}-\frac{1}{r_{10}}\right)}, \qquad (11.135)$$

$$t = \sqrt{\frac{r_{10}^3}{2Gm(r_1)}}\left[\frac{r_1 v_r}{\sqrt{2Gm(r_1)r_{10}}} + \operatorname{arctg}\left(v_r\sqrt{\frac{r_{10}}{2Gm(r_1)}}\right)\right]. \qquad (11.136)$$

Let's consider that the bodies, begun the motion from the radius r_{10}, will continue to move until their collision with central mass will happen. At the first stage it is possible to consider what density of central mass during an accretion does not vary and is equal to some mean density, for example, the density of the Earth ρ_f = 5000 kg/m^3. At this density the radius of central mass, with which the body m_1 will come in collision, will be

$$r_{1f} = \left(\frac{3m(r_1)}{4\pi\rho_f}\right)^{1/3} = r_{10}\left(\frac{\rho_0}{\rho_f}\right)^{1/3}. \qquad (11.137)$$

After a substitution (11.137) in (11.135) and (11.136) with allowance for (11.134) we obtain the radial velocity of motion of mass m_1 at its collision with a central body and time of motion up to the collision:

$$v_r(r_{10}) = r_{10}\sqrt{\frac{8\pi G\rho_0}{3}\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1\right]}, \qquad (11.138)$$

$$t = \sqrt{\frac{3}{8\pi G\rho_0}}\left[\frac{v_r(\rho_0/\rho_f)^{1/3}}{r_{10}\sqrt{8\pi G\rho_0/3}} + \operatorname{arctg}\left(v_r\sqrt{\frac{3}{8\pi G\rho_0 r_{10}^2}}\right)\right]. \qquad (11.139)$$

With velocity v_r, according to (11.138), the spherical layer of mass $\mathbf{d}m = 4\pi r_{1f}^2\rho_f\mathbf{d}r_f$ will come nearer to a central body with a radius r_{1f}. As the result of collision it transmits to body a kinetic energy, which will turn in the thermal one:

$$\mathbf{d}E_t = \frac{\mathbf{d}m\,v_r^2}{2} = \frac{16}{3}\pi^2 G\rho_0^2\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1\right]r_{10}^4\mathbf{d}r_{10}. \qquad (11.140)$$

The full energy of an accretion is obtained as the result of integration on the whole array of substance:

$$E_t = \int_0^{r_0}\mathbf{d}E_t\mathbf{d}r_{10} = \frac{16\pi^2 G\rho_0^2 r_0^5}{15}\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1\right]. \qquad (11.141)$$

The time of a full accretion is determined at a substitution $r_{10} = r_0$ in expression (11.139). Energy E_t can still be written through the mass of a central body equal to the mass of an accretion area $m = 4\pi\rho_f r_f^3/3 = 4\pi\rho_0 r_0^3/3$, expressing in (11.141) density through it:

$$E_t = \frac{3Gm^2}{5r_0}\left(\frac{r_0}{r_f} - 1\right). \qquad (11.142)$$

The energy of an accretion, as it is seen from (11.142), is increased with the increase of the value of area r_0. However at $r_0/r_f > 100$ unit in brackets can be neglected and the energy E_t will not depend from the value of area:

$$E_t = \frac{3Gm^2}{5r_f}. \qquad (11.143)$$

Kant and Laplace considered a hypothesis, according to which the Sun and the planets were formed as the result of a condensation of initial gas nebula. On this basis Helmholtz and L. Tomson have advanced the impactive theory of the Sun formation. Kolosovsky [23] gives a derivation of the expression (11.143), investigating the activity, executed in gravity during a mass $\mathbf{d}m$, falling from a full-sphere surface in its centre:

$$\mathbf{d}A = \frac{Gm\mathbf{d}m}{r^2}r. \qquad (11.144)$$

After a substitution of mass of a full-sphere m, the spherical volume $\mathbf{d}m$ and the integration on a radius from 0 up to r we obtained the value of activity A, equal to E_t according to (11.143). In this derivation there is a defect, as the force is not constant at the motion of a volume to the centre. Therefore, the activity is necessary to note as:

$$\mathbf{d}\mathbf{d}A = \mathbf{d}F\mathbf{d}r = \frac{Gm\mathbf{d}m}{r^2}\mathbf{d}r.$$

Apparently, this inaccuracy is insignificant at $r_0 >> r_f$.

The expression (11.143) includes the radius r_f. It is a final distance up to centre passed by an attracted particle. A visible radius of the Sun $R_c > r_f$ and it determines a gas shell of the Sun. The attracted body moves with large velocity and penetrates in the Sun on significant depth. Therefore, the transferred mechanical energy to a central body is necessary to calculate at a final depth of penetration. As r_f is determined by mean density ρ_f, then, varying the last one, we will receive different thermal energy of an accretion.

We adduce a calculated final (at $r_{10} = r_0$) velocity (11.138), time (11.139) and energy of an accretion (11.142) with reference to the Sun in three versions below:

1) The radius of initial area of substance is taken equal to the double semimajor axis of Pluto orbit ($r_0 = 1.2\cdot10^{13}$ m) and the mean density of a central body at an accretion ρ_f = 5000 kg/ m^3;

2) Unlike first version density ρ_f is increased in 2 times;

3) Unlike first version the radius of area is equal to a half of a distance up to the nearest star α - Centaurus ($r_0 = 1.2\cdot10^{16}$ m). The obtained results of an accretion of the Sun are shown in a table:

N	$r_{0,}$ m	ρ_f, kg/m^3	$v_{r,}$ km/s	t, years	E_t, J	t°, C$^\circ$	ρ, kg/m^3
1	$1.2\cdot10^{13}$	5 000	760	127.7	$3.42\cdot10^{41}$	$4.14\cdot10^{7}$	$2.72\cdot10^{-10}$
2	$1.2\cdot10^{13}$	10 000	853	127.7	$4.30\cdot10^{41}$	$5.22\cdot10^{7}$	$2.72\cdot10^{-10}$
3	$1.2\cdot10^{16}$	5 000	760	$4.04\cdot10^{6}$	$3.42\cdot10^{41}$	$4.14\cdot10^{7}$	$2.72\cdot10^{-19}$
4	$1.4\cdot10^{18}$	5 000	760	$5.1\cdot10^{9}$	$3.42\cdot10^{41}$	$4.14\cdot10^{7}$	$1.71\cdot10^{-25}$

The, conditional temperature of a central body is normalised here, provided that the mean thermal capacity of a substance is equal to thermal capacity of water: C = 1 kcal/(kg·grad).

As it is seen from a table, the energy of an accretion E_t reaches rather large value. The final velocity of a falling substance is equal to 760 km/s. Actually even before reaching $r_f = 5.55\cdot10^8$ m (in the first version) the body will confront with the Sun atmosphere ($r_s = 6.945\cdot10^8$ m). Therefore the actual velocity of falling will be less, however transferred energy to the Sun will not change the value. The accretion will be completed for 127.7 years, and the mean temperature will exceed 40 million degrees. With the increase of density ρ_f in 2 times (version 2) the energy of an accretion will increase at 1.26 times. The increase in 1000 times of an accretion area (version 3) results in the increase of time of an accretion up to 4 million years. In a table there is version 4, where such value of initial density ρ_0 areas is given, at which accretion will proceed 5 billion years. It follows from shown versions that the time of an accretion can vary over a wide range and the energy of an accretion in the considered statement is more conservative.

N.A.Kolosovsky [23] gives timing, for which the energy of an accretion will be radiated by the Sun. Full radiation of the Sun $L = 3{,}86\cdot10^{26}$ W calculate on the basis of measured quantity of heat obtained by the unit of an earth surface. Then the time of radiation of the Sun the energy of an accretion:

$$t_s = E_t/L = 0{,}89\cdot10^{15} \text{ c} = 28 \text{ million years}$$

As the age of a Solar system is supposed to be equal to 5 billion of years, it forms the basis for an inference about the existence of other energy sources of the Sun.

However there are circumstances, which are not taken into account by considered centrally symmetric statement of an accretion problem. The actual accretion happens no symmetric, that, as our calculations under the program "Galactica" have shown, results in the formation of a substance, circulating along the closed orbits. As the result of collisions, the particle can lose the orbital velocity and be attracted by a central body, so that the accretion happens not for 127.7 years, as is obtained for version 1, and is stretched for long years.

In a solar system the accretion was not finished: all planets, satellites and asteroids are covered with impact craters, fireballs. The large objects, such as meteorites from time to time, penetrate the Earth atmosphere. The recent falling fragments of Levi-Shumecer comet on Jove are a convincing. In this connection we consider the conditions, at which the energy of a modern accretion of the Sun would coincide with its radiation.

The expressions (11.138), (11.139) and (11.141) are obtained for the substance distributed with density ρ_0 in area with the radius r_0, which is going in mass m. Let's consider a problem, when the substance is distributed with density ρ_0 in area with the radius $r_{20} < r_0$. Then after an accretion of substance made in an orb with the radius r_{20}, the particles found at the distance $r_{10} > r_{20}$ will be besieged on a central body. The process of an accretion is described by the same expressions, but in the expression (11.141) instead of r_0 there will be r_{10}:

$$E_t = \frac{16\pi^2 G\rho_0^2 r_{10}^5}{15}\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1\right]. \qquad (11.145)$$

It is the energy of an accretion of substance made in the orb r_{10}.

Substituting (11.138) in (11.139), we will record the time of an accretion as follows:

$$t = \sqrt{\frac{3}{8\pi G\rho_0}}\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3}\sqrt{\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1} + \operatorname{arctg}\left[\left(\frac{\rho_f}{\rho_0}\right)^{1/3} - 1\right]\right]. \qquad (11.146)$$

As we can see, the time of an accretion does not depend on the value of area, and is determined only by ratio of density: initial and final. That is in considered statement the external layers simultaneously approach place, where they must confront with a central body, and the confluence happens instantly. It is stipulated by the fact that with removing a particle from the centre of area, the force of action on it, and consequently, and acceleration are increased due to increase of the mass of the substance internal volume. So, according to (11.133), with allowance for masses (11.134) of internal layers the acceleration of a particle at the initial radius r_{01} is equal to

$$w = \frac{4\pi G\rho_0}{3} r_{01}, \qquad (11.147)$$

i.e. the initial acceleration of a particle is proportional to its removing from the centre.

In central symmetric statement we summarized the action of all remaining particles on a considered particle and have received the force (11.129) action of substance in a volume with the radius r_{10}. However besides the universal action there are local interactions, during which the close located particles direct to each other and merge. From numerical calculations of an accretion at different quantities of particles and different forms of areas it is seen, that the accretion at the beginning starts in a rim: at the cubic form in the field of main diagonals the multiple centres of an accretion will be formed. More massive bodies swallow more small-valued. Then the most massive, merging, form a central body, to which then for a long time the remaining bodies aim, which remained in a rim because of small

velocity of motion. So, the actual accretion happens continuously in the whole volume and the energy of an accretion is reserved in separate fragments. Therefore, the full energy of an accretion will exceed the value, calculated in a solution of a spherical accretion. To decide how much large this excess is, it is necessary to conduct the further numerical researches of an accretion process.

By the results of a spherical accretion we will evaluate the mean energy of an accretion as $N = E_t/t$. At large values (ρ_f/ρ_0) expressions for energy (11.145) and time (11.146) become simpler:

$$E_t \approx \frac{16\pi^2 G\rho_0^2 r_{10}^5}{15}\left(\frac{\rho_f}{\rho_0}\right)^{1/3}, \quad t \approx \sqrt{\frac{3\pi}{32G\rho_0}}. \qquad (11.148)$$

Then mean energy of an accretion:

$$N = \frac{64\sqrt{2}(0{,}75)^{5/3}\rho_0^{5/6}G^{1.5}m^{5/3}}{15\sqrt{3}\pi^{1/6}}. \qquad (11.149)$$

The given expression is recorded depending on density of the area ρ_0 and mass m of a central body, and the initial radius r_{10} is eliminated. For the considered earlier values of density ρ_0 we will calculate the power of an accretion for the Sun by this formula

N	ρ, kg/m^3	N, W
1	$2.72\cdot10^{-10}$	$3.2\cdot10^{27}$
2	$2.72\cdot10^{-19}$	$1.0\cdot10^{20}$
3	$1.71\cdot10^{-25}$	$6.9\cdot10^{14}$

As we can see, the energy of radiation of the Sun ($L = 3{,}86\cdot10^{26}$ W) is inside the range of a rated power of an accretion. That is the power, allocated by the Sun, could be filled up due to an accretion, if there are actual values of mass of substance acting on it. At velocity of an accretion υ_r = 760 km/s the influx of substance per one year

$$m_{yr} = \frac{LT_{yr}}{0.5\upsilon_r^2} = 4.24\cdot10^{22} \text{ kg/year}.$$

It makes 0,7 % from mass of the Earth. On evaluations the influx of meteoric substance on the Earth makes $5\cdot10^7$ kg/years [7]. If to consider that the flow of meteoric substance in a Solar system is stipulated by the attraction of the Sun, and only those meteorites fall on the Earth which are overlapped by its section πR_E^2, the annual influx of meteoric substance on the Sun can be appreciated by value

$$m_m = 5\cdot10^7 4\pi r_E^2/(\pi R_E^2) = 1.2\cdot10^{17} \text{ kg/years},$$

where r_E is the distance of the Earth up to the Sun; R_E is the radius of the Earth. Obtained value is times less than the necessary value of the influx of a mass to maintain a stable radiant emittance of the Sun.

The made calculations show, that the power of an accretion can be rather large at the formation of the Sun, but is not sufficient (according to the of observation) for maintenance in a stable condition of its heat storage. The centrally symmetric model of an accretion gives the underestimated values of the power of an accretion and the further researches of this process are necessary to straighten out the initial reserves of the Sun energy and to determine a role of other energy sources.

We will note an interesting outcome, following from a problem about a spherical accretion. It follows from (11.135), that the radial velocity of a subtended substance is increased in accordance with approaching r_1 to a radius of an accretion r_f. As we have already repeatedly noted, the results do not depend on a direction of the integration, therefore they can be applied to a retraction of a substance, for example to explosion of the body with the radius r_f. If at the moment of explosion the velocity of body particles is subjected to the law (11.138), i.e. the same one as at the moment of an accretion, then on reaching by particles the distances $r_1 > r_f$ their velocity will be determined by expression (11.135). Therefore, the velocity of particles during removing from the centre of the explosion will decrease and, reaching the radius $r = r_{10}$, will be equal to zero.

The obtained result is completely natural, as at the retraction of the substance it moves against a resultant gravity and is broken. In the theories of the extending universe and "of large explosion" the velocity of particles removing from the centre of explosion grows. These theories are based on the explanation of "reddening" of galaxies light by action of Doppler, according to which (see formula (8.75)) the frequency of light of a removed source decreases. As it is seen, the given explanation contradicts the laws of a nature: if there is gravitation the velocity of removed objects should drop. " The Reddening " of light is stipulated by the other reasons. The universe was not born by the explosion and there is no basis to consider it the extending one. In this plan, the works of the Finnish astronomer Toyvo Jaakola about the equilibrium universe [95] are represented interesting.

DEFINITIONS OF BASIC CONCEPTS

Definitions of concepts and some consequences following from them are below submitted.

All consists of two things: the ambient world and its description.

The ambient world is a part of everything that does not depend on the person's reasoning.

The description of the ambient world is an understanding, explanation of the world around, i.e. its conceptualization.

The knowledge is the understanding about the world around, which will not change with the passage of time.

Truth is the knowledge, which help the people's activities to of operations of the person correspond his intentions.

The truth is knowledge, which the man uses at his activity and receive results, which corresponds, to his intentions.

The basic part of the knowledge, which plays the important role always or at the certain moment of time, usually is accepted as truth.

Science is the area of human activity, directed for obtaining the new knowledge about the ambient world.

The theory is the description of objects properties of the ambient world, of the methods of human activity and their results.

The time characterizes a variability of objects and is determined as the result of comparison the changing of objects with the change of a standard body or object.

Time of existence, time of life, duration of the phenomenon or object is quantity of cycles of the standard change, which is equivalent to the change of the considered object or the phenomenon.

Instant in a change of object is a binding of some stage of its change to a certain stage or a cycle of a standard change.

Time interval between two different stages of object change is quantity of cycles of the standard change, which have occurred between these stages.

Mathematical time is the result of comparing the changes of objects with stabile cyclical change of the envisioned measurement standard.

The mathematical time is used at the theoretical description of the ambient world.

Size of object (body) of object (body) is the result of comparison at imposing the standard on object; it is expressed by quantity of standards or quantity of shares of the standard, which can be imposed, on object.

The size of the object is determined in three mutually perpendicular directions, which in accordance with decrease are named: length, width and thickness.

The name of size of object may be adhered to a vertical: height (depth); to a horizontal: width (thickness); to the parties of light: breadth, a longitude, etc.

Size of an interval between objects is the result of a comparison, by accommodation of the measurement standards between objects; it is expressed by a quantity of the measurement standards or their parts.

The intervals between objects are situated in three mutually perpendicular directions.

The sizes of intervals are called distances.

Space is a combination of objects and intervals between them.

This physical determination of space is necessary to distinguish from a word "space" used in mathematics, poetry, fantasy and other areas of human activity.

Mathematical space is imagined coordinate system, in which three numbers fix the position of points.

The coordinate system should be considered by a synonym of mathematical space

The system of linear and mutually perpendicular axes of coordinates is called Cartesian.

Mathematical space is used for the theoretical description of the ambient world.

Velocity of motion of one object relatively the other is a change of a distance between them per the unit of time.

The velocity characterizes the motion of an object relative to the second one. The same object in relation to several ones can have different velocities.

The inertial system is a imagined coordinate system, which moves without acceleration, i.e. on inertia.

Acceleration of an object motion is the change of its velocity in relation to an inertial system per the unit of time, provided that at the initial moment of measurement the velocities coincide, and the change of velocities is considered for infinitely small period.

The acceleration of motion of object characterizes its motion irrespectively of the other objects, i.e. the acceleration is the own characteristic of the object (if to neglect dependence on a choice of inertial system). This property of acceleration is mathematical. At measurement of acceleration instead of mathematical coordinates and time the actual measurement standards of length and time are applied, and instead of an inertial system the real object is used, for example, the Earth surface, which moves with acceleration. The measured acceleration is expressed in relation to the used measurement standards.

Action of one object on the other is an ability of an object to move the second one or to change its motion, i.e. to give acceleration to the second object.

If there is no action to the object that it has no acceleration, i.e. it rests or moves rectilinearly and is uniform (the first law of mechanics).

The force of action of one object on the other is a characteristic of action, expressed in change of properties of the third object, which counteracts the interaction of the first two ones.

For example, the spring, which is located between body and Earth surface attracting body, reduces the length on magnitude Δl, which characterizes the force of action of the Earth on the body.

The force is directed along the acceleration of the body.

The interacting bodies have opposite directed accelerations.

The force of action has the relation to two bodies, and its magnitude is the same. From here, the forces of interaction of two bodies, which at the description of interaction by the person mentally are applied to bodies, are equal on the magnitude and are opposite on a direction (the third law of mechanics).

Mass of a body is a quantity of standard bodies, which at action are characterized by some acceleration, drive the same change of properties of counteractive body as the well as considered body.

For example, the Earth acts on a stone and on standard weights with identical acceleration. The mass of a stone is equal to such quantity of standard weights, which deform a spring that counteracts their falling, on the same magnitude, as stone.

Mass of a body, in other words, is quantities of standard bodies, which at action are characterized by identical force, get the same acceleration as the body.

At action on the standard (kg), which is characterized by acceleration 1 m/s^2, the change of counteractive body is accepted for unit of force F in 1 Newton (N).

The action on the standard, characterized by the acceleration w m/s^2, is also determined by the magnitude of force

$$F = w.$$

The action on a group of m standards, which is characterized by acceleration w m/s^2, is described also by force

$$F = mw.$$

At actions on body and on a group of m standards, which are characterized by identical acceleration w, the force of action on the body is equal to

$$F = Mw \text{ (the second law of mechanics).}$$

It follows from determination of mass that it does not depend on a kind of action.

There is no gravitational or inertial mass. The body has one mass, which is in accordance with its determination.

Only those objects have mass, which can be acted, i. e. can be accelerated.

The particles, which are not acted, for example photon, graviton, neutrino etc., should not be given a mass.

Mass of body is a factor of the conformity between the force of action on a body and its acceleration, which shows in how many times the acceleration of body is less than the acceleration of the measurement standard at identical on force action.

The force of action on a body is the acceleration of a body, but expressed in other units.

The connection between the force and the acceleration can be only as $F = mw$ and there is no other relation of force to acceleration.

The laws of mechanics are not the laws of nature, they are the consequence of an adopted method of the interaction description.

The laws of mechanics are identical to any interactions. They cannot be changed without change of a manner of the interactions description.

Work of force $\vec{F}$ at moving body on distance $\mathbf{d}\vec{l}$ is multiplication of distance $\mathbf{d}\vec{l}$ on force projection to it: $\mathbf{d}A = \vec{F}\,\mathbf{d}\vec{l}$.

Potential energy of interaction of bodies is the value, equal to the work with a converse sign.

Kinetic energy of a body is a half of multiplication of a body mass on square of its velocity.

Electrical charge is a force of action between uniformly electrified bodies, located at a distance in one unit of length.

The electromagnetic wave created by charged or magnetized body is a variable action in each point remote from body which will test other charged or magnetized body placed in it.

The field is a mathematical term for a designation of distribution of any function A at the spatial coordinate system x, y, z at an instant t, which is noted $A\,(x, y, z, t)$.

Electromagnetic field is a mathematical term for a designation of the characteristics of electromagnetic action, for example, of electrical intensity $E(x, y, z, t)$.

Gravitational field is a mathematical term for a designation of the characteristics of gravitational action, for example, the distributions of acceleration of Earth gravitation $g(x, y, z, t)$.

EPILOGUE

We have analysed the basis of mechanics: time, space, speed, acceleration, mass, force also have determined their essence. Space, time and mass cannot depend on speed of motion, therefore, the introduction of such relation in the Theory of Relativity was an error.

We were defined with knowledge of the world: there is an outside ambient world and its description. We invent the description, and the outside ambient world does not depend on our reasoning. This position allows simply deciding a problem of reality of various theoretical representations. Everything, which is invented by imagination, is unreal, i.e. it is not present in the outside ambient world. There are no substations of a type: space, time, mass, force, energy, field, ether. Mass cannot pass in energy, and time in substance. There is no spatially-temporary continuum, there is no curvilinear world, there was no "Big Bang" and there is no extending universe.

We come to a conclusion, that many constructions of modern physics are the curved description of the ambient world. The large activity off correcting the mistakes and creating the new description is necessary. We hope that it will be constructed on the unhypothetical base. The physical sciences will not any more be the convention of sacral positions, unintelligible even for devoted people. They will give people the precise and clear knowledge of the world, which will allow them to organize the life consciously, purposefully and with optimum.

We are sure, that in the world there is nothing afterworld and inexplicable. There is much unknown and interesting to us. We hope that together with a relativity theory, there will leave for inexistence world the mystical perception of the ambient and a prime target of a society will become rushing to new knowledge of the world. The nearest problems in a direction of motion to this are: obtaining the superluminal particles and physics revision, starting with E. Rutherford experiments. Apparently, Ernst Rutherford in 20-ht century was the last scientist, who, like Isaac Newton, trued not to introduce the hypothetical constructions in the scientific results. For the description of phenomena we call to use the unhypothetical approach, which we here tried to demonstrate on the examples of various interactions. The ways, considered in the book, of acceleration of particles up to superluminal velocities we offer to the future physicists-experimentalists and below we present our appeal to them, published in two dissident scientific journals:

1. Smulsky J.J. Appeal to Physicists-Experimentalists // Apeiron. -1998.- Vol. 5, N.1-2.-P. 107;

2. Smulsky J.J. Appeal to Physicists-Experimentalists // Galilean Electrodynamics.- 1998.- Vol. 9, N.5.-P. 88.

Lately, from the moment of the Russian edition of the book, I discussed modern scientific understanding of the world with many colleagues. These discussions have convinced me, that unhypothetical method of studying of the world will allow us to get rid of many lacks and mistakes. Whether and there are mistakes, and what they? As the answer to this question I am too attaching my report at the conference “The Main Mistakes of Modern Science”.

APPEAL TO PHYSICISTS-EXPERIMENTALISTS

Dear colleagues!

I want to attract your attention in acceleration of elementary particles up to superluminal speed. There are no obstacles for reaching such speeds, except outlook stipulated by a relativity theory.

The essence of STR is obstacle in the following. In nature objectively exists the action of bodies against each other. The electromagnetic interactions depend not only on a distance between bodies, but also on their relative speed. The description of interactions in STR is constructed so, that equations for interactions of motionless from each other bodies, and the equations for interactions of relatively moving bodies were identical. Therefore, to satisfy the empirical data, it is necessary to transform parameters at rest to the parameters at motion on known relativistic transformations.

If the interactions between moving bodies to describe, as they are, that is dependent from the speed of their relative motion, transformations of space, time and the mass are not necessary. It is first.

Secondly, the founders of STR, which carried along by ether, have run into fallacy, assuming, that they create not the description of interactions, but create the world, in which the material bodies are subjected to changes according to relativistic rates. And as the relativistic transformations at superluminal speed became imaginary, in STR superluminal motions have prohibited.

But a relativistic description of interactions is not the only possible one. Elementary descriptions, based on classical physics, have been published by G.I. Sukhorukov and co-authors (Russia) [64], T.G. Bames and co-authors [83], C.W. Lucas, Jr. (USA) [102], and many others. Oleg D. Jefimenko, Professor of Physics

at West Virginia University (USA) in his book [98] has presented a method of retarded fields, whose origin can be traced to Oliver Heaviside, which is capable to replace completely the SRT.

Due to the researches I have developed a load-carrying method of the descriptions, based on the expression for force of interaction of two bodies, dependent from a distance and a speed between bodies. The superluminal motions exist in nature: jets of substance and separate fragments of galaxies in far space move with speeds, exceeding the speed of light in some times; the space particles with superluminal speeds are introduced into atmosphere of the Earth. I call to receive superluminal motions on the Earth.

I ask the organizations and scientist to take part in this activity. Many organizations have all necessary to speed up the particles up to the superluminal speed according to the scheme, offered in my foob. If necessary, the scheme of experiment can be changed and adapted to necessary conditions. I offer also to use the methods, developed by me, for calculation of boosters and nuclear transformations. They are more exact relativistic ones.

Why does the Earth need the superluminal motions?

1. They are new drivers for intersidereal rockets.
2. It is the high-power tool in antiasteroidal protection of the Earth.
3. These are new ground technologies.
4. These are the new purposes and perspectives for mankind.

THE MAIN MISTAKES OF MODERN SCIENCE

The Report at VIII International Scientific Conference: Space, Time and Gravitation.
August, 16-20, 2004, Saint-Petersburg, Russia.

J.J.Smulsky

Institute of the Earth's Cryosphere of the Siberian Branch of the Russian Academy of Science, 625000, Tyumen, P.O.Box 1230, Russia.

1. THE CONCEPTION OF THE WORLD AND WHAT THE REAL WORLD IS

Let's address to Homer. Achilles throws a javelin in Trojan's Aeneas, son of Anhis, which, having pierced periphery of a shield, sticks into the ground. Achilles with surprise exclaims:

"Gods! By my eyes I see a great miracle:
Spear in front of me lies on the ground; but do not see the man,

Against which has thrown, which I desired to overthrow.
Truly and this Anhisid to deities Olympic is kind!"

The reason of rescue from inevitable, in opinion of Achilles, destruction of Aeneas the Achilles sees in operation of gods. It is not a metaphor or Homer's literary reception. Reading ancient authors, we see, that the gods controlled the winds and a rain, gave us light of the Sun and of the Moon, directed people and supervised their activities. Now we consider these ideas naive. We know what air is and what its structure is, what the reasons of air velocity are and when air becomes wind or even a terrible hurricane. Our knowledge is so reliable, that we undoubtedly consider mistaken the ancient people's vision of the world.

In this connection there are some questions. Are true our conceptions of the world? Which of them will not be found naive and therefore rejected by our descendants? Are there some true conceptions among our ones, which will never be rejected? Can we find out our mistaken conceptions? If we may, let's reveal them and we shall reject, and we shall not lead up themselves to a shame before the future generations.

There is a surrounding world round us. It is the sky, the stars, the trees, our house, the subjects in it and so on. These objects of world around are changed and influence each on other. We investigate them, explain the reasons of objects change, in this way we create the world understanding. If the ancient people involved the gods' power or demons' ones for the explanations the world, we explain everything with the help of forces, fields, ether, energy, space-time etc. As we can see, the explanations and the understanding of the world constantly change, but the world practically remains changeless.

2. THE UNHYPOTHETICAL DESCRIPTION OF ELECTROMAGNETIC INTERACTIONS

2.1. Interaction of the motionless electrified bodies and magnets

The modern physical understanding of the world is based on the Theory of Relativity. The approach of the description of electromagnetic interactions lays in a basis of it. This approach consists that the field, the space and time, which express interaction of bodies, are changed and deformed in dependence on relative velocity of movement of interacting bodies. Whether so it is?

Let's consider how the magnetic and electrified bodies interact, being based on those laws of electromagnetism, which are received as a result of measurements. The motionless electrified bodies with charges q_1 and q_2 and the magnets with magnetic charges M_1 and M_2 (see Fig. 1, *a*, *b*) act one on other with force according to expressions:

$$\vec{F}_{1e} = \frac{q_1 q_2 \vec{R}}{\varepsilon R^3}; \qquad \vec{F}_{1M} = \frac{\mu M_1 M_2 \vec{R}}{R^3}, \tag{1}$$

Which name Colombos' laws for electrostatic and magnetic interactions.

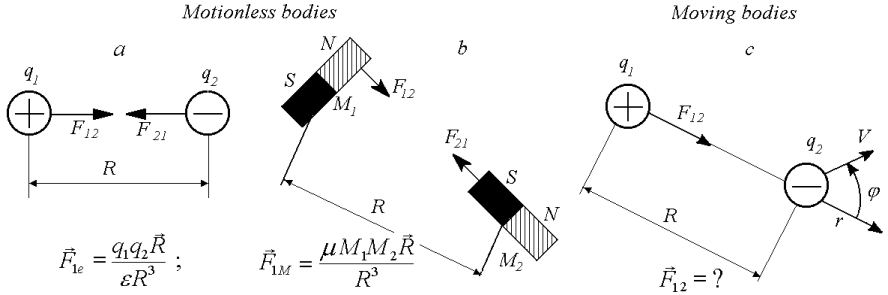


Fig. 1. As magnets and the electrified bodies interact.

These forces are received as a result of measurements. Under action of force the body will start movement and will get velocity *v*. There is a question: the moving charged body q_2 (see Fig. 1, *a*) will act on motionless body with the same force (1) or force will be another? Unfortunately, in electrodynamics of 18 and 19 centuries for an explanation of interactions the concept a field and its determining values: scalar φ and vector potential $\vec{A}$, intensities $\vec{E}$ and $\vec{H}$, inductions $\vec{D}$ и $\vec{B}$ and other values was entered. And the problem of force of interaction till now has remained open. However all measurements of interactions of bodies one on other were carried out. Let's not use a field and its values, and, using of measurements, we shall define force of interaction between the moving electrified bodies.

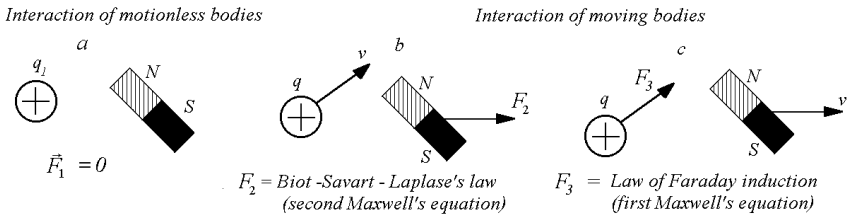


Fig. 2. Than the interaction of motionless and moving bodies is defined?

With this purpose we shall consider interaction of the electrified body *q* and magnet *M*. If they rest (see Fig. 2, *a*), then one body do not acts on other, i.e. the force of interaction *F=0*. If the body *q* (see Fig. 2, *b*) moves relatively magnet *M* we understand it that according to Biot-Savart-Laplace's law in the point of the magnet presence the magnetic field is induced. The magnetic field acts on a magnet with force F_2. We shall reject this interpretation of action, and we shall leave only result: the moving charge acts on magnet with force, which is defined by the mentioned law

F_2 = Biot-Savart-Laplace's law. (2)

If the magnet *M* moves relatively a charge *q* (see Fig. 2, *c*) we understand it that according to the Faraday's law of induction in the point of body *q* the electric field is induced. The electric field acts on a charge *q* with force F_3. Again we shall reject this interpretation of action, and we shall leave only its result: the moving magnet *M* acts on the electrified body *q* with force, which is determined by the above named law:

F_3 = the Faraday's law of induction. (3)

So, three these experimental facts testify, that a motionless charge and they interact when move from one on other. From here, the important conclusion follows: interaction of a charge and a magnet depends on their velocity of relative movement.

2.2 The Interaction Of The Electrified Bodies Moving One Relatively Other

Now we shall return to the answer to a question about interaction of the moving electrified bodies. If the charged body q_2 moves with velocity *v* relatively motionless q_1 (see Fig. 3) that the three above-considered measurements define their interaction. The first component of force F_1 is caused by own interaction of charged bodies F_{1e}. Due to movement of a charge q_2 there is an action on magnet F_2, which is situated at the point of charge q_1. It is the second component. As the distance from the charge q_2 up to this imagined magnet changes, the action on it changes also. The change of this action we shall present the movement of the magnet, which is disposing on the charge *q2* place. So, the third component F_3 will present action of a moving magnet on a charge q_1.
Additional actions F_2 and F_3 depend on velocity of movement and as we already mentioned, it is written as experimental laws Biot-Savart-Laplace

($\mathbf{d}\vec{H} = \frac{I}{R^3 c}\left[\mathbf{d}\vec{l} \times \vec{R}\right]$) and induction of Faraday ($u = -\frac{1}{c}\frac{\mathbf{d}\Phi}{\mathbf{d}t}$). For infinitesimal sizes of a charge and a magnet and at distributed on coordinates the characteristics of action these experimental laws are accordingly: second

(rot $\vec{H} = \frac{\varepsilon}{c q_2}\frac{\partial \vec{F}}{\partial t} + \frac{4\pi}{c}\rho\vec{v}$) and first ((rot $\frac{\vec{F}}{q_2} = -\frac{\mu}{c}\frac{\partial \vec{H}}{\partial t}$) Maxwell's equations.

After exception intensity *H* from them it is received differential equation for interaction force of the moving charge q_2 on motionless q_1 as:

$$\Delta\vec{F} - \frac{1}{c_1^2}\frac{\partial^2 \vec{F}}{\partial t^2} = \frac{4\pi q_2}{\varepsilon}\left[\frac{1}{c_1^2}\frac{\partial(\rho\vec{v})}{\partial t} + \text{grad}\,\rho\right], \quad (4)$$

where $c_1 = c/\sqrt{\mu\varepsilon}$ is speed of light in the considered media, and ρ is density of a charge, which is defined from the condition $q_2 = \int_V \rho \mathbf{d}V$.

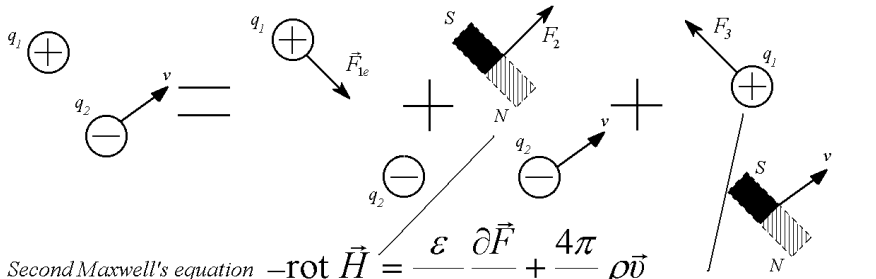


Fig. 3. How the force between moving bodies are defined.

As a result of the decision of this differential equation we have received the following expression for force

$$\vec{F} = \frac{q_2 q_1 \left(1-\beta^2\right)\vec{R}}{\varepsilon\left\{R^2 - \left[\vec{\beta}\times\vec{R}\right]^2\right\}^{3/2}}, \qquad (5)$$

where $\vec{\beta} = \vec{v}/c_1$.

This force describes the all electromagnetic interactions. And as it is seen, the force depends on velocity of movement. If the velocity comes nearer to speed of light ($\beta \to 1$), the force tends to zero. Naturally, the movement of the charged bodies does not lead to the change of space, time and mass as it is accepted in the Theory of Relativity. These positions of the Theory of Relativity are erroneous and should be rejected.

3. MISTAKES OF THE BASES OF THE GENERAL THEORY OF RELATIVITY

By the end of 19 centuries in physics the explanation of the world phenomena was formed with assistance by ether and the field. It was supposed, that waves of light are propagated in ether. The smallest particles of substance consist of ether. The charged bodies and magnets create around of themselves the appropriate fields, which then act on other bodies. Became tempting the idea to present gravitational action as a field. Then all interactions in a nature will express the same as fields. It will be possible to create the united theory of a field and thus as it was supposed, all picture of the world will be constructed.

It is necessary to pay earnest heed. Physicists-theorists aspired to create, construct a picture of the world, i.e. to explain for themselves the behavior of the objects of world around. However, for other people this explanation became understanding. The world around began to be perceived and interpreted by people in

invented images. As these images were represented as real ones, the subsequent generations perceived them as objects of world around. So, we shall note that ether and the field are the concepts entered for an explanation of the world. These are dreamed up objects.

Let's continue consideration of gravitational action. In the Special Theory of Relativity the electromagnetic interaction of bodies moving one relatively other was explained by changing of space-time dependences. In this connection there was a logic contradiction as it was not required space-time transformations for an explanation of gravitational interaction. The desire to create a uniform picture of the world was so strong, that final speed of gravitation propagation, equal to speed of light was accepted. By analogy to the description of electromagnetic interaction the description of gravitational interaction relatively moving bodies was constructed. In addition, this description of gravitational interaction was submitted in the depersonalized four-dimensional curvilinear coordinates. So there was the General Theory of Relativity (GTR) as the new images completely imagined on the basis of mathematical concepts. These imagined objects of GTR are do not like to one of the objects of world around. Therefore this science for the person became non-comparable to world around. Many positions of GTR are not joined to that, which the man sees in world around. GTR is full of logic contradictions. I do not doubt that contradictions would be less, if founders of the Theory of Relativity as ancient men the electromagnetic and gravitational interactions would explain by actions of gods.

For confirmation of GTR the three possible phenomena were involved: perihelion precession of Mercury, a deviation of light and change its frequency at passage near gravitating body. I think that it is necessary to be guided by an unshakable rule: **the ungrounded statement should not even be checked on confirmation.** The unique “basis” for GTR is a desire to create the uniform theory of a field. But the world around is not accommodated to man’s desires.

We may not follow above to the formulated rule and we may consider this desire as the basis. The following statement the GTR is that the speed of gravitation propagation is equal to light speed. This idea was continuously checked from the moment of the formulation by Newton of the universal law gravitation. And each time it was rejected by more exact solving of the equations or taking into account of additional action not taken before into account body. The most difficult calculation of gravitational interaction - the calculation of the Moon movement was executed. On its basis Laplace in 1787 has come to a conclusion, if gravitation speed is final it should exceed speed of light in 100 million times.

Apparently, I have walked on all chain of statements GTR, and have come to a conclusion: any of them has no the bases. The analysis of some these statements is given in my works and, at desire, everyone may be convinced of it independently.

4. THE IMAGINED COMPREHENSIONS OF A MACROCOSM

So, as for as the Special Theory of Relativity the concepts and positions of GTR are erroneous and should be rejected. We shall consider most popular of them.

Gravitational waves. They follow from GTR. If force of gravitation between two bodies would depend on velocity, such waves similar electromagnetic, might exist. But the bases for this purpose are not present. Therefore the gravitational waves do not exist. I call the researchers, almost half-centuries engaged in their detection, to try to understand the above analysis and to refuse from searching of what does not exist.

The closed and open universe, spatial "wormholes", transition through zero-hyperspace etc. These imagined objects are caused GTR. As already I have noted, GTR are formed by two positions: 1) gravitation speed is equal to speed of light; 2) the interaction and movements is considered in four-dimensional curvilinear coordinates. The identification of curvilinear coordinates with some substance also has caused appearance of the above-mentioned imagined objects. Here I shall note that the paraphrasing of the GTR interactions in rectilinear Cartesian three-dimensional coordinates will not result to appearance of these objects.

So, such universes, "wormholes" and zero of hyperspace are not present in the world around. They need to be thrown out from scientific use and to be as more soon neglected. Their existence in household use is fraught with psychological illnesses and traumas.

"Black holes". It is illogical construction of GTR. The essence of this imagined object consists in the following. So as the body has departed the Earth on infinity, its velocity, according to the Newton's law of gravitation, should be not less than 11.2 km/sec, and from the Sun - 500 km/sec. It is possible to imagine body with mass and radius at which escaping velocity will be equal to speed of light: c = 300000 km/sec. Body with such parameters have been named the "black hole". It is supposed, that light from such body cannot leave to the far observer, and therefore this body will been looked as a black hole on a roof of heaven.

The concept of the "black hole" is offered in frameworks of GTR. Here again there is a logic mistake. At approach of body velocity to the speed of light the force of action on it follows to zero (in interpretation of the Theory of Relativity: the mass follows to infinity). Therefore the body with light velocity will not be slowed down and will leave the body, which escaping velocity, according to the Newton's law of gravitation, is equal to speed of light. That is, within the framework of GTR the "black holes" are in principle impossible. If they were really found out, it would confirm the Newton's law of gravitation.

So, the bases for proposing of concepts of "black hole" are not presented. To astrophysicists, which occupy of searching of "black hole", I advise to take into account these arguments and to direct their endeavor on studying of real properties of new found out objects.

The expanding Universe and Big Bang. The object, for example the galaxy, is more distanced from our Earth, the smaller frequency has its spectrum of light. It

is spoken there is "reddening" of light with increasing of distance up to its source. It is known, that at movement of a source of light its frequency changes according to the Doppler's formula. If the source leaves the receiver there is a reddening of light. Therefore at an explanation of the reddening of light of far galaxies by the Doppler effect it is follows that we are in the center from which in all directions Galaxies leave. The conclusion about the expanding Universe and as consequence, that at some moment of time the Universe was collected in one point and the Big Bang has resulted in its expansion, follows from here.

As we see, the concepts of the expanding universe and Big Bang directly do not follow from GTR. However, due to the usual method of GTR of studying of the world as proposing of hypotheses and constructions on them explanations of the world, there was possible an appearance of these paradoxical designs.

The expanding Universe and Big Bang contradict many our knowledge of the world. We shall stop on one of contradictions. As bodies are attracted to each other, then at removing from each other their relative velocities are decreased. This interaction of bodies can be described by mechanical energy E. It is equal to the sum of kinetic energy T and potential energy P. At increase of the distance between bodies their potential energy is increased, and kinetic due to reduction of bodies' velocity is decreased. The increase The expanding Universe and Big Bang. The object, for example the galaxy, is more distanced from our Earth, the smaller frequency has its spectrum of light. It is spoken there is "reddening" of light with increasing of distance up to its source. It is known, that at movement of a source of light its frequency changes according to the Doppler's formula. If the source leaves the receiver there is a reddening of light. Therefore at an explanation of the reddening of light of far galaxies by the Doppler effect it is follows that we are in the centre from which in all directions Galaxies leave. The conclusion about the expanding Universe and as consequence, that at some moment of time the Universe was collected in one point and the Big Bang has resulted in its expansion, follows from here.

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$$E = П + T = \text{const}. \qquad (6)$$

This position, the law of conservation of mechanical energy, is a basis of our civilization. All engineering, the heavenly mechanics, astronautics etc. are based on this law.

At interpretation of a reddening of light by Doppler effect it is received that with removing of object its velocity grows. Therefore, the galaxies located on the big distances have the big both kinetic and potential energy, i.e. their full mechanical energy E with removing from the Earth grows. Thus, in the extending universe there is a continuous increase of mechanical energy.

If scientists did not have sample as the paradoxical Theory of Relativity, they would act absolutely in another way: «As the increase of mechanical energy in the closed system is impossible, the reddening of light of far galaxies Doppler effect does not explained", - to such conclusion would come scientists. And for past 70-80 years other explanation of the phenomenon "reddening of light" would be found.

5. THE CREATION OF IMAGINED OBJECTS INSTEAD OF STUDYING WORLD AROUND IS THE MAIN MISTAKE OF MODERN SCIENCE.

So, the extending Universe and Big Bang there are erroneous conceptions about the world of modern science. We have stopped on the objects created within the framework of the Theory of Relativity. However, on methodology of the Theory of Relativity the quantum mechanics, the theory of a nucleus, the theory of elementary particles and modern astrophysics were constructed. In their frameworks the many imagined objects were created, which are accepted as objects of world around. By these objects the explanation as micro- and a macrocosm is constructed. These objects do not exist, and a modern physical picture of the world is a fruit of human imagination.

I think many with me will agree that an explanation of the world by contemporaries of Homer by the actions of gods is more attractive. Zeus, Hera, Poseidon, Hephaestus etc. for us are more nice than the ether, a field, space-time, Big Bang, the captivated quark etc. Actions of gods we may predict as people have created them on the similarity. The behaviour of the created modern physical objects in any logic frameworks is not stacked. To not amaze our descendants with the naivety, let's more soon get rid of such fantastic explanation of the world.

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Appendix 1

THE PROGRAM IN MATHCAD ENVIRONMENT OF INTEGRATIONS OF THE MOTION EQUATIONS FOR ELLIPSELIKE ORBITS

Initial parameters

$rn := 1.00001$ - initial radius of an integration;

$n := 10$ - quantity of integration segments;

$b := 1.21996$ - radius of apocentre - final radius of an integration. Is set approximately, and th calculated from a condition of equality to zero of radial velocity.

$al1 := -.70 \qquad bt := .70$

Finding of an apocentre radius

$$root\left[\left[1-\frac{bt^2}{b^2}-\left(1-bt^2\right)\cdot\exp\left[2\cdot al1\cdot bt^2\cdot\left(\frac{1}{\sqrt{b^2-bt^2}}-\frac{1}{\sqrt{1-bt^2}}\right)\right]\right],b\right]=1.21996726$$

Evaluation of a step h of segments and other parameters

$$h := \frac{b-rn}{n} \qquad i := 1..n \qquad j := 0..n \qquad j1 := 0..n+1 \qquad r_0 := rn$$

$$z := rn+h, rn+2\cdot h..b \qquad r_{j1+1} := r_{j1}+h \qquad a_i := r_i - h$$

$$bt0 := \sqrt{\left(1-al1^2\right)} \qquad bt0 = 0.7141 \qquad al := 2\cdot al1\cdot bt^2$$

Calculation of radial velocity

$$vr(z) := \frac{\sqrt{1-\frac{bt^2}{z^2}-\left(1-bt^2\right)\cdot\exp\left[2\cdot al1\cdot bt^2\cdot\left(\frac{1}{\sqrt{z^2-bt^2}}-\frac{1}{\sqrt{1-bt^2}}\right)\right]}}{bt}$$

Evaluation of the correction in an initial point rn

$$t_0 := \left[\frac{1}{\sqrt{1+\frac{al1}{\sqrt{1-bt^2}}}}\right]\cdot\left(\ln\left(\sqrt{rn^2-1}+rn\right)\right) \qquad fi_0 := \left[\frac{1}{\sqrt{1+\frac{al1}{\sqrt{1-bt^2}}}}\right]\cdot\frac{\sqrt{rn^2-1}}{rn}$$

Integration of time t and angular coordinates fi

$$f(z) := \frac{1}{z^2\cdot vr(z)} \qquad ft(z) := \frac{1}{vr(z)} \qquad t_0 = 3.1779\cdot 10^{-2} \qquad fi_0 = 3.1779\cdot 10^{-2}$$

$$dt_i := \int_{a_i}^{r_i} ft(z)\,dz \qquad dfi_i := \int_{a_i}^{r_i} f(z)\,dz \qquad vor_j := vr\left(r_j\right)$$

Evaluation of the correction in apocentre

$$dfi_{n+1} := \operatorname{acos}\left[\frac{\sqrt{(al1+1)^2-(vor_n)^2}}{1+al1}\right]\cdot\left[\sqrt{\frac{al1+1}{1+\frac{al1}{\sqrt{1-(bt)^2}}}}\right] \qquad dt_{n+1} := dfi_{n+1}\cdot b^2$$

Summation on segments

$t_i := t_{i-1} + dt_i$ $fi_i := fi_{i-1} + dfi_i$ $t_{n+1} := t_n + dt_{n+1}$ $fi_{n+1} := fi_n + dfi_{n+1}$

$vor_{n+1} := 0$ $r_{n+1} := b$ $x_{j1} := r_{j1}\cdot\cos(fi_{j1})$ $y_{j1} := r_{j1}\cdot\sin(fi_{j1})$

Outcomes of calculations

al1 = −0.7 bt = 0.7 rn = 1 b = 1.22

r_{j1}	vor_j	fi_{j1}	x_{j1}	y_{j1}	dfi_{j1}	t_{j1}
1	$6.2934\cdot10^{-4}$	$3.1779\cdot10^{-2}$	0.9995	$3.1774\cdot10^{-2}$	0	$3.1779\cdot10^{-2}$
1.022	$2.8816\cdot10^{-2}$	1.4797	$9.3004\cdot10^{-2}$	1.0178	1.4479	1.5014
1.044	$3.9224\cdot10^{-2}$	2.0831	-0.5118	0.91	0.6034	2.1445
1.066	$4.5617\cdot10^{-2}$	2.5473	-0.8832	0.5969	0.4642	2.6609
1.088	$4.9288\cdot10^{-2}$	2.9455	-1.0671	0.2119	0.3983	3.1227
1.11	$5.0668\cdot10^{-2}$	3.3087	-1.0945	-0.1846	0.3631	3.5612
1.132	$4.9866\cdot10^{-2}$	3.6557	-0.9857	-0.5566	0.347	3.9971
1.154	$4.6751\cdot10^{-2}$	4.0028	-0.7519	-0.8754	0.3471	4.4507
1.176	$4.0825\cdot10^{-2}$	4.3711	-0.3936	-1.1081	0.3683	4.9506
1.198	$3.0589\cdot10^{-2}$	4.805	0.1108	-1.1928	0.4339	5.5625
1.22	$5.8579\cdot10^{-4}$	5.7342	1.0407	-0.6366	0.9292	6.9282
1.22		5.7418	1.0455	-0.6287	$7.5999\cdot10^{-3}$	6.9395

Formation of an output matrix

$P_{0,0} := al1$ $P_{0,1} := bt$ $P_{0,2} := vor_0$ $P_{0,3} := vor_n$

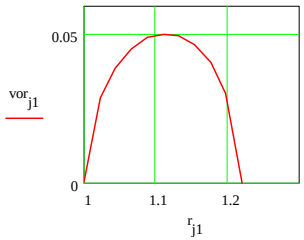
$P_{0,4} := al$ $P_{0,5} := rn$ $P_{0,6} := b$

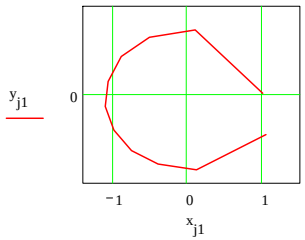
$P_{j1+1,0} := r_{j1}$ $P_{j1+1,1} := vor_{j1}$ $P_{j1+1,2} := fi_{j1}$ $P_{j1+1,3} := x_{j1}$

$P_{j1+1,4} := y_{j1}$ $P_{j1+1,5} := dfi_{j1}$ $P_{j1+1,6} := t_{j1}$

Record of outcomes in the file R_7070.prn WRITEPRN("R_7070.prn") := P

Plotting of the graphics





Appendix 2

INITIAL PARAMETERS OF TRAJECTORIES AND MEANINGS IN FINAL CALCULATED POINTS

Main symbols

Al1 = α_1, Bt = β_p; Btc = $\beta_{pc} = (1 - \alpha_1^2)^{0.5}$;
Al10 = α_1^0; Bto = β_{t0}; Btc0 = $\beta_{pc0} = (1 - (\alpha_1^0)^2)^{0.5}$; Br0 = β_{r0};
Al = $\alpha = 2\alpha_1\beta_p^2$; ficl = φ_{cl}; Tcl = $\bar{t}_{cl}$; Racl =. $\bar{R}_{acl} = R_{acl}/R_p$.

The index "cl" designates the parameters of classical trajectory:
for hyperbolic and parabolic (α_1 >= -0.5) φ_c and $\bar{t}_{cl}$ - angle and relative time at final values R_r;
for elliptical orbits (α_1 < -0.5) φ_{cl}, $\bar{t}_{cl}$ and $\bar{R}_{acl}$ - angle, time and relative radius at the apocentres.

Rr = R/R_p; Vor = $\bar{v}_r = v_r/v_p$; Vor0 = $\bar{v}_r^0 = v_r/v_{t0}$; Fi = φ;
X = x/R_p; Y = y/R_p; dFi = $\Delta\varphi$; T = $\bar{t} = t\,v_p/R_p$.

At trajectories with Bt = Btc the floor level of integration is equal to 1.001.
Numbers of trajectories correspond to numbers of trajectories in tables of values of all computational points [59].

Rr Vor Fi X Y dFi T

1. Attraction (Al1 < 0)

1.1. Al1 = var

1.1.1. Al1 = -.1

1.1.1.1. Al1 = -.100 Bt = .100 Btc = .995
Al = -.002 Ficl = 1.681 Tcl = .1117E+04
.1000E+04 .8950E+00 .1680E+01-.1085E+03 .9941E+03 .1228E-03 .1117E+04

1.1.1.2. Al1 = -.100 Bt = .300 Btc = .995
Al = -.018 Ficl = 1.681 Tcl = .1117E+04
.1000E+04 .8990E+00 .1681E+01-.1095E+03 .9940E+03 .1223E-03 .1112E+04

1.1.1.3. Al1 = -.100 Bt = .500 Btc = .995
Al = -.050 Ficl = 1.681 Tcl = .1117E+04
.1000E+04 .9070E+00 .1683E+01-.1125E+03 .9937E+03 .1212E-03 .1102E+04

1.1.1.4. Al1 = -.100 Bt = .700 Btc = .995
Al = -.098 Ficl = 1.681 Tcl = .1117E+04
.1000E+04 .9200E+00 .1685E+01-.1144E+03 .9934E+03 .1194E-03 .1086E+04

1.1.1.5. Al1 = -.100 Bt = .900 Btc = .995
Al = -.162 Ficl = 1.681 Tcl = .1117E+04
.1000E+04 .9460E+00 .1693E+01-.1223E+03 .9925E+03 .1162E-03 .1057E+04

1.1.1.6. Al1 = -.100 Bt = .995 Btc = .995
Al = -.198 Ficl = 1.681 Tcl = .1117E+04
.1000E+01 .0000E+00 - - - - -
.1000E+04 .9680E+00 .1828E+01-.2547E+03 .9670E+03 .1135E-03 .1033E+04

1.1.2. AL1 = -0.2

1.1.2.1. Al1 = -.200 Bt = .100 Btc = .980

Rr Vor Fi X Y dFi T
Al = -.004 Ficl = 1.822 Tcl = .1288E+04
.1000E+04 .7760E+00 .1822E+01-.2485E+03 .9686E+03 .1417E-03 .1287E+04
1.1.2.2. Al1 = -.200 Bt = .300 Btc = .980
Al = -.036 Ficl = 1.822 Tcl = .1288E+04
.1000E+04 .7820E+00 .1825E+01-.2513E+03 .9679E+03 .1405E-03 .1277E+04
1.1.2.3. Al1 = -.200 Bt = .500 Btc = .980
Al = -.100 Ficl = 1.822 Tcl = .1288E+04
.1000E+04 .7960E+00 .1829E+01-.2552E+03 .9669E+03 .1381E-03 .1255E+04
1.1.2.4. Al1 = -.200 Bt = .700 Btc = .980
Al = -.196 Ficl = 1.822 Tcl = .1288E+04
1000E+04 .8190E+00 .1839E+01-.2648E+03 .9643E+03 .1341E-03 .1219E+04
1.1.2.5. Al1 = -.200 Bt = .900 Btc = .980
Al = -.324 Ficl = 1.822 Tcl = .1288E+04
.1000E+04 .8610E+00 .1890E+01-.3135E+03 .9496E+03 .1276E-03 .1161E+04
1.1.2.6. Al1 = -.200 Bt = .980 Btc = .980
Al = -.384 Ficl = 1.822 Tcl = .1288E+04
.1000E+01 .0000E+00 - - - - -
.1000E+04 .8700E+00 .2412E+01-.7452E+03 .6669E+03 .1262E-03 .1149E+04
1.1.3. AL1 = -.3
1.1.3.1. Al1 = -.300 Bt = .100 Btc = .954
Al = -.006 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .6340E+00 .2011E+01-.4264E+03 .9045E+03 .1733E-03 .1572E+04
1.1.3.2. Al1 = -.300 Bt = .300 Btc = .954
Al = -.054 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .6420E+00 .2017E+01-.4318E+03 .9020E+03 .1712E-03 .1553E+04
1.1.3.3. Al1 = -.300 Bt = .500 Btc = .954
Al = -.150 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .6580E+00 .2028E+01-.4416E+03 .8972E+03 .1669E-03 .1515E+04
1.1.3.4. Al1 = -.300 Bt = .700 Btc = .954
Al = -.294 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .6860E+00 .2061E+01-.4709E+03 .8822E+03 .1602E-03 .1455E+04
1.1.3.5. Al1 = -.300 Bt = .900 Btc = .954
Al = -.486 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .7210E+00 .2259E+01-.6352E+03 .7724E+03 .1524E-03 .1385E+04
1.1.3.6. Al1 = -.300 Bt = .930 Btc = .954
Al = -.519 Ficl = 2.012 Tcl = .1574E+04
.1000E+04 .7180E+00 .2438E+01-.7625E+03 .6469E+03 .1530E-03 .1391E+04
1.1.3.7. Al1 = -.300 Bt = .954 Btc = .954
Al = -.546 Ficl = 2.012 Tcl = .1574E+04
.1000E+01 .0000E+00 - - - - -
.1000E+04 .6990E+00 .3238E+01-.9953E+03-.9635E+02 .1572E-03 .1429E+04
1.1.3.8. Al10 = -.300 Bt0 = .960 Btc0 = .954 Br0 = .100
Al = -.576 Al1 = -.288 Ficl = 1.844 Tcl = .1416E+02
.1042E+02 .7540E+00 .2413E+01-.7772E+01 .6936E+01 .1300E-01 .1465E+02
1.1.3.9. Al10 = -.300 Bt0 = .960 Btc0 = .954 Br0 = .200
Al = -.576 Al1 = -.288 Ficl = 1.844 Tcl = .1416E+02
.1042E+02 .8890E+00 .1785E+01-.2214E+01 .1018E+02 .1100E-01 .1215E+02
1.1.3.10. Al10 = -.300 Bt0 = .960 Btc0 = .954 Br0 = .250
Al = -.576 Al1 = -.288 Ficl = 1.844 Tcl = .1416E+02
.1042E+02 .9790E+00 .1575E+01-.4379E-01 .1042E+02 .1000E-01 .1100E+02
1.1.3.11. A10 = -.300 Bt0 = .960 Btc0 = .954 Br0 = .280
Al = -.576 Al1 = -.288 Ficl = 1.844 Tcl = .1416E+02
.1042E+02 .1037E+01 .1476E+01 .9860E+00 .1037E+02 .1000E-01 .1037E+02
1.1.3.12. Al10 = -.300 Bt0 = .970 Btc0 = .954 Br0 = .100
Al = -.582 Al1 = -.291 Ficl = 1.848 Tcl = .1406E+02
.1031E+02 .7420E+00 .2413E+01-.7692E+01 .6864E+01 .1300E-01 .1458E+02
1.1.3.13. Al10 = -.300 Bt0 = .970 Btc0 = .954 Br0 = .200

Rr Vor Fi X Y dFi T
Al = -.582 Al1 = -.291 Ficl = 1.848 Tcl = .1406E+02
.1031E+02 .9260E+00 .1666E+01-.9800E+00 .1026E+02 .1100E-01 .1246E+02
1.1.3.14. Al10 = -.300 Bt0 = .970 Btc0 = .954 Br0 = .243
Al = -.582 Al1 = -.291 Ficl = 1.848 Tcl = .1406E+02
.1031E+02 .1026E+01 .1473E+01 .1007E+01 .1026E+02 .1000E-01 .1026E+02
1.1.3.15. Al10 = -.300 Bt0 = .980 Btc0 = .954 Br0 = .100
Al = -.588 Al1 = -.294 Ficl = 1.852 Tcl = .1397E+02
.1020E+02 .7110E+00 .2603E+01-.8760E+01 .5234E+01 .1400E-01 .1503E+02
1.1.3.16. Al10 = -.300 Bt0 = .980 Btc0 = .954 Br0 = .199
Al = -.588 Al1 = -.294 Ficl = 1.852 Tcl = .1397E+02
.1020E+02 .1015E+01 .1467E+01 .1057E+01 .1015E+02 .1000E-01 .1015E+02
1.1.3.17. Al10 = -.300 Bt0 = .987 Btc0 = .954 Br0 = .100
Al = -.592 Al1 = -.296 Ficl = .332 Tcl = .3410E+00
.1040E+01 .3000E-02 .7190E+00 .7822E+00 .6847E+00 .1690E+00 .7545E+00
1.1.3.18. Al10 = -.300 Bt0 = .987 Btc0 = .954 Br0 = .161
Al = -.592 Al1 = -.296 Ficl = 1.855 Tcl = .1391E+02
.1013E+02 .1008E+01 .1471E+01 .1009E+01 .1008E+02 .1000E-01 .1008E+02
1.1.4. Al1 = -.4
1.1.4.1. Al1 = -.400 Bt = .100 Btc = .917
Al = -.008 Ficl = 2.298 Tcl = .2211E+04
.1000E+04 .4490E+00 .2279E+01-.6506E+03 .7594E+03 .2447E-03 .2207E+04
1.1.4.2. Al1 = -.400 Bt = .300 Btc = .917
Al = -.072 Ficl = 2.298 Tcl = .2211E+04
.1000E+04 .4560E+00 .2311E+01-.6745E+03 .7382E+03 .2408E-03 .1959E+04
1.1.4.3. Al1 = -.400 Bt = .500 Btc = .917
Al = -.200 Ficl = 2.298 Tcl = .2211E+04
.1000E+04 .4710E+00 .2341E+01-.6963E+03 .7177E+03 .2335E-03 .2109E+04
1.1.4.4. Al1 = -.400 Bt = .700 Btc = .917
Al = -.392 Ficl = 2.298 Tcl = .2211E+04
.1000E+04 .4890E+00 .2434E+01-.7599E+03 .6500E+03 .2245E-03 .2050E+04
1.1.4.5. Al1 = -.400 Bt = .900 Btc = .917
Al = -.648 Ficl = 2.298 Tcl = .2211E+04
.1000E+04 .4450E+00 .3333E+01-.9817E+03-.1904E+03 .2469E-03 .2231E+04
1.1.4.6. Al1 = -.400 Bt = .917 Btc = .917
Al = -.673 Ficl = 2.298 Tcl = .2211E+04
.1000E+01 .0000E+00 - - - - -
.1000E+04 .4110E+00 .4512E+01-.1987E+03-.9801E+03 .2671E-03 .2412E+04
1.1.4.7. Al10 = -.400 Bt0 = .960 Btc0 = .917 Br0 = .200
Al = -.768 Al1 = -.384 Ficl = 2.242 Tcl = .2144E+04
.1042E+04 .7110E+00 .2142E+01-.5632E+03 .8763E+03 .1545E-03 .1523E+04
1.1.5. Al10 = -.498
1.1.5.1. Al10 = -.498 Bt0 = .500 Btc0 = .867 Br0 = .800
Al = -.498 Al1 = -.249 Ficl = 1.902 Tcl = .2791E+03
.2000E+03 .1848E+01 .1742E+01-.3407E+02 .1971E+03 .5352E-03 .2308E+03
1.1.5.2. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .100
Al = -.926 Al1 = -.463 Ficl = .599 Tcl = .6400E+00
.1103E+01 .8979E-03 .1044E+01 .5547E+00 .9537E+00 .1490E+00 .3501E+01
1.1.5.3. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .120
Al = -.926 Al1 = -.463 Ficl = .674 Tcl = .7340E+00
.1133E+01 .8174E-03 .1433E+01 .1557E+00 .1123E+01 .2890E+00 .1688E+01
1.1.5.4. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .128
Al = -.926 Al1 = -.463 Ficl = .765 Tcl = .8550E+00
.1176E+01 .8828E-03 .2366E+01-.8399E+00 .8236E+00 .6650E+00 .4139E+01
1.1.5.5. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .129
Al = -.926 Al1 = -.463 Ficl = 1.811 Tcl = .4384E+01
.2981E+01 .5892E-03 .9652E+01-.2904E+01-.6715E+00 .5450E+00 .2434E+02
1.1.5.6. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .130

Rr Vor Fi X Y dFi T

Al = -.926 Al1 = -.463 Ficl = 1.822 Tcl = .4488E+01
.3035E+01 .9198E-03 .7565E+01 .8650E+00 .2910E+01 .5320E+00 .2150E+02

1.1.5.7. Al10 = -.498 Bt0 = .930 Btc0 = .867 Br0 = .200

Al = -.926 Al1 = -.463 Ficl = 2.611 Tcl = .3961E+06
.1172E+04 .1420E+00 .3420E+01-.1127E+04-.3221E+03 .7300E-01 .1076E+05

1.1.6. Al1 = -.5

1.1.6.1. Al1 = -.500 Bt = .100 Btc = .866

Al = -.010 Ficl = 3.137 Tcl = .7354E+08
.2300E+06 .3434E-03 .3145E+01-.2300E+06-.8302E+03 .8335E-03 .1429E+09

1.1.6.2. Al1 = -.500 Bt = .300 Btc = .866

Al = -.090 Ficl = 3.102 Tcl = .8691E+05
.2570E+04 .7453E-03 .3180E+01-.2568E+04-.9868E+02 .1100E-01 .1802E+06

1.1.6.3. Al1 = -.500 Bt = .500 Btc = .866

Al = -.250 Ficl = 3.015 Tcl = .2651E+04
2500E+03 .3000E-03 .3260E+01-.2482E+03-.2953E+02 .3500E-01 .5842E+04

1.1.6.4. Al1 = -.500 Bt = .700 Btc = .866

Al = -.490 Ficl = 2.831 Tcl = .1577E+03
.3728E+02 .1000E-02 .3502E+01-.3488E+02-.1315E+02 .9200E-01 .3700E+03

1.1.6.5. Al1 = -.500 Bt = .800 Btc = .866

Al = -.640 Ficl = 2.583 Tcl = .3524E+02
.1316E+02 .0000E+00 .3915E+01-.9417E+01-.9194E+01 .7000E-02 .8627E+02

1.1.6.6. Al1 = -.500 Bt = .866 Btc = .866

Al = -.750 Ficl = 2.257 Tcl = .1049E+02
.1000E+01 .0000E+00 - - - - -
.5456E+01 .0000E+00 .6695E+01 .5000E+01 .2184E+01 .5000E-01 .3193E+02

1.1.6.7. Al10 = -.500 Bt0 = .900 Btc0 = .866 Br0 = .200

Al = -.900 Al1 = -.450 Ficl = 2.529 Tcl = .3503E+05
.1111E+06 .2170E+00 .3268E+01-.1102E+06-.1399E+05 .5111E-04 .1010E+06

1.1.7. Al1 = -.6

1.1.7.1. Al1 = -.600 Bt = .100 Btc = .800

Al = -.012 Ficl = 3.142 Tcl = .2107E+02 Racl = .5000E+01
.4969E+01 .0000E+00 .3146E+01-.4969E+01-.2190E-01 .1000E-01 .2101E+02

1.1.7.2. Al1 = -.600 Bt = .300 Btc = .800

Al = -.108 Ficl = 3.142 Tcl = .2107E+02 Racl = .5000E+01
.4712E+01 .0000E+00 .3197E+01-.4705E+01-.2609E+00 .8000E-02 .1968E+02

1.1.7.3. Al1 = -.600 Bt = .500 Btc = .800

Al = -.300 Ficl = 3.142 Tcl = .2107E+02 Racl = .5000E+01
.4100E+01 .0000E+00 .3341E+01-.4019E+01-.8122E+00 .1700E-01 .1694E+02

1.1.7.4. Al1 = -.600 Bt = .700 Btc = .800

Al = -.588 Ficl = 3.142 Tcl = .2107E+02 Racl = .5000E+01
.2867E+01 .0000E+00 .3858E+01-.2162E+01-.1883E+01 .1600E-01 .1205E+02

1.1.7.5. Al1 = -.600 Bt = .800 Btc = .800

Al = -.768 Ficl = 3.142 Tcl = .2107E+02 Racl = .5000E+01
.1000E+01 .0000E+00 - - - - -
.1762E+01 .5518E-03 .9063E+01-.1648E+01 .6236E+00 .7020E+00 .1329E+02

1.1.8. Al1 = -.7

1.1.8.1. Al1 = -.700 Bt = .100 Btc = .714

Al = -.014 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01
.2482E+01 .0000E+00 .3148E+01-.2482E+01-.1590E-01 .2000E-01 .8632E+01

1.1.8.2. Al1 = -.700 Bt = .300 Btc = .714

Al = -.126 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01
.2334E+01 .0000E+00 .3220E+01-.2327E+01-.1828E+00 .2100E-01 .8172E+01

1.1.8.3. Al1 = -.700 Bt = .500 Btc = .714

Al = -.350 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01
.1991E+01 .0000E+00 .3446E+01-.1899E+01-.5968E+00 .1700E-01 .7203E+01

1.1.8.4. Al1 = -.700 Bt = .700 Btc = .714

Al = -.686 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01

Rr Vor Fi X Y dFi T
.1220E+01 .0000E+00 .5726E+01 .1035E+01-.6451E+00 .2600E-01 .6914E+01

1.1.8.5. Al1 = -.700 Bt = .714 Btc = .714

Al = -.714 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01
.1000E+01 .0000E+00 - - - - -
.1031E+01 .1925E-03 .2338E+02-.1845E+00-.1014E+01 .2877E+01 .2404E+02

1.1.8.6. Al10 = -.700 Bt0 = .800 Btc0 = .714 Br0 = .400

Al =-1.120 Al1 = -.560 Ficl = 3.142 Tcl = .4232E+02 Racl = .8333E+01
.1125E+05 .4140E+00 .2439E+01-.8586E+04 .7271E+04 .3350E-04 .3765E+05

1.1.9. Al1 = -.707

1.1.9.1. Al1 = -.707 Bt = .707 Btc = .707

Al = -.707 Ficl = 3.142 Tcl = .8338E+01 Racl = .2415E+01
Rn = 1.0001 Ra = 1.00045
.1000E+01 .0000E+00 - - - - -
.1000E+01 .0000E+00 .1061E+03 .7596E+00-.6504E+00 .9882E-04 .1062E+03

1.1.10. Al1 = -.8

1.1.10.1. Al1 = -.800 Bt = .100 Btc = .436

Al = -.016 Ficl = 3.142 Tcl = .5408E+01 Racl = .1667E+01
.1653E+01 .0000E+00 .3151E+01-.1653E+01-.1555E-01 .3000E-01 .5373E+01

1.1.10.2 Al1 = -.800 Bt = .300 Btc = .600

Al = -.144 Ficl = 3.142 Tcl = .5408E+01 Racl = .1667E+01
.1540E+01 .0000E+00 .3243E+01-.1532E+01-.1559E+00 .2800E-01 .5112E+01

1.1.10.3. Al1 = -.800 Bt = .500 Btc = .600

Al = -.400 Ficl = 3.142 Tcl = .5408E+01 Racl = .1667E+01
.1270E+01 .0000E+00 .3601E+01-.1138E+01-.5631E+00 .5700E-01 .4616E+01

1.1.10.4. Al1 = -.800 Bt = .599 Btc = .600

Al = -.574 Ficl = 3.142 Tcl = .8338E+01 Racl = .2415E+01
Rn = 1.001 Ra = 1.0042
.1004E+01 .0000E+00 .4366E+01-.3409E+00-.9444E+00 .2100E-01 .4362E+01

1.1.11. Al1 = -.9

1.1.11.1. Al1 = -.900 Bt = .100 Btc = .436

Al = -.018 Ficl = 3.142 Tcl = .3951E+01 Racl = .1250E+01
.1239E+01 .0000E+00 .3151E+01-.1239E+01-.1166E-01 .3100E-01 .3929E+01

1.1.11.2. Al1 = -.900 Bt = .300 Btc = .436

Al = -.162 Ficl = 3.142 Tcl = .3951E+01 Racl = .1250E+01
.1142E+01 .0000E+00 .3280E+01-.1131E+01-.1576E+00 .2700E-01 .3764E+01

1.1.11.3. Al1 = -.900 Bt = .400 Btc = .436

Al = -.288 Ficl = 3.142 Tcl = .3951E+01 Racl = .1250E+01
Rn = 1.001 Ra = 1.0462
.1046E+01 .0000E+00 .3411E+01-.1008E+01-.2784E+00 .2400E-01 .3565E+01

1.1.11.4. Al1 = -.900 Bt = .435 Btc = .436

Al = -.341 Ficl = 3.142 Tcl = .3951E+01 Racl = .1250E+01
Rn = 1.001 Ra = 1.00124
.1001E+01 .0000E+00 .2925E+01-.9776E+00 .2151E+00 .1300E-01 .2915E+01

1.1.11.5. Al10 = -.900 Bt0 = .600 Btc0 = .436 Br0 = .500

Al =-1.080 Al1 = -.540 Ficl = 3.142 Tcl = .7497E+02 Racl = .1250E+02
.1667E+05 .5850E+00 .2455E+01-.1289E+05 .1057E+05 .1896E-04 .4741E+05

1.2. Al = -0.3 = const

1.2.1. Al1 = -.900 Bt = .408 Btc = .436

Al = -.300 Ficl = 3.142 Tcl = .3951E+01 Racl = .1250E+01
.1036E+01 .0000E+00 .3315E+01-.1020E+01-.1788E+00 .1060E+00 .3426E+01

1.2.2. Al1 = -.700 Bt = .463 Btc = .714

Al = -.300 Ficl = 3.142 Tcl = .8693E+01 Racl = .2500E+01
.2074E+01 .0000E+00 .3377E+01-.2017E+01-.4837E+00 .1200E-01 .7417E+01

1.2.3. Al1 = -.500 Bt = .548 Btc = .866

Al = -.300 Ficl = 2.982 Tcl = .1324E+04
.1570E+03 .0000E+00 .3301E+01-.1550E+03-.2485E+02 .5679E-03 .3123E+04

1.2.4. Al1 = -.497 Bt = .550 Btc = .868

```
  Rr          Vor          Fi           X            Y           dFi          T
                    Al =  -.301     Ficl =  2.889     Tcl =  .5838E+03
.1000E+03 .1030E+00 .3095E+01-.9989E+02 .4658E+01 .9000E-02 .6656E+03
                 1.2.5.   Al1 =  -.300     Bt =   .707     Btc =   .954
                    Al =  -.300     Ficl =  1.998     Tcl =  .1536E+03
.1000E+03 .6900E+00 .2050E+01-.4611E+02 .8874E+02 .1000E-02 .1430E+03
                 1.2.6.   Al1 =  -.200     Bt =   .866     Btc =   .980
                    Al =  -.300     Ficl =  1.811     Tcl =  .1274E+03
.1000E+03 .8530E+00 .1861E+01-.2861E+02 .9582E+02 .1000E-02 .1169E+03
                 1.2.7.   Al1 =  -.180     Bt =   .913     Btc =   .984
                    Al =  -.300     Ficl =  1.780     Tcl =  .1236E+03
.1000E+03 .8860E+00 .1786E+01-.2135E+02 .9769E+02 .1000E-02 .1128E+03
                 1.2.8.   Al1 =  -.154     Bt =   .988     Btc =   .988
                    Al =  -.301     Ficl =  1.742     Tcl =  .1191E+03
  .1000E+01 .0000E+00          -            -            -           -           -
.1000E+03 .9240E+00 .2105E+01-.5092E+02 .8607E+02 .1000E-02 .1086E+03
              1.2.9.   Al10 =  -.152   Bt0 =   .992   Btc0 =   .988   Br0 =   .100
                 Al =  -.302   Al1 =  -.151   Ficl =  1.751  Tcl =  .1208E+05
.1008E+05 .9760E+00 .1698E+01-.1278E+04 .9999E+04 .1138E-04 .1042E+05
                 1.2.10.  Al1 =  -.150     Bt =  1.000     Btc =   .989
                    Al =  -.300     Ficl =  1.736     Tcl =  .1185E+03
.1000E+03 .1000E+01 .1562E+01 .8807E+00 .1000E+03 .9891E-03 .1000E+03
                              2. Repulsion (Al1 > 0)
                                 2.1. Al1 = .3
                       2.1.1.    Al1 =   .300     Bt =   .100
                    Al =   .006     Ficl =  1.337     Tcl =  .7916E+03
.1000E+04 .1263E+01 .1338E+01 .2309E+03 .9730E+03 .8701E-04 .7930E+03
                       2.1.2.    Al1 =   .300     Bt =   .500
                    Al =   .150     Ficl =  1.337     Tcl =  .7916E+03
.1000E+04 .1215E+01 .1342E+01 .2269E+03 .9739E+03 .9042E-04 .8235E+03
                       2.1.3.    Al1 =   .300     Bt =   .900
                    Al =   .486     Ficl =  1.337     Tcl =  .7916E+03
.1000E+04 .1076E+01 .1380E+01 .1895E+03 .9819E+03 .1021E-03 .9294E+03
                       2.1.4.    Al1 =   .300     Bt =  1.000
                    Al =   .600     Ficl =  1.337     Tcl =  .7916E+03
.1000E+04 .1000E+01 .1569E+01 .1539E+01 .1000E+04 .1099E-03 .1000E+04
                                 2.2. Al1 = .7
                       2.2.1.    Al1 =   .700     Bt =   .100
                    Al =   .014     Ficl =  1.146     Tcl =  .6468E+03
.1000E+04 .1543E+01 .1147E+01 .4108E+03 .9117E+03 .7120E-04 .6490E+03
                       2.2.2.    Al1 =   .700     Bt =   .500
                    Al =   .350     Ficl =  1.146     Tcl =  .6468E+03
.1000E+04 .1413E+01 .1163E+01 .3970E+03 .9178E+03 .7776E-04 .7084E+03
                       2.2.3.    Al1 =   .700     Bt =   .900
                    Al =  1.134     Ficl =  1.146     Tcl =  .6468E+03
.1000E+04 .1103E+01 .1277E+01 .2895E+03 .9572E+03 .9960E-04 .9062E+03
                                 2.3. Al1 = 1.5
                       2.3.1.    Al1 =  1.500     Bt =   .100
                    Al =   .030     Ficl =   .927     Tcl =  .5013E+03
.1000E+04 .1984E+01 .9289E+00 .5987E+03 .8010E+03 .5537E-04 .5051E+03
                       2.3.2.    Al1 =  1.500     Bt =   .500
                    Al =   .750     Ficl =   .927     Tcl =  .5013E+03
.1000E+04 .1654E+01 .9703E+00 .5651E+03 .8251E+03 .6641E-04 .6050E+03
                       2.3.3.    Al1 =  1.500     Bt =   .900
                    Al =  2.430     Ficl =   .927     Tcl =  .5013E+03
.1000E+04 .1111E+01 .1205E+01 .3577E+03 .9339E+03 .9893E-04 .9132E+10
```


Appendix 3

THE PROGRAM IN MATHCAD ENVIRONMENT OF INTEGRATIONS OF THE EQUATIONS OF NEARLUMINAL PARTICLE MOVEMENT

The initial parameters (Are adduced to values at an initial point R0).

$rn := 1$ is the initial radius of an integration;;

$n := 10$ is a quantity of integration segments;

b is a final radius of an integration.

$al10 := -4 \qquad bt := .7 \qquad br := 0.3 \qquad b := bt + .000002$

Evaluation of a step h of segments and other parameters

$h := \frac{b - rn}{n} \qquad i := 1..n \qquad j := 0..n \qquad r_0 := rn \qquad r_{j+1} := r_j + h \qquad z := rn + h, rn + 2 \cdot h .. b$

$a_i := r_i - h \qquad bp := \sqrt{1 - bt^2} \qquad fi_0 := 0 \qquad dfi_0 := 0 \qquad dfi_{n+1} := 0 \qquad t_0 := 0 \qquad al1 := al10 \cdot bt \qquad al := 2 \cdot al$

$ba := \sqrt{1 - bt^2 - \exp\left[2 \cdot al10 \cdot bt^2 \cdot \left(1 - bt^2\right)^{-.5}\right]} \qquad ba = 0.7112 \quad bp = 0.7141$

Calculation of radial velocity

$$vor(z) := \frac{\sqrt{1 - \frac{bt^2}{z^2} - \left(1 - br^2 - bt^2\right) \cdot \exp\left[2 \cdot al10 \cdot bt^2 \cdot \left(\frac{1}{\sqrt{z^2 - bt^2}} - \frac{1}{\sqrt{1 - bt^2}}\right)\right]}}{bt}$$

Integration of time t and angular coordinates fi

$$f(z) := \frac{1}{z^2 \cdot vor(z)} \qquad ft(z) := \frac{1}{vor(z)} \qquad dt_i := \int_{a_i}^{r_i} ft(z)\,dz \qquad dfi_i := \int_{a_i}^{r_i} f(z)\,dz$$

Summation on segments The correction at the pericentre at luminal velocity

$t_i := t_{i-1} + dt_i \qquad fi_i := fi_{i-1} + dfi_i \qquad t_n := t_n - .002 \cdot bt^2 \qquad fi_n := fi_n - .002$

Reduction to parameters at the pericentre

$z1_j := r_j \qquad r_j := \frac{r_j}{bt} \qquad at := t_n \qquad afi := fi_n \qquad fi_j := fi_j - afi \qquad t_j := \frac{t_j - at}{bt^2}$

$bt0 := bt \qquad br0 := br \qquad x_j := r_j \cdot \cos\left(fi_j\right) \qquad y_j := r_j \cdot \sin\left(fi_j\right)$

$vr_j := vor\left(z1_j\right) \cdot bt \qquad k := n..0 \qquad k1 := n + 1..1 \qquad dfi_i := -dfi_i$

Outcomes of calculation

al10 = −4 bt0 = 0.7 br0 = 0.3 b = 0.7

al = −5.6 al1 = −2.8 vor(rn) = 0.4286

r_k	vr_k	fi_k	x_k	y_k	dfi_{k1}	t_k
1	$2.3905\cdot10^{-3}$	0	1	0	0	0
1.0429	0.2837	0.2873	1.0001	0.2955	0.2853	0.2956
1.0857	0.3892	0.3997	1.0001	0.4225	0.1124	0.4226
1.1286	0.4611	0.4818	1.0001	0.523	$8.2152\cdot10^{-2}$	0.5231
1.1714	0.5107	0.5484	0.9996	0.6107	$6.6587\cdot10^{-2}$	0.6111
1.2143	0.5402	0.6056	0.9983	0.6912	$5.7194\cdot10^{-2}$	0.6924
1.2571	0.5495	0.657	0.9955	0.7678	$5.1368\cdot10^{-2}$	0.7708
1.3	0.5369	0.7051	0.99	0.8425	$4.8101\cdot10^{-2}$	0.8495
1.3429	0.4989	0.7523	0.9805	0.9176	$4.7194\cdot10^{-2}$	0.9319
1.3857	0.4276	0.8017	0.9638	0.9957	$4.9421\cdot10^{-2}$	1.0239
1.4286	0.3	0.8606	0.9314	1.0832	$5.8909\cdot10^{-2}$	1.1407

Formation of an output matrix

$P_{0,0} := al10$ $P_{0,1} := bt0$ $P_{0,2} := br0$ $P_{0,3} := al1$

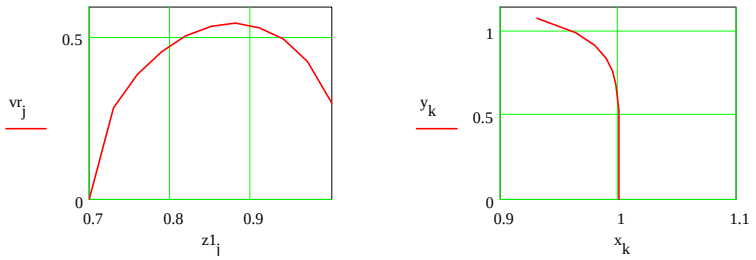
$P_{0,4} := al$ $P_{0,5} := rn$ $P_{0,6} := b$

$P_{j+1,0} := r_j$ $P_{j+1,1} := vr_j$ $P_{j+1,2} := fi_j$ $P_{j+1,3} := x_j$

$P_{j+1,4} := y_j$ $P_{k+1,5} := dfi_k$ $P_{j+1,6} := t_j$

Record of outcomes in the file R_407003.prn WRITEPRN("R_40703.prn") := P

Plotting of the graphics



TRAJECTORIES OF MOVEMENT INSIDE «BLACK HOLES»

1.1.1. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0$ $\alpha_1 = -1$ $\alpha = -2$

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1	0.002	0	1	0	0	0
1.1	0.3785	0.437	0.9966	0.4656	0.436	0.4701
1.2	0.4346	0.6209	0.976	0.6982	0.1839	0.7124
1.3	0.4405	0.7667	0.9363	0.9019	0.1457	0.9397
1.4	0.4251	0.8932	0.8777	1.091	0.1265	1.17
1.5	0.3972	1.009	0.7994	1.269	0.1155	1.413
1.6	0.3599	1.119	0.6992	1.439	0.1098	1.677
1.7	0.3137	1.227	0.5722	1.601	0.1089	1.973
1.8	0.2567	1.342	0.4086	1.753	0.1143	2.324
1.9	0.1814	1.475	0.1821	1.891	0.133	2.78
2	0	1.759	-0.3746	1.965	0.2844	3.879

1.1.2. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.1$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.3791	0.4369	0.9967	0.4654	0.4359	0.4699
1.2	0.4364	0.6203	0.9764	0.6976	0.1834	0.7116
1.3	0.4437	0.7652	0.9376	0.9005	0.1449	0.9376
1.4	0.43	0.8906	0.8805	1.088	0.1254	1.166
1.5	0.4038	1.005	0.8047	1.266	0.114	1.405
1.6	0.3687	1.112	0.7083	1.435	0.1077	1.664
1.7	0.3253	1.218	0.5874	1.595	0.1057	1.952
1.8	0.2725	1.327	0.4344	1.747	0.1091	2.286
1.9	0.2052	1.449	0.2305	1.886	0.1221	2.705
2	0.1	1.621	-0.0997	1.998	0.1715	3.361
2.003	0.09486	1.629	-0.1166	2	0.008032	3.393
2.006	0.08942	1.637	-0.1337	2.002	0.008466	3.427
2.009	0.08364	1.646	-0.1519	2.004	0.008987	3.463
2.013	0.07743	1.656	-0.1715	2.005	0.009627	3.502
2.016	0.07067	1.667	-0.1927	2.006	0.01044	3.545
2.019	0.0632	1.678	-0.2161	2.007	0.01151	3.592
2.022	0.05473	1.691	-0.2426	2.007	0.01302	3.645
2.025	0.04468	1.706	-0.2739	2.006	0.0154	3.708
2.028	0.03159	1.727	-0.3145	2.004	0.02001	3.79
2.031	0.0002414	1.774	-0.4107	1.989	0.04789	3.987

1.1.3. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.3$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.3833	0.4359	0.9971	0.4645	0.4349	0.4688
1.2	0.4504	0.6157	0.9797	0.693	0.1797	0.7055
1.3	0.4688	0.7545	0.9472	0.8904	0.1389	0.922
1.4	0.4667	0.8717	0.901	1.072	0.1172	1.135
1.5	0.4533	0.975	0.8418	1.242	0.1033	1.352
1.6	0.4325	1.069	0.7697	1.403	0.09393	1.578
1.7	0.4063	1.156	0.6844	1.556	0.08754	1.816
1.8	0.3755	1.24	0.5847	1.702	0.08351	2.072
1.9	0.3402	1.322	0.4686	1.841	0.08162	2.351
2	0.3	1.404	0.3326	1.972	0.08212	2.664
2.036	0.2843	1.434	0.2779	2.016	0.02985	2.786
2.071	0.2677	1.464	0.22	2.059	0.03051	2.914
2.107	0.2501	1.496	0.1579	2.101	0.03143	3.051

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.142	0.2312	1.528	0.09064	2.14	0.03269	3.199
2.178	0.2107	1.563	0.01717	2.177	0.03444	3.359
2.213	0.1881	1.6	-0.06428	2.212	0.03694	3.538
2.249	0.1626	1.641	-0.1566	2.243	0.04067	3.74
2.284	0.1325	1.687	-0.2656	2.269	0.04683	3.981
2.32	0.0935	1.747	-0.4057	2.284	0.05926	4.295
2.355	0.0004051	1.885	-0.727	2.24	0.138	5.053

1.1.4. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.5$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.3916	0.4341	0.998	0.4627	0.4331	0.4667
1.2	0.4772	0.607	0.9856	0.6845	0.1729	0.6943
1.3	0.5152	0.7358	0.9637	0.8725	0.1288	0.895
1.4	0.5327	0.8404	0.934	1.043	0.1047	1.085
1.5	0.5389	0.9292	0.8977	1.202	0.08877	1.272
1.6	0.538	1.007	0.8557	1.352	0.07732	1.457
1.7	0.5325	1.075	0.8085	1.495	0.06864	1.644
1.8	0.5238	1.137	0.7566	1.633	0.06184	1.833
1.9	0.5128	1.193	0.7002	1.766	0.05639	2.026
2	0.5	1.245	0.6395	1.895	0.05195	2.224
2.191	0.4722	1.335	0.5129	2.13	0.08952	2.617
2.382	0.4413	1.415	0.3707	2.353	0.08002	3.035
2.573	0.4081	1.488	0.2132	2.564	0.07329	3.484
2.764	0.3729	1.557	0.03943	2.763	0.0687	3.973
2.955	0.3357	1.622	-0.1526	2.951	0.06593	4.512
3.145	0.296	1.687	-0.3661	3.124	0.06499	5.117
3.336	0.2526	1.754	-0.6069	3.281	0.06628	5.814
3.527	0.2032	1.825	-0.8865	3.414	0.07113	6.652
3.718	0.1415	1.909	-1.234	3.507	0.08434	7.762
3.909	0.0006827	2.093	-1.949	3.389	0.1835	10.47

1.1.5. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.659$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.401	0.4321	0.9989	0.4606	0.4311	0.4614
1.2	0.5064	0.5981	0.9917	0.6757	0.166	0.6798
1.3	0.564	0.7176	0.9794	0.8548	0.1195	0.8659
1.4	0.5997	0.8119	0.9634	1.016	0.09431	1.037
1.5	0.623	0.8897	0.9444	1.165	0.07783	1.201
1.6	0.6383	0.9558	0.9232	1.307	0.06604	1.359
1.7	0.6482	1.013	0.9	1.442	0.05714	1.515
1.8	0.6543	1.063	0.8751	1.573	0.05017	1.668
1.9	0.6576	1.108	0.8489	1.7	0.04456	1.821
2	0.659	1.148	0.8213	1.824	0.03997	1.972
3.8	0.5926	1.519	0.1966	3.795	0.371	4.825
5.6	0.522	1.67	-0.5559	5.572	0.1512	8.069
7.4	0.4695	1.758	-1.375	7.271	0.08749	11.71
9.2	0.4297	1.816	-2.238	8.924	0.05877	15.73
11	0.3983	1.859	-3.131	10.54	0.04295	20.08
12.8	0.3728	1.893	-4.048	12.14	0.03315	24.76
14.6	0.3516	1.919	-4.984	13.72	0.02659	29.73
16.4	0.3336	1.941	-5.935	15.29	0.02194	34.99
18.2	0.3181	1.96	-6.9	16.84	0.01851	40.52
20	0.3046	1.976	-7.875	18.38	0.01588	46.31
201.8	0.09918	2.196	-118.1	163.6	0.22	1366
401.6	0.07044	2.225	-244.4	318.7	0.02906	3815
601.4	0.0576	2.238	-372.2	472.4	0.01292	6978

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
801.2	0.04992	2.246	-500.6	625.5	0.007714	10720
1001	0.04467	2.251	-629.6	778.2	0.005268	14960
1201	0.04079	2.255	-758.9	930.6	0.00389	19650
1401	0.03777	2.258	-888.4	1083	0.003025	24750
1600	0.03533	2.26	-1018	1235	0.002439	30220
1800	0.03332	2.262	-1148	1387	0.00202	36050
2000	0.03161	2.264	-1278	1538	0.001709	42210

1.1.6. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.7$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.4038	0.4315	0.9992	0.46	0.4305	0.4637
1.2	0.5149	0.5956	0.9934	0.6732	0.1641	0.6795
1.3	0.5779	0.7126	0.9836	0.85	0.1171	0.8619
1.4	0.6186	0.8044	0.971	1.009	0.09174	1.029
1.5	0.6463	0.8796	0.9561	1.156	0.07524	1.187
1.6	0.6656	0.9431	0.9396	1.295	0.06349	1.339
1.7	0.6793	0.9978	0.9217	1.428	0.05466	1.488
1.8	0.6889	1.046	0.9026	1.557	0.04776	1.634
1.9	0.6955	1.088	0.8825	1.683	0.04223	1.778
2	0.7	1.125	0.8615	1.805	0.03771	1.921
3.8	0.6728	1.465	0.4017	3.779	0.3399	4.516
5.6	0.6279	1.595	-0.1333	5.598	0.1297	7.289
7.4	0.5947	1.666	-0.7005	7.367	0.071	10.24
9.2	0.5703	1.711	-1.285	9.11	0.04537	13.33
11	0.5517	1.743	-1.881	10.84	0.0317	16.54
12.8	0.5372	1.766	-2.485	12.56	0.02348	19.85
14.6	0.5255	1.784	-3.093	14.27	0.01813	23.24
16.4	0.5159	1.799	-3.706	15.98	0.01444	26.7
18.2	0.5079	1.81	-4.321	17.68	0.01178	30.22
20	0.5011	1.82	-4.938	19.38	0.009802	33.78
201.8	0.4279	1.917	-68.49	189.8	0.097	446
401.6	0.4232	1.923	-138.5	377	0.005793	916.1
601.4	0.4216	1.925	-208.5	564.1	0.001958	1389
801.2	0.4208	1.926	-278.5	751.2	0.0009845	864
1001	0.4203	1.926	-348.5	938.4	0.0005924	2339
1201	0.42	1.927	-418.5	1126	0.0003956	2814
1401	0.4197	1.927	-488.5	1313	0.000283	3290
1600	0.4196	1.927	-558.5	1500	0.0002124	3766
1800	0.4194	1.927	-628.6	1687	0.0001653	4243
2000	0.4193	1.928	-698.6	1874	0.0001323	4719

1.1.7. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.8$ $\alpha_1 = -1$ $\alpha = -2$

1	0.002	0	1	0	0	0
1.1	0.4112	0.4299	0.9999	0.4585	0.4289	0.4619
1.2	0.5371	0.5892	0.9977	0.6668	0.1593	0.6713
1.3	0.6139	0.7004	0.994	0.8379	0.1112	0.8445
1.4	0.6667	0.7861	0.9892	0.9907	0.08575	1
1.5	0.7051	0.8555	0.9837	1.132	0.06939	1.146
1.6	0.7342	0.9134	0.9777	1.267	0.05788	1.285
1.7	0.7566	0.9627	0.9712	1.395	0.04931	1.419
1.8	0.7744	1.005	0.9643	1.52	0.04268	1.55
1.9	0.7885	1.043	0.9572	1.641	0.03741	1.677
2	0.8	1.076	0.9498	1.76	0.03313	1.803
3.8	0.8535	1.36	0.7948	3.716	0.2841	3.948
5.6	0.8517	1.459	0.6237	5.565	0.0991	6.057
7.4	0.8462	1.51	0.4471	7.386	0.05115	8.178

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
9.2	0.8413	1.542	0.2678	9.196	0.03133	10.31
11	0.8372	1.563	0.08718	11	0.02119	12.46
12.8	0.834	1.578	-0.09438	12.8	0.0153	14.61
14.6	0.8313	1.59	-0.2765	14.6	0.01157	16.77
16.4	0.8291	1.599	-0.4591	16.39	0.009055	18.94
18.2	0.8272	1.606	-0.6419	18.19	0.007282	21.11
20	0.8257	1.612	-0.825	19.98	0.005983	23.29
201.8	0.8089	1.667	-19.41	200.9	0.055	247.2
401.6	0.8079	1.67	-39.85	399.6	0.00305	494.4
601.4	0.8075	1.671	-60.3	598.4	0.001024	741.8
801.2	0.8074	1.672	-80.74	797.1	0.0005135	989.2
1001	0.8073	1.672	-101.2	995.9	0.0003086	1237
1201	0.8072	1.672	-121.6	1195	0.0002059	1484
1401	0.8071	1.672	-142.1	1393	0.0001472	1732
1600	0.8071	1.673	-162.5	1592	0.0001104	1979
1800	0.8071	1.673	-182.9	1791	8.593e-005	2227
2000	0.807	1.673	-203.4	1990	6.876e-005	2474

1.2.1. $\alpha_1^0 = -2$ $\beta_{t0} = 0.5516$ $\beta_{r0} = 0$ α -1.103

1.813	0	1.572	-0.002003	1.813	0	3.051
1.772	0.138	1.39	0.3186	1.743	-0.1819	2.462
1.732	0.1949	1.311	0.4455	1.673	-0.07944	2.218
1.691	0.2383	1.247	0.5387	1.603	-0.06401	2.03
1.65	0.2745	1.19	0.6136	1.532	-0.05676	1.872
1.61	0.3058	1.137	0.6764	1.461	-0.0527	1.732
1.569	0.3335	1.087	0.7301	1.389	-0.05032	1.604
1.528	0.3581	1.038	0.7766	1.316	-0.04899	1.487
1.488	0.3799	0.9894	0.8171	1.243	-0.04842	1.377
1.447	0.3992	0.941	0.8524	1.169	-0.04843	1.272
1.406	0.4158	0.892	0.8831	1.095	-0.04897	1.173
1.366	0.4297	0.842	0.9096	1.019	-0.05002	1.077
1.325	0.4403	0.7904	0.9324	0.9417	-0.05159	0.9833
1.285	0.4473	0.7366	0.9515	0.8629	-0.05376	0.8918
1.244	0.4495	0.6799	0.9672	0.7821	-0.05668	0.8012
1.203	0.4456	0.6193	0.9797	0.6985	-0.0606	0.7105
1.163	0.4332	0.5533	0.9891	0.611	-0.06601	0.6182
1.122	0.408	0.4794	0.9954	0.5175	-0.0739	0.5218
1.081	0.3615	0.3927	0.999	0.4138	-0.08673	0.4167
1.041	0.2742	0.2799	1	0.2875	-0.1128	0.29
1	0.001904	0	1	0	-0.2789	0

1.3.1. $\alpha_1^0 = -2$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0$ $\alpha_1 = -1.4$ $\alpha = -2.8$

1	0.00169	0	1	0	0	0
1.043	0.2826	0.2872	1	0.2954	0.2862	0.2964
1.086	0.3756	0.4014	0.9994	0.4242	0.1142	0.4255
1.129	0.4211	0.4887	0.9965	0.5298	0.0873	0.5324
1.171	0.4366	0.5639	0.99	0.6261	0.07526	0.6318
1.214	0.4308	0.6331	0.9789	0.7185	0.06921	0.7303
1.257	0.4076	0.6999	0.9616	0.8098	0.06675	0.8322
1.3	0.3683	0.7673	0.9358	0.9024	0.06737	0.9424
1.343	0.3107	0.8393	0.897	0.9993	0.07205	1.068
1.386	0.2254	0.9248	0.8342	1.106	0.08549	1.228
1.429	0	1.111	0.6338	1.28	0.1863	1.6

1.3.2. $\alpha_1^0 = -2$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.1$ $\alpha_1 = -1.4$ $\alpha = -2.8$

1	0.00169	0	1	0	0	0

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.043	0.2827	0.2872	1	0.2954	0.2862	0.2964
1.086	0.3759	0.4013	0.9994	0.4241	0.1141	0.4254
1.129	0.4219	0.4885	0.9966	0.5297	0.08719	0.5322
1.171	0.4384	0.5635	0.9903	0.6258	0.07503	0.6314
1.214	0.4339	0.6324	0.9795	0.7177	0.06883	0.7293
1.257	0.4125	0.6985	0.9627	0.8084	0.06613	0.8303
1.3	0.3755	0.7649	0.9379	0.9002	0.06636	0.9388
1.343	0.3215	0.8351	0.9012	0.9955	0.07021	1.061
1.386	0.2433	0.9163	0.8436	1.099	0.08119	1.213
1.429	0.1	1.041	0.7218	1.233	0.1248	1.461
1.429	0.0949	1.045	0.7172	1.236	0.004166	1.47
1.43	0.08951	1.05	0.7122	1.24	0.004398	1.478
1.431	0.08376	1.054	0.7068	1.244	0.004676	1.488
1.432	0.07757	1.059	0.7009	1.249	0.005016	1.498
1.433	0.07084	1.065	0.6945	1.253	0.005446	1.51
1.434	0.06338	1.071	0.6874	1.258	0.006015	1.522
1.434	0.05491	1.078	0.6792	1.263	0.006817	1.536
1.435	0.04485	1.086	0.6693	1.27	0.008073	1.553
1.436	0.03173	1.096	0.6564	1.277	0.0105	1.574
1.437	0.0009264	1.121	0.625	1.294	0.02466	1.625

1.3.3. $\alpha_1^0 = -2$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.3$ $\alpha_1 = -1.4$ $\alpha = -2.8$

1	0.00169	0	1	0	0	0
1.043	0.2828	0.2872	1	0.2954	0.2862	0.2964
1.086	0.3781	0.401	0.9996	0.4238	0.1138	0.4251
1.129	0.4289	0.4873	0.9972	0.5284	0.08628	0.5307
1.171	0.4526	0.5606	0.9921	0.6228	0.07329	0.6276
1.214	0.4579	0.6266	0.9836	0.712	0.066	0.7214
1.257	0.4491	0.6883	0.9709	0.7986	0.06176	0.8157
1.3	0.4287	0.748	0.953	0.8842	0.05963	0.9132
1.343	0.3976	0.8073	0.9286	0.9701	0.0593	1.017
1.386	0.3555	0.8683	0.8954	1.058	0.06103	1.13
1.429	0.3	0.9342	0.8493	1.149	0.06589	1.261
1.438	0.2857	0.9494	0.8371	1.169	0.01537	1.293
1.447	0.2704	0.9653	0.8236	1.19	0.01598	1.326
1.456	0.2538	0.9821	0.8087	1.211	0.01673	1.361
1.466	0.2358	0.9998	0.7921	1.233	0.01769	1.399
1.475	0.2159	1.019	0.7735	1.256	0.01893	1.44
1.484	0.1937	1.039	0.7521	1.279	0.02062	1.485
1.493	0.1683	1.062	0.727	1.304	0.02304	1.536
1.503	0.1378	1.089	0.6959	1.332	0.02691	1.596
1.512	0.0977	1.124	0.6535	1.363	0.03454	1.675
1.521	0.0002855	1.206	0.543	1.421	0.08192	1.864

1.3.4. $\alpha_1^0 = -2$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.5$ $\alpha_1 = -1.4$ $\alpha = -2.8$

1	0.00169	0	1	0	0	0
1.043	0.2832	0.2871	1	0.2953	0.2861	0.2964
1.086	0.3825	0.4004	0.9998	0.4232	0.1133	0.4244
1.129	0.4424	0.4849	0.9984	0.5261	0.08455	0.5279
1.171	0.4798	0.5551	0.9955	0.6174	0.07016	0.6206
1.214	0.5024	0.6164	0.9908	0.7019	0.06126	0.7077
1.257	0.5145	0.6715	0.9842	0.7822	0.05515	0.7919
1.3	0.5189	0.7222	0.9755	0.8593	0.0507	0.8747
1.343	0.5173	0.7695	0.9645	0.9344	0.04734	0.9574
1.386	0.5107	0.8143	0.9511	1.008	0.04477	1.041
1.429	0.5	0.8571	0.9352	1.08	0.04281	1.125

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.477	0.4836	0.9037	0.9139	1.16	0.04668	1.224
1.526	0.463	0.9491	0.8885	1.24	0.04542	1.326
1.574	0.4385	0.9939	0.8586	1.319	0.04476	1.433
1.622	0.41	1.039	0.8233	1.398	0.04471	1.548
1.671	0.3773	1.084	0.7817	1.477	0.04539	1.671
1.719	0.3396	1.131	0.7321	1.556	0.04703	1.806
1.768	0.2956	1.181	0.6716	1.635	0.05016	1.958
1.816	0.2423	1.237	0.5947	1.716	0.05606	2.139
1.865	0.1718	1.306	0.4877	1.8	0.06899	2.373
1.913	0.0008075	1.462	0.2075	1.902	0.1559	2.934

1.3.5. α_1^0 = -2 β_{t0} = 0.7 β_{r0} = 0.6676 α_1 = -1.4 α = -2.8

1	0.00378	0	1	0	0	0
1.043	0.2836	0.287	1	0.2952	0.284	0.2953
1.086	0.3877	0.3995	1	0.4223	0.1126	0.4224
1.129	0.4584	0.4821	1	0.5232	0.08258	0.5235
1.171	0.511	0.5489	0.9993	0.6112	0.06683	0.6118
1.214	0.5519	0.6056	0.9984	0.6912	0.05666	0.6924
1.257	0.5847	0.655	0.997	0.7658	0.04938	0.7677
1.3	0.6115	0.6988	0.9953	0.8363	0.04383	0.8393
1.343	0.6336	0.7382	0.9933	0.9037	0.03942	0.9081
1.386	0.6521	0.774	0.9909	0.9686	0.03582	0.9748
1.429	0.6676	0.8068	0.9883	1.032	0.0328	1.04
2.714	0.7312	1.265	0.8179	2.588	0.4577	2.801
4	0.6724	1.432	0.5516	3.962	0.1678	4.632
5.286	0.6174	1.527	0.2338	5.281	0.09409	6.629
6.571	0.5719	1.589	-0.1179	6.57	0.06219	8.795
7.857	0.5344	1.634	-0.4941	7.842	0.04499	11.12
9.143	0.5032	1.668	-0.8892	9.1	0.03449	13.6
10.43	0.4767	1.696	-1.299	10.35	0.02752	16.23
11.71	0.4539	1.718	-1.722	11.59	0.02262	19
13	0.434	1.737	-2.155	12.82	0.01902	21.89
14.29	0.4165	1.754	-2.597	14.05	0.01628	24.92
144.1	0.1385	1.977	-56.91	132.4	0.223	701.7
286.9	0.0985	2.006	-120.9	260.1	0.02911	1953
429.6	0.08057	2.019	-186	387.2	0.01293	3569
572.3	0.06984	2.026	-251.8	513.9	0.007719	5479
715	0.0625	2.032	-318	640.4	0.005271	7645
857.7	0.05708	2.036	-384.5	766.7	0.003892	10040
1000	0.05286	2.039	-451.1	892.9	0.003026	12640
1143	0.04945	2.041	-518	1019	0.002439	15430
1286	0.04663	2.043	-585	1145	0.002021	18410
1429	0.04424	2.045	-652.1	1271	0.00171	21550

1.3.6. α_1^0 = -2 β_{t0} = 0.7 β_{r0} = 0.7 α_1 = -1.4 α = -2.8

1	0.00169	0	1	0	0	0
1.043	0.2837	0.2871	1	0.2953	0.2861	0.2963
1.086	0.3889	0.3995	1	0.4223	0.1124	0.4233
1.129	0.4619	0.4816	1	0.5228	0.08215	0.5239
1.171	0.5178	0.5478	1	0.6101	0.06614	0.6113
1.214	0.5625	0.6035	0.9998	0.6892	0.05576	0.6906
1.257	0.5995	0.6519	0.9994	0.7627	0.04831	0.7643
1.3	0.6305	0.6945	0.9989	0.832	0.04263	0.8339
1.343	0.6571	0.7326	0.9983	0.8981	0.03813	0.9005
1.386	0.68	0.7671	0.9977	0.9617	0.03445	0.9645
1.429	0.7	0.7984	0.9969	1.023	0.03137	1.027

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.714	0.8727	1.212	0.9536	2.541	0.4138	2.597
4	0.8868	1.346	0.8912	3.899	0.1343	4.053
5.286	0.8848	1.415	0.8217	5.221	0.06861	5.504
6.571	0.8801	1.457	0.7485	6.529	0.04194	6.961
7.857	0.8755	1.485	0.6732	7.828	0.02836	8.426
9.143	0.8714	1.506	0.5965	9.123	0.02049	9.898
10.43	0.8679	1.521	0.519	10.42	0.01551	11.38
11.71	0.8648	1.533	0.4408	11.71	0.01215	12.86
13	0.8622	1.543	0.3622	12.99	0.009777	14.35
14.29	0.8599	1.551	0.2832	14.28	0.00804	15.84
144.1	0.8335	1.625	-7.864	143.9	0.0074	170.7
286.9	0.8318	1.63	-16.84	286.4	0.004145	342.1
429.6	0.8312	1.631	-25.81	428.8	0.001393	513.8
572.3	0.8309	1.632	-34.79	571.2	0.0006986	685.5
715	0.8307	1.632	-43.76	713.7	0.0004198	857.3
857.7	0.8306	1.632	-52.73	856.1	0.0002802	1029
1000	0.8305	1.633	-61.71	998.5	0.0002003	1201
1143	0.8304	1.633	-70.68	1141	0.0001503	1373
1286	0.8304	1.633	-79.66	1283	0.0001169	1545
1429	0.8303	1.633	-88.63	1426	9.356e-005	1717

1.3.7. α_1^0 = -2 β_{t0} = 0.7 β_{r0} = 0.7141 α_1 = -1.4 α = -2.8

1	0.00169	0	1	0	0	0
1.043	0.2837	0.2871	1	0.2953	0.2861	0.2963
1.086	0.3894	0.3994	1	0.4222	0.1123	0.4233
1.129	0.4635	0.4814	1	0.5225	0.08196	0.5236
1.171	0.5208	0.5472	1	0.6095	0.06584	0.6106
1.214	0.5673	0.6026	1	0.6882	0.05536	0.6893
1.257	0.606	0.6504	1	0.7612	0.04785	0.7623
1.3	0.639	0.6926	1.001	0.8301	0.04212	0.8311
1.386	0.6923	0.764	1.001	0.9587	0.03387	0.9597
1.429	0.7141	0.7948	1.001	1.02	0.03078	1.021
2.714	0.9297	1.193	1.001	2.523	0.3981	2.524
4	0.9682	1.318	1.002	3.873	0.1246	3.874
5.286	0.9819	1.38	1.002	5.19	0.06234	5.191
6.571	0.9884	1.418	1.003	6.494	0.03757	6.496
7.857	0.9919	1.443	1.003	7.793	0.02515	7.794
9.143	0.994	1.461	1.004	9.088	0.01802	9.089
10.43	0.9954	1.474	1.004	10.38	0.01356	10.38
11.71	0.9963	1.485	1.005	11.67	0.01057	11.67
13	0.997	1.493	1.005	12.96	0.008471	12.96
14.29	0.9975	1.5	1.006	14.25	0.006942	14.25
144.1	1	1.563	1.057	144.1	0.063	144.1
286.9	1	1.567	1.114	286.9	0.003452	286.9
429.6	1	1.568	1.171	429.6	0.001158	429.6
572.3	1	1.569	1.228	572.3	0.0005805	572.3
715	1	1.569	1.285	715	0.0003488	715
857.7	1	1.569	1.342	857.7	0.0002327	857.7
1000	1	1.569	1.399	1000	0.0001663	1000
1143	1	1.57	1.456	1143	0.0001248	1143
1286	1	1.57	1.513	1286	9.709e-005	1286
1429	1	1.57	1.57	1429	7.769e-005	1429

1.4.1. α_1^0 = -2 β_{t0} = 0.722 β_{r0} = 0 α_1 = -1.444 α = -2.888

1	0.001664	0	1	0	0.2716	0
1.039	0.2692	0.2726	1	0.2796	0.1085	0.2796

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.077	0.3616	0.3811	0.9997	0.4006	0.08263	0.4008
1.116	0.41	0.4637	0.9977	0.499	0.07096	0.5
1.154	0.4294	0.5347	0.9929	0.5881	0.06509	0.5913
1.193	0.4271	0.5998	0.9844	0.6731	0.06268	0.6809
1.231	0.4069	0.6625	0.9706	0.7572	0.06322	0.7729
1.27	0.3698	0.7257	0.9497	0.8425	0.06761	0.8718
1.308	0.3135	0.7933	0.9176	0.9322	0.08024	0.9841
1.347	0.2285	0.8735	0.8646	1.032	0.1745	1.126
1.385	0	1.048	0.6915	1.2	0	1.454

1.5.1. α_1^0 = -2 β_{t0} = 0.815 β_{r0} = 0 α_1 = -1.63 α = -3.26

1	0.001567	0	1	0	0	0
1.023	0.2095	0.2105	1	0.2137	0.2095	0.2138
1.045	0.2902	0.2954	1	0.3043	0.08482	0.3044
1.068	0.3434	0.3593	0.9999	0.3756	0.06398	0.3758
1.091	0.3755	0.4134	0.9989	0.4382	0.05401	0.4387
1.113	0.3888	0.4621	0.9967	0.4964	0.04872	0.4979
1.136	0.3838	0.5083	0.9925	0.553	0.04626	0.5564
1.182	0.3137	0.6035	0.9729	0.6706	0.04897	0.6843
1.204	0.2343	0.6612	0.9505	0.7396	0.05778	0.7666
1.227	0	0.7859	0.8672	0.8681	0.1247	0.9519

1.6.1. α_1^0 = -2 β_{t0} = 0.9 β_{r0} = 0 α_1 = -1.8 α = -3.6

1	0.001491	0	1	0	0	0
1.011	0.1478	0.1479	1	0.149	0.1469	0.1494
1.022	0.2073	0.2084	1	0.2115	0.06051	0.2119
1.033	0.2513	0.2543	1	0.2599	0.04584	0.2603
1.044	0.285	0.2926	1	0.3013	0.03833	0.3016
1.056	0.3081	0.3265	0.9998	0.3386	0.0339	0.339
1.067	0.3184	0.3579	0.9991	0.3737	0.03139	0.3744
1.078	0.3125	0.3884	0.9975	0.4082	0.03049	0.4094
1.089	0.2846	0.4199	0.9943	0.4439	0.0315	0.4464
1.1	0.2215	0.4562	0.9875	0.4845	0.03626	0.4898
1.111	0	0.532	0.9575	0.5637	0.07589	0.5829

1.6.2. α_1^0 = -2 β_{t0} = 0.9 β_{r0} = 0.1 α_1 = -1.8 α = -3.6

1	0.001	0	1	0	0	0
1.011	0.1478	0.1479	1	0.149	0.1469	0.1494
1.022	0.2074	0.2084	1	0.2115	0.06051	0.2119
1.033	0.2513	0.2543	1	0.2599	0.04583	0.2603
1.044	0.2852	0.2926	1	0.3013	0.03832	0.3016
1.056	0.3087	0.3265	0.9998	0.3385	0.03386	0.3389
1.067	0.32	0.3577	0.9991	0.3735	0.03128	0.3742
1.078	0.316	0.388	0.9977	0.4078	0.03025	0.409
1.089	0.2915	0.419	0.9947	0.443	0.03098	0.4453
1.1	0.2358	0.4538	0.9887	0.4822	0.03485	0.4871
1.111	0.1	0.5066	0.9716	0.5391	0.05277	0.5516
1.111	0.095	0.5087	0.9706	0.5412	0.001654	0.554
1.112	0.0897	0.5104	0.9698	0.543	0.001745	0.5562
1.112	0.08402	0.5123	0.969	0.5449	0.001855	0.5585
1.112	0.0779	0.5142	0.9681	0.5469	0.00199	0.5609
1.112	0.07121	0.5164	0.9671	0.5491	0.00216	0.5636
1.112	0.06378	0.5188	0.9659	0.5515	0.002385	0.5666
1.113	0.05532	0.5215	0.9646	0.5542	0.002702	0.5699
1.113	0.04523	0.5247	0.963	0.5574	0.003199	0.5739
1.113	0.03203	0.5289	0.9609	0.5615	0.004162	0.579

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.113	0.0002331	0.539	0.9553	0.5713	0.01016	0.5916

1.6.3. α_1^0 = -2 β_{t0} =0.9 β_{r0} = 0.3 α_1 = -1.8 α = -3.6

1	0.001491	0	1	0	0	0
1.011	0.1478	0.147	1	0.149	0.1469	0.1494
1.022	0.2074	0.2084	1	0.2115	0.06051	0.2119
1.033	0.2516	0.2543	1	0.2599	0.04582	0.2603
1.044	0.2867	0.2925	1	0.3011	0.03822	0.3015
1.056	0.3138	0.326	1	0.338	0.03352	0.3384
1.067	0.3327	0.3565	0.9996	0.3722	0.03047	0.3727
1.078	0.3425	0.385	0.9989	0.4048	0.02856	0.4056
1.089	0.3418	0.4126	0.9975	0.4366	0.0276	0.438
1.1	0.3287	0.4402	0.9951	0.4687	0.02758	0.471
1.111	0.3	0.469	0.9911	0.5022	0.0288	0.5062
1.114	0.2897	0.4766	0.9898	0.511	0.007618	0.5154
1.117	0.278	0.4845	0.9882	0.5201	0.007874	0.5252
1.119	0.2644	0.4927	0.9863	0.5295	0.0082	0.5355
1.122	0.2489	0.5013	0.9841	0.5393	0.008621	0.5463
1.125	0.2309	0.5105	0.9816	0.5497	0.009178	0.5579
1.128	0.2097	0.5204	0.9785	0.5608	0.009944	0.5705
1.131	0.1844	0.5315	0.9746	0.573	0.01106	0.5846
1.133	0.1528	0.5444	0.9695	0.5869	0.01286	0.6011
1.136	0.1096	0.5608	0.9621	0.6043	0.01643	0.6222
1.139	0.0001998	0.5997	0.9402	0.6428	0.03894	0.6727

1.6.4. α_1^0 = -2 β_{t0} = 0.9 β_{r0} = 0.4 α_1 = -1.8 α = -3.6

1	0.0014	0	1	0	0	0
1.011	0.1478	0.1479	1	0.149	0.1469	0.1494
1.022	0.2074	0.2084	1	0.2115	0.06051	0.2119
1.033	0.2518	0.2542	1	0.2599	0.0458	0.2602
1.044	0.288	0.2924	1	0.301	0.03812	0.3014
1.056	0.3183	0.3256	1	0.3376	0.03323	0.338
1.067	0.3435	0.3554	1	0.3712	0.02981	0.3716
1.078	0.3641	0.3827	0.9998	0.4025	0.0273	0.4029
1.089	0.3803	0.4081	0.9995	0.4322	0.02542	0.4328
1.1	0.3923	0.4321	0.9989	0.4607	0.02399	0.4615
1.111	0.4	0.455	0.9981	0.4883	0.02293	0.4895
1.123	0.4035	0.4791	0.9967	0.5178	0.0241	0.5201
1.135	0.4019	0.5026	0.9949	0.5469	0.02353	0.5501
1.147	0.395	0.5259	0.9924	0.576	0.02328	0.5804
1.16	0.3824	0.5493	0.9889	0.6054	0.02337	0.6115
1.172	0.3635	0.5731	0.9844	0.6353	0.02385	0.6439
1.184	0.3374	0.598	0.9783	0.6664	0.02486	0.6784
1.196	0.3022	0.6247	0.97	0.6993	0.02668	0.7162
1.208	0.2545	0.6547	0.9582	0.7355	0.03002	0.7595
1.22	0.1852	0.6919	0.9395	0.7783	0.03719	0.8144
1.232	0.001152	0.7766	0.8789	0.8635	0.08469	0.9421

1.6.5. α_1^0 = -2 β_{t0} = 0.9 β_{r0} = 0.435 α_1 = -1.8 α = -3.6

1	0.001491	0	1	0	0	0
1.011	0.1478	0.1479	1	0.149	0.1469	0.1491
1.022	0.2074	0.2084	1	0.2115	0.06051	0.2116
1.033	0.2519	0.2542	1	0.2599	0.04579	0.26
1.044	0.2886	0.2923	1	0.301	0.03809	0.3011
1.056	0.3201	0.3254	1	0.3375	0.03311	0.3376
1.067	0.3479	0.355	1	0.3707	0.02954	0.3709

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.078	0.3728	0.3818	1	0.4016	0.02682	0.4017
1.089	0.3953	0.4064	1	0.4305	0.02465	0.4306
1.1	0.416	0.4293	1	0.4579	0.02287	0.458
1.111	0.435	0.4507	1	0.484	0.02136	0.4841
2.111	0.7946	1.099	0.9594	1.881	0.6481	1.923
3.111	0.7759	1.291	0.858	2.99	0.1923	3.188
4.111	0.7317	1.395	0.7198	4.048	0.1034	4.514
5.111	0.6886	1.462	0.5563	5.081	0.06692	5.924
6.111	0.6503	1.51	0.3742	6.1	0.04779	7.419
7.111	0.6169	1.546	0.1776	7.109	0.0363	8.999
8.111	0.5877	1.575	-0.03086	8.111	0.02878	10.66
9.111	0.5621	1.598	-0.249	9.108	0.02353	12.4
10.11	0.5394	1.618	-0.4755	10.1	0.01971	14.22
11.11	0.5192	1.635	-0.709	11.09	0.01681	16.11
112.1	0.1775	1.861	-32.08	107.4	0.226	428.2
223.1	0.1264	1.89	-70.06	211.8	0.02918	1187
334.1	0.1035	1.903	-109	315.8	0.01295	2165
445.1	0.08972	1.911	-148.5	419.6	0.007726	3322
556.1	0.08031	1.916	-188.3	523.3	0.005275	4633
667.1	0.07335	1.92	-228.3	626.8	0.003894	6082
778.1	0.06793	1.923	-268.5	730.3	0.003027	7657
889.1	0.06356	1.926	-308.8	833.8	0.00244	9347
1000	0.05993	1.928	-349.3	937.1	0.002022	11150
1111	0.05687	1.929	-389.8	1040	0.00171	13050

2.1.1. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5224	0.6397	0.9895	0.7363	0.6377	0.743
1.467	0.5731	0.8724	0.9431	1.123	0.2326	1.163
1.7	0.5623	1.036	0.866	1.463	0.164	1.572
1.933	0.5279	1.166	0.7612	1.777	0.1298	1.999
2.167	0.4813	1.276	0.629	2.073	0.1101	2.461
2.4	0.4267	1.375	0.4672	2.354	0.09862	2.975
2.633	0.3648	1.468	0.2703	2.619	0.0931	3.565
2.867	0.2934	1.562	0.0261	2.867	0.09373	4.274
3.1	0.204	1.667	-0.2977	3.086	0.1053	5.214
3.333	0	1.884	-1.027	3.171	0.2171	7.512

2.1.2. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.1$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5232	0.6395	0.9896	0.7361	0.6375	0.7427
1.467	0.5751	0.8716	0.9439	1.123	0.2321	1.161
1.7	0.5656	1.035	0.8681	1.462	0.1632	1.569
1.933	0.5326	1.164	0.7655	1.775	0.1289	1.993
2.167	0.4876	1.273	0.6365	2.071	0.1089	2.45
2.633	0.3755	1.461	0.2894	2.617	0.09093	3.532
2.4	0.4349	1.37	0.4793	2.352	0.09708	2.956
2.867	0.3078	1.551	0.05691	2.866	0.09028	4.215
3.1	0.2259	1.649	-0.2425	3.091	0.09815	5.091
3.333	0.1	1.787	-0.715	3.256	0.1379	6.529
3.339	0.09483	1.792	-0.7342	3.258	0.005474	6.59
3.345	0.08936	1.798	-0.7543	3.259	0.00577	6.654
3.351	0.08356	1.804	-0.7756	3.26	0.006124	6.723
3.357	0.07732	1.811	-0.7984	3.261	0.006559	6.797
3.363	0.07056	1.818	-0.823	3.261	0.007111	6.877
3.369	0.06308	1.826	-0.8501	3.26	0.007841	6.966

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
3.375	0.0546	1.835	-0.8805	3.258	0.008873	7.067
3.381	0.04456	1.845	-0.9163	3.254	0.01049	7.187
3.387	0.0315	1.859	-0.9622	3.247	0.01363	7.343
3.393	0.0002197	1.892	-1.07	3.22	0.03265	7.718

2.1.3. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.3$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.529	0.638	0.9907	0.7345	0.636	0.7406
1.467	0.5906	0.8659	0.9503	1.117	0.2279	1.152
1.7	0.5913	1.024	0.8846	1.452	0.1576	1.545
1.933	0.5688	1.146	0.7976	1.761	0.122	1.946
2.167	0.5353	1.246	0.6909	2.054	0.1007	2.368
2.4	0.4958	1.333	0.565	2.333	0.0869	2.821
2.633	0.4523	1.411	0.4192	2.6	0.07778	3.313
2.867	0.4055	1.483	0.2516	2.856	0.07199	3.857
3.1	0.3551	1.552	0.05863	3.099	0.06896	4.471
3.333	0.3	1.621	-0.1664	3.329	0.06885	5.184
3.399	0.2832	1.641	-0.2381	3.391	0.01991	5.41
3.465	0.2657	1.661	-0.313	3.451	0.02034	5.649
3.531	0.2474	1.682	-0.3925	3.509	0.02096	5.906
3.596	0.2279	1.704	-0.4777	3.565	0.0218	6.182
3.662	0.207	1.727	-0.5696	3.618	0.02296	6.485
3.728	0.1843	1.752	-0.6703	3.667	0.02462	6.821
3.794	0.1588	1.779	-0.783	3.712	0.0271	7.205
3.86	0.1291	1.81	-0.914	3.75	0.0312	7.662
3.925	0.09081	1.849	-1.079	3.774	0.03948	8.261
3.991	0.0001922	1.942	-1.446	3.72	0.09218	9.713

2.1.4. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.5$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5404	0.635	0.9929	0.7316	0.633	0.7365
1.467	0.6207	0.8552	0.9623	1.107	0.2202	1.133
1.7	0.6395	1.003	0.914	1.433	0.148	1.502
1.933	0.6351	1.114	0.8522	1.735	0.1112	1.867
2.167	0.6198	1.203	0.7791	2.022	0.08865	2.239
2.4	0.5992	1.277	0.6961	2.297	0.07354	2.622
2.633	0.5759	1.339	0.6042	2.563	0.06279	3.019
2.867	0.5511	1.394	0.5039	2.822	0.05482	3.433
3.1	0.5257	1.443	0.3955	3.075	0.04874	3.866
3.333	0.5	1.487	0.2794	3.322	0.04401	4.321
3.671	0.4628	1.544	0.09733	3.67	0.05728	5.023
4.009	0.4257	1.596	-0.1006	4.007	0.05162	5.783
4.346	0.3887	1.643	-0.3156	4.335	0.04757	6.613
4.684	0.3514	1.688	-0.549	4.652	0.0448	7.526
5.022	0.3135	1.731	-0.8032	4.957	0.04316	8.543
5.359	0.2743	1.774	-1.082	5.249	0.04267	9.693
5.697	0.2325	1.818	-1.393	5.524	0.04363	11.03
6.035	0.1859	1.865	-1.748	5.776	0.04693	12.64
6.372	0.1288	1.92	-2.183	5.987	0.05575	14.8
6.71	0.0006003	2.042	-3.046	5.979	0.1216	20.08

2.1.5. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.6632$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5537	0.6315	0.9954	0.7282	0.6295	0.7319
1.467	0.6545	0.8436	0.975	1.096	0.212	1.114
1.7	0.6924	0.9822	0.9439	1.414	0.1386	1.459

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.933	0.7057	1.084	0.9052	1.708	0.1014	1.792
2.167	0.7072	1.162	0.8606	1.988	0.07878	2.122
2.4	0.7025	1.226	0.8113	2.259	0.06362	2.453
2.633	0.6946	1.279	0.7581	2.522	0.05283	2.787
2.867	0.6849	1.324	0.7015	2.78	0.0448	3.125
3.1	0.6742	1.362	0.6419	3.033	0.03863	3.468
3.333	0.6632	1.396	0.5798	3.283	0.03376	3.817
6.333	0.542	1.629	-0.3699	6.323	0.2332	8.849
9.333	0.4652	1.73	-1.478	9.216	0.1005	14.85
12.33	0.4133	1.789	-2.671	12.04	0.05928	21.71
15.33	0.3754	1.829	-3.919	14.82	0.04021	29.34
18.33	0.3463	1.859	-5.208	17.58	0.02957	37.67
21.33	0.323	1.882	-6.527	20.31	0.02292	46.65
24.33	0.3038	1.9	-7.871	23.03	0.01844	56.24
27.33	0.2877	1.915	-9.235	25.73	0.01525	66.39
30.33	0.2739	1.928	-10.62	28.42	0.01288	77.08
33.33	0.2619	1.939	-12.01	31.09	0.01107	88.29
336.3	0.08427	2.094	-168.2	291.3	0.155	2673
669.3	0.05981	2.115	-346.5	572.7	0.02053	7478
1002	0.04889	2.124	-526.7	852.8	0.009132	13690
1335	0.04237	2.129	-707.8	1132	0.005452	21040
1668	0.03791	2.133	-889.6	1411	0.003724	29370
2001	0.03462	2.136	-1072	1690	0.00275	38570
2334	0.03205	2.138	-1254	1969	0.002138	48580
2667	0.02999	2.14	-1437	2247	0.001724	59330
3000	0.02828	2.141	-1620	2525	0.001428	70780
3333	0.02683	2.142	-1803	2803	0.001209	82880

2.1.6. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.7$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5572	0.6307	0.9961	0.7273	0.6287	0.7307
1.467	0.6631	0.8407	0.9782	1.093	0.21	1.109
1.7	0.7057	0.9771	0.9511	1.409	0.1364	1.448
1.933	0.7232	1.076	0.9175	1.702	0.09923	1.774
2.167	0.7285	1.153	0.8791	1.98	0.07668	2.095
2.4	0.7273	1.215	0.8369	2.249	0.06161	2.416
2.633	0.7227	1.266	0.7915	2.512	0.0509	2.738
2.867	0.716	1.308	0.7434	2.769	0.04296	3.062
3.1	0.7083	1.345	0.6931	3.022	0.03686	3.39
3.333	0.7	1.377	0.6407	3.271	0.03206	3.721
6.333	0.6053	1.593	-0.1395	6.332	0.2158	8.349
9.333	0.5466	1.681	-1.025	9.277	0.08797	13.58
12.33	0.5084	1.73	-1.957	12.18	0.04938	19.29
15.33	0.4817	1.762	-2.917	15.05	0.03204	25.36
18.33	0.462	1.785	-3.894	17.92	0.02261	31.72
21.33	0.4467	1.802	-4.882	20.77	0.01688	38.33
24.33	0.4346	1.815	-5.879	23.61	0.01311	45.14
27.33	0.4248	1.825	-6.882	26.45	0.0105	52.13
30.33	0.4166	1.834	-7.89	29.29	0.008601	59.26
33.33	0.4096	1.841	-8.901	32.12	0.007182	66.52
336.3	0.3361	1.913	-113	316.8	0.072	934.2
669.3	0.3314	1.918	-227.7	629.4	0.004432	1934
1002	0.3298	1.919	-342.4	942	0.001501	2941
1335	0.329	1.92	-457.1	1255	0.0007554	3952
1668	0.3285	1.921	-571.8	1567	0.0004547	4966

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2001	0.3282	1.921	-686.5	1880	0.0003038	5980
2334	0.3279	1.921	-801.2	2193	0.0002173	6995
2667	0.3277	1.921	-915.9	2505	0.0001631	8011
3000	0.3276	1.921	-1031	2818	0.000127	9027
3333	0.3275	1.922	-1145	3130	0.0001017	10040

2.1.7. $\alpha_1^0 = -4$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.9$ $\alpha_1 = -1.2$ $\alpha = -2.4$

1	0.003651	0	1	0	0	0
1.233	0.5787	0.6253	1	0.7219	0.6233	0.7235
1.467	0.7158	0.8238	0.9965	1.076	0.1985	1.08
1.7	0.7854	0.9483	0.9913	1.381	0.1245	1.39
1.933	0.8262	1.036	0.9848	1.664	0.08804	1.679
2.167	0.852	1.103	0.9777	1.934	0.06635	1.957
2.4	0.8692	1.155	0.9699	2.195	0.05212	2.228
2.633	0.8811	1.197	0.9617	2.451	0.04218	2.494
2.867	0.8895	1.232	0.9531	2.704	0.03491	2.758
3.1	0.8956	1.261	0.9442	2.953	0.02941	3.019
3.333	0.9	1.286	0.9352	3.199	0.02515	3.279
6.333	0.9111	1.443	0.8103	6.281	0.1565	6.579
9.333	0.9079	1.498	0.6761	9.309	0.05578	9.877
12.33	0.9047	1.527	0.5394	12.32	0.02875	13.19
15.33	0.9022	1.545	0.4016	15.33	0.01756	16.51
18.33	0.9003	1.556	0.2631	18.33	0.01184	19.84
21.33	0.8987	1.565	0.1243	21.33	0.008527	23.17
24.33	0.8975	1.571	-0.01484	24.33	0.006434	26.51
27.33	0.8966	1.576	-0.1541	27.33	0.005028	29.86
30.33	0.8958	1.58	-0.2935	30.33	0.004038	33.21
33.33	0.8951	1.584	-0.433	33.33	0.003314	36.56
336.3	0.8881	1.614	-14.56	336	0.03	377.3
669.3	0.8877	1.616	-30.09	668.7	0.001666	752.3
1002	0.8876	1.616	-45.61	1001	0.0005592	1127
1335	0.8875	1.617	-61.14	1334	0.0002803	1503
1668	0.8875	1.617	-76.67	1667	0.0001684	1878
2001	0.8874	1.617	-92.2	1999	0.0001124	2253
2334	0.8874	1.617	-107.7	2332	8.032e-005	2628
2667	0.8874	1.617	-123.3	2664	6.027e-005	3004
3000	0.8874	1.617	-138.8	2997	4.689e-005	3379
3333	0.8874	1.617	-154.3	3330	3.752e-005	3754

2.2.1. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4151	0.429	1	0.4576	0.427	0.458
1.2	0.5361	0.5872	0.999	0.6648	0.1582	0.6661
1.3	0.5891	0.7005	0.9939	0.838	0.1133	0.8426
1.4	0.602	0.7924	0.983	0.9969	0.09186	1.01
1.5	0.587	0.8722	0.9647	1.149	0.07981	1.177
1.6	0.5499	0.9453	0.9369	1.297	0.07305	1.353
1.7	0.4924	1.016	0.8961	1.445	0.07031	1.544
1.8	0.412	1.088	0.8364	1.594	0.07199	1.765
1.9	0.2967	1.17	0.742	1.749	0.08206	2.046
2	0	1.341	0.4559	1.947	0.1712	2.708

2.2.2. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.1$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4152	0.429	1	0.4576	0.427	0.458
1.2	0.5363	0.5872	0.999	0.6648	0.1581	0.666

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.3	0.5898	0.7004	0.9939	0.8379	0.1132	0.8425
1.4	0.6034	0.7921	0.9833	0.9966	0.09171	1.009
1.5	0.5894	0.8717	0.9653	1.148	0.07956	1.176
1.6	0.5536	0.9444	0.938	1.296	0.07267	1.351
1.7	0.4979	1.014	0.8983	1.443	0.0697	1.541
1.8	0.4204	1.085	0.8405	1.592	0.07091	1.758
1.9	0.3107	1.165	0.7508	1.745	0.07959	2.031
2	0.1	1.291	0.5524	1.922	0.1264	2.516
2.001	0.09488	1.294	0.5471	1.925	0.002876	2.528
2.002	0.08947	1.297	0.5416	1.928	0.003037	2.54
2.003	0.0837	1.3	0.5356	1.93	0.00322	2.553
2.004	0.0775	1.304	0.5292	1.933	0.00346	2.567
2.006	0.07076	1.307	0.5222	1.936	0.003763	2.582
2.007	0.06329	1.312	0.5145	1.94	0.004157	2.598
2.008	0.05482	1.316	0.5056	1.943	0.00471	2.617
2.009	0.04477	1.322	0.495	1.947	0.005584	2.64
2.01	0.03166	1.329	0.4811	1.952	0.007267	2.669
2.11	0.000289	1.347	0.447	1.961	0.0176	2.74

2.2.3. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.3$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4153	0.429	1	0.4576	0.427	0.458
1.2	0.5381	0.5869	0.9992	0.6646	0.1579	0.6657
1.3	0.5953	0.6995	0.9947	0.837	0.1126	0.841
1.4	0.6146	0.79	0.9854	0.9945	0.09049	1.006
1.5	0.6082	0.8676	0.97	1.144	0.07765	1.169
1.6	0.5824	0.9374	0.947	1.29	0.0698	1.336
1.7	0.5402	1.003	0.9146	1.433	0.06533	1.514
1.8	0.482	1.067	0.8697	1.576	0.06376	1.71
1.9	0.4053	1.132	0.8068	1.72	0.0657	1.935
2	0.3	1.207	0.7125	1.869	0.07431	2.218
2.011	0.285	1.217	0.6974	1.887	0.009724	2.257
2.023	0.269	1.227	0.6821	1.904	0.01015	2.298
2.034	0.2519	1.238	0.6655	1.922	0.01067	2.342
2.046	0.2335	1.249	0.6473	1.941	0.01133	2.39
2.057	0.2134	1.261	0.6271	1.959	0.01217	2.441
2.069	0.1911	1.274	0.6043	1.978	0.01329	2.497
2.08	0.1656	1.289	0.578	1.998	0.01491	2.561
2.092	0.1354	1.307	0.5459	2.019	0.01747	2.637
2.103	0.09582	1.329	0.5031	2.042	0.02249	2.736
2.114	0.0002458	1.383	0.3952	2.077	0.05359	2.975

2.2.4. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.5$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4156	0.429	1	0.4576	0.427	0.458
1.2	0.5417	0.5864	0.9995	0.6641	0.1574	0.665
1.3	0.6062	0.6977	0.9962	0.8352	0.1113	0.8383
1.4	0.6363	0.7859	0.9895	0.9904	0.08819	0.9987
1.5	0.6441	0.8601	0.9786	1.137	0.07421	1.155
1.6	0.6362	0.925	0.9629	1.278	0.06497	1.31
1.7	0.6161	0.9837	0.9418	1.415	0.05862	1.47
1.8	0.5862	1.038	0.9144	1.55	0.05428	1.636
1.9	0.5474	1.089	0.8796	1.684	0.05151	1.812
2	0.5	1.14	0.8359	1.817	0.05017	2.003
2.044	0.4764	1.162	0.8126	1.875	0.02184	2.092

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.087	0.4509	1.184	0.7876	1.933	0.02204	2.186
2.131	0.4231	1.206	0.7596	1.991	0.02242	2.286
2.174	0.3928	1.229	0.7281	2.049	0.02305	2.393
2.218	0.3595	1.253	0.6923	2.107	0.02402	2.509
2.262	0.3222	1.279	0.6509	2.166	0.02549	2.637
2.305	0.2795	1.307	0.6018	2.225	0.02778	2.782
2.349	0.2285	1.338	0.5411	2.286	0.03168	2.953
2.392	0.1617	1.378	0.4583	2.348	0.03972	3.177
2.436	0.0004409	1.47	0.2453	2.424	0.09188	3.714

2.2.5. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.7$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4161	0.4289	1	0.4575	0.4269	0.4579
1.2	0.547	0.5857	1	0.6633	0.1567	0.664
1.3	0.6222	0.6951	0.9984	0.8326	0.1094	0.8344
1.4	0.6676	0.7801	0.9952	0.9847	0.08506	0.989
1.5	0.6946	0.85	0.99	1.127	0.06983	1.136
1.6	0.7092	0.9093	0.9829	1.262	0.05931	1.278
1.7	0.7151	0.9609	0.9738	1.393	0.05159	1.418
1.8	0.7145	1.007	0.9626	1.521	0.04569	1.558
1.9	0.7091	1.048	0.9493	1.646	0.04105	1.698
2	0.7	1.085	0.934	1.769	0.03733	1.84
2.202	0.673	1.152	0.896	2.012	0.06684	2.135
2.405	0.6377	1.21	0.8488	2.25	0.05824	2.443
2.607	0.5964	1.262	0.7916	2.484	0.05226	2.771
2.81	0.5502	1.31	0.7232	2.715	0.04815	3.124
3.012	0.4992	1.356	0.642	2.943	0.04554	3.51
3.215	0.4431	1.4	0.5453	3.168	0.04432	3.94
3.417	0.3804	1.445	0.4285	3.39	0.04471	4.432
3.62	0.3076	1.493	0.2829	3.609	0.04751	5.021
3.822	0.2153	1.548	0.0855	3.821	0.05585	5.796
4.025	0.001043	1.669	-0.3943	4.005	0.1205	7.681

2.2.6. $\alpha_1^0 = -4$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.8$ $\alpha_1 = -2$ $\alpha = -4$

1	0.002828	0	1	0	0	0
1.1	0.4164	0.4289	1	0.4575	0.4269	0.4579
1.2	0.5504	0.5852	1	0.6629	0.1563	0.6634
1.3	0.6319	0.6935	0.9997	0.831	0.1083	0.832
1.4	0.6864	0.7768	0.9985	0.9814	0.08327	0.9834
1.5	0.7243	0.8442	0.9965	1.121	0.06746	1.125
1.6	0.7512	0.9007	0.9937	1.254	0.05645	1.26
1.7	0.7705	0.949	0.9903	1.382	0.0483	1.392
1.8	0.7841	0.991	0.9861	1.506	0.04203	1.52
1.9	0.7936	1.028	0.9813	1.627	0.03705	1.647
2	0.8	1.061	0.9759	1.746	0.03302	1.772
5.693	0.6532	1.484	0.4917	5.672	0.4233	6.774
9.386	0.5169	1.602	-0.2896	9.381	0.1173	13.17
13.08	0.4233	1.666	-1.237	13.02	0.06384	21.09
16.77	0.3526	1.709	-2.308	16.61	0.04335	30.67
20.46	0.2949	1.742	-3.488	20.16	0.03321	42.14
24.16	0.2448	1.77	-4.774	23.68	0.02767	55.89
27.85	0.1986	1.794	-6.178	27.16	0.02475	72.62
31.54	0.1531	1.818	-7.73	30.58	0.02389	93.72
35.24	0.1028	1.844	-9.518	33.93	0.02595	122.8
38.93	0.0005904	1.896	-12.46	36.88	0.05216	196.5

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.2.7. α_1^0 = -4	β_{t0} = 0.5	β_{r0} = 0.8066	α_1 = -2	α = -4		
1	0.002828	0	1	0	0	0
1.1	0.4164	0.4289	1	0.4575	0.4269	0.4579
1.2	0.5506	0.5852	1	0.6628	0.1563	0.6633
1.3	0.6326	0.6934	0.9998	0.8309	0.1082	0.8318
1.4	0.6877	0.7765	0.9987	0.9811	0.08315	0.983
1.5	0.7264	0.8438	0.9969	1.121	0.0673	1.124
1.6	0.7541	0.9001	0.9945	1.253	0.05626	1.259
1.7	0.7743	0.9482	0.9914	1.381	0.04809	1.39
1.8	0.7889	0.99	0.9877	1.505	0.0418	1.518
1.9	0.7993	1.027	0.9834	1.626	0.03681	1.644
2	0.8066	1.06	0.9785	1.744	0.03277	1.768
3.8	0.7713	1.354	0.8174	3.711	0.294	4.014
5.6	0.6959	1.469	0.5703	5.571	0.1148	6.473
7.4	0.6342	1.534	0.2724	7.395	0.0652	9.187
9.2	0.5852	1.577	-0.05992	9.2	0.04333	12.14
11	0.5456	1.609	-0.4174	10.99	0.03144	15.33
12.8	0.5128	1.633	-0.7945	12.78	0.02415	18.74
14.6	0.4852	1.652	-1.187	14.55	0.0193	22.35
16.4	0.4616	1.668	-1.593	16.32	0.01588	26.16
18.2	0.4411	1.681	-2.01	18.09	0.01336	30.15
20	0.4231	1.693	-2.436	19.85	0.01144	34.31
201.8	0.14	1.85	-55.65	194	0.157	971.2
401.6	0.09952	1.871	-118.7	383.7	0.02058	2705
601.4	0.0814	1.88	-183	572.9	0.009144	4943
801.2	0.07056	1.885	-247.9	761.9	0.005457	7591
1001	0.06314	1.889	-313.3	950.7	0.003727	10590
1201	0.05766	1.892	-378.9	1139	0.002752	13910
1401	0.0534	1.894	-444.8	1328	0.002139	17510
1600	0.04996	1.896	-510.9	1517	0.001725	21390
1800	0.04711	1.897	-577.1	1705	0.001429	25510
2000	0.0447	1.898	-643.5	1894	0.001209	29860
2.3.1. α_1^0 = -4	β_{t0} = 0.7	β_{r0} = 0	α_1 = -2.8	α = -5.6		
1	0.00378	0	1	0	0	0
1.043	0.2837	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3892	0.3993	1	0.4221	0.1124	0.4222
1.129	0.4605	0.4815	1	0.5226	0.08219	0.5228
1.171	0.5085	0.5482	0.9998	0.6105	0.06675	0.611
1.214	0.5343	0.6058	0.9982	0.6915	0.05761	0.6929
1.257	0.5366	0.6581	0.9946	0.7689	0.05223	0.7726
1.3	0.5124	0.7078	0.9877	0.8452	0.04976	0.854
1.343	0.4547	0.7583	0.9749	0.9235	0.05046	0.9421
1.386	0.3455	0.8153	0.9501	1.009	0.05701	1.048
1.429	0	0.9328	0.8509	1.148	0.1175	1.283
2.3.2. α_1^0 = -4	β_{t0} = β_{r0} = 0.1	α_1 = -2.8	-2.8	α = -5.6		
1	0.00239	0	1	0	0	0
1.043	0.2837	0.2873	1	0.2955	0.2853	0.2956
1.086	0.3892	0.3997	1	0.4225	0.1124	0.4226
1.129	0.4606	0.4819	1	0.523	0.08219	0.5232
1.171	0.5087	0.5486	0.9995	0.6109	0.06673	0.6113
1.214	0.5349	0.6062	0.998	0.6918	0.05757	0.6932
1.257	0.5381	0.6583	0.9944	0.7691	0.05214	0.7728
1.3	0.5152	0.7079	0.9877	0.8453	0.04957	0.8538

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.343	0.4598	0.7579	0.9753	0.9231	0.05006	0.9413
1.386	0.3555	0.8139	0.9515	1.007	0.05598	1.046
1.429	0.1	0.9062	0.8811	1.124	0.09225	1.229
1.429	0.09491	0.9076	0.8797	1.126	0.001617	1.232
1.429	0.08952	0.9093	0.8779	1.128	0.001708	1.236
1.43	0.08378	0.9111	0.8761	1.13	0.001817	1.24
1.43	0.0776	0.9131	0.8741	1.132	0.001951	1.243
1.43	0.07087	0.9152	0.8719	1.134	0.002119	1.248
1.431	0.06341	0.9176	0.8694	1.136	0.002342	1.253
1.431	0.05494	0.9202	0.8666	1.139	0.002656	1.258
1.431	0.04488	0.9234	0.8632	1.142	0.003148	1.264
1.431	0.03175	0.9275	0.8587	1.145	0.004098	1.273
1.432	0.0004817	0.9373	0.8475	1.154	0.009873	1.293
2.3.3. α_1^0 = -4	β_{t0} = 0.7	β_{r0} = 0.3	α_1 = -2.8	α = -5.6		
1	0.00239	0	1	0	0	0
1.043	0.2837	0.2873	1	0.2955	0.2853	0.2956
1.086	0.3892	0.3997	1	0.4225	0.1124	0.4226
1.129	0.4611	0.4818	1	0.523	0.08215	0.5231
1.171	0.5107	0.5484	0.9996	0.6107	0.06659	0.6111
1.214	0.5402	0.6056	0.9983	0.6912	0.05719	0.6924
1.257	0.5495	0.657	0.9955	0.7678	0.05137	0.7708
1.3	0.5369	0.7051	0.99	0.8425	0.0481	0.8495
1.343	0.4989	0.7523	0.9805	0.9176	0.04719	0.9319
1.386	0.4276	0.8017	0.9638	0.9957	0.04942	1.024
1.429	0.3	0.8606	0.9314	1.083	0.05891	1.141
1.432	0.2858	0.8665	0.9271	1.091	0.005529	1.152
1.435	0.2707	0.8723	0.9229	1.099	0.005793	1.164
1.439	0.2543	0.8784	0.9183	1.107	0.006113	1.177
1.442	0.2364	0.8849	0.9132	1.116	0.00651	1.19
1.445	0.2167	0.892	0.9074	1.125	0.007017	1.205
1.448	0.1946	0.8997	0.9008	1.134	0.007694	1.221
1.452	0.1693	0.9083	0.8929	1.145	0.008657	1.239
1.455	0.1388	0.9185	0.8833	1.156	0.01018	1.261
1.458	0.09852	0.9316	0.87	1.171	0.01315	1.289
1.462	0.0003	0.9632	0.8345	1.2	0.03152	1.356
2.3.4. α_1^0 = -4	β_{t0} = 0.7	β_{r0} = 0.5	α_1 = -2.8	α = -5.6		
1	0.00239	0	1	0	0	0
1.043	0.2837	0.2873	1	0.2955	0.2853	0.2956
1.086	0.3893	0.3997	1	0.4225	0.1123	0.4226
1.129	0.462	0.4817	1	0.5229	0.08208	0.523
1.171	0.5146	0.548	0.9999	0.6103	0.0663	0.6106
1.214	0.5507	0.6045	0.9991	0.6902	0.05647	0.6909
1.257	0.5717	0.6545	0.9974	0.7653	0.04993	0.7671
1.3	0.5779	0.7	0.9943	0.8375	0.04553	0.8415
1.343	0.569	0.7427	0.9892	0.9081	0.04271	0.9161
1.386	0.5438	0.784	0.9812	0.9785	0.04128	0.9929
1.429	0.5	0.8253	0.969	1.05	0.04135	1.075
1.442	0.4819	0.8381	0.9644	1.072	0.01313	1.102
1.455	0.4614	0.8515	0.9587	1.095	0.01342	1.13
1.468	0.4379	0.8654	0.9521	1.118	0.01382	1.16
1.482	0.4112	0.8797	0.9444	1.142	0.01437	1.191
1.495	0.3806	0.8949	0.9353	1.166	0.01514	1.225

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.508	0.3449	0.9111	0.9244	1.192	0.01623	1.261
1.522	0.3025	0.929	0.9109	1.219	0.01787	1.302
1.535	0.25	0.9495	0.8934	1.248	0.02056	1.35
1.548	0.1789	0.9756	0.868	1.282	0.02602	1.412
1.561	0.0002995	1.036	0.7953	1.344	0.06083	1.559

2.3.5. $\alpha_1^0 = -4$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.7$ $\alpha_1 = -2.8$ $\alpha = -5.6$

1	0.00239	0	1	0	0	0
1.043	0.2837	0.2873	1	0.2955	0.2853	0.2956
1.086	0.3894	0.3997	1	0.4225	0.1123	0.4226
1.129	0.4634	0.4816	1	0.5228	0.08197	0.5229
1.171	0.5204	0.5475	1	0.6098	0.06587	0.6099
1.214	0.566	0.603	1	0.6886	0.05545	0.6887
1.257	0.6034	0.6509	1	0.7618	0.048	0.762
1.3	0.6345	0.6933	0.9999	0.8308	0.04236	0.8312
1.343	0.6604	0.7312	0.9996	0.8967	0.03791	0.8974
1.386	0.682	0.7655	0.9991	0.9602	0.03431	0.9612
1.429	0.7	0.7968	0.9985	1.022	0.03132	1.023
1.637	0.75	0.9196	0.9925	1.302	0.1226	1.309
1.846	0.7586	1.011	0.9805	1.564	0.09125	1.585
2.055	0.7412	1.084	0.9612	1.816	0.07318	1.863
2.263	0.7055	1.146	0.9331	2.062	0.06189	2.151
2.472	0.6555	1.201	0.8945	2.305	0.05469	2.457
2.681	0.5927	1.251	0.843	2.545	0.05035	2.791
2.89	0.5164	1.299	0.7747	2.784	0.04847	3.167
3.098	0.4228	1.349	0.6819	3.022	0.04951	3.612
3.307	0.2989	1.405	0.5454	3.262	0.05623	4.19
3.516	0.001635	1.523	0.1697	3.512	0.1174	5.58

2.3.6. $\alpha_1^0 = -4$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.7112$ $\alpha_1 = -2.8$ $\alpha = -5.6$

1	0.00239	0	1	0	0	0
1.043	0.2837	0.2873	1	0.2955	0.2853	0.2956
1.086	0.3894	0.3997	1	0.4225	0.1123	0.4226
1.129	0.4635	0.4816	1	0.5228	0.08197	0.5229
1.171	0.5207	0.5475	1	0.6098	0.06585	0.6099
1.214	0.567	0.6028	1	0.6885	0.05538	0.6886
1.257	0.6055	0.6507	1	0.7615	0.04788	0.7617
1.3	0.638	0.6929	1	0.8304	0.04217	0.8306
1.343	0.666	0.7305	1	0.8961	0.03765	0.8963
1.386	0.6901	0.7645	1	0.9592	0.03396	0.9594
1.429	0.7112	0.7954	1	1.02	0.03089	1.021
2.714	0.8692	1.203	0.9761	2.533	0.408	2.572
4	0.8378	1.341	0.9118	3.895	0.1379	4.072
5.286	0.7901	1.415	0.8184	5.222	0.07452	5.652
6.571	0.7447	1.464	0.7036	6.534	0.04817	7.329
7.857	0.7045	1.498	0.5726	7.836	0.03434	9.105
9.143	0.6694	1.524	0.4287	9.133	0.02604	10.98
10.43	0.6386	1.545	0.2741	10.42	0.02061	12.95
11.71	0.6114	1.561	0.1107	11.71	0.01684	15
13	0.5873	1.575	-0.06021	13	0.01408	17.15
14.29	0.5657	1.587	-0.2377	14.28	0.01201	19.38
144.1	0.1951	1.748	-25.45	141.9	0.161	502.1
286.9	0.139	1.769	-56.47	281.2	0.02065	1390
429.6	0.1138	1.778	-88.41	420.4	0.009162	2534

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
572.3	0.09866	1.784	-120.8	559.4	0.005465	3886
715	0.08832	1.787	-153.6	698.3	0.003731	5419
857.7	0.08066	1.79	-186.5	837.2	0.002754	7113
1000	0.07471	1.792	-219.7	976	0.002141	8954
1143	0.0699	1.794	-252.9	1115	0.001726	10930
1286	0.06592	1.795	-286.3	1254	0.00143	13030
1429	0.06254	1.797	-319.8	1392	0.00121	15260

2.4.1. $\alpha_1^0 = -4$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0$ $\alpha_1 = -3.6$ $\alpha = -7.2$

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3012	0.03808	0.3012
1.056	0.3197	0.3256	1	0.3377	0.03313	0.3377
1.067	0.345	0.3553	1	0.371	0.02966	0.3711
1.078	0.3607	0.3826	0.9999	0.4024	0.02731	0.4024
1.089	0.3557	0.4088	0.9991	0.4329	0.02625	0.4333
1.1	0.3033	0.4365	0.9968	0.4651	0.0277	0.4664
1.111	0	0.4882	0.9813	0.5211	0.05166	0.5298

2.4.2. $\alpha_1^0 = -4$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.1$ $\alpha_1 = -3.6$ $\alpha = -7.2$

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3012	0.03808	0.3012
1.056	0.3197	0.3256	1	0.3377	0.03312	0.3377
1.067	0.3452	0.3553	1	0.371	0.02965	0.371
1.078	0.3613	0.3825	0.9999	0.4023	0.02728	0.4024
1.089	0.3579	0.4087	0.9992	0.4327	0.02616	0.4331
1.1	0.3103	0.4361	0.9971	0.4646	0.02735	0.4659
1.111	0.1	0.4775	0.9868	0.5106	0.04143	0.5166
1.111	0.095	0.4787	0.9863	0.5119	0.0007199	0.5179
1.111	0.08969	0.4795	0.986	0.5127	0.00076	0.5188
1.111	0.08402	0.4803	0.9856	0.5135	0.0008079	0.5198
1.111	0.07789	0.4812	0.9853	0.5144	0.0008666	0.5209
1.112	0.07121	0.4821	0.9849	0.5154	0.0009409	0.5221
1.112	0.06378	0.4831	0.9844	0.5164	0.001039	0.5233
1.112	0.05531	0.4843	0.9839	0.5176	0.001178	0.5248
1.112	0.04523	0.4857	0.9832	0.519	0.001395	0.5265
1.112	0.03204	0.4875	0.9824	0.5209	0.001814	0.5288
1.112	0.001263	0.4919	0.9801	0.5252	0.00441	0.5342

2.4.3. $\alpha_1^0 = -4$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.3$ $\alpha_1 = -3.6$ $\alpha = -7.2$

1	0.002108	0	1	0	0	0
1.011	0.1479	0.1483	1	0.1494	0.1463	0.1494
1.022	0.2074	0.2088	1	0.2119	0.0605	0.2119
1.033	0.2519	0.2546	1	0.2603	0.04579	0.2603
1.044	0.2886	0.2927	1	0.3014	0.03809	0.3014
1.056	0.3199	0.3258	1	0.3379	0.03312	0.3379
1.067	0.3464	0.3554	1	0.3712	0.0296	0.3712
1.078	0.3666	0.3825	0.9999	0.4023	0.02707	0.4023
1.089	0.3752	0.4079	0.9995	0.432	0.02544	0.4322
1.1	0.3614	0.433	0.9985	0.4615	0.02501	0.4621
1.111	0.3	0.46	0.9956	0.4933	0.02708	0.4952

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.112	0.2892	0.463	0.9951	0.4968	0.003018	0.4987
1.113	0.277	0.4662	0.9945	0.5004	0.003135	0.5026
1.114	0.2632	0.4694	0.9939	0.5041	0.003279	0.5067
1.116	0.2475	0.4729	0.9931	0.5081	0.003461	0.511
1.117	0.2295	0.4766	0.9922	0.5122	0.003698	0.5156
1.118	0.2085	0.4806	0.9911	0.5167	0.004018	0.5206
1.119	0.1834	0.4851	0.9897	0.5217	0.004482	0.5262
1.12	0.1521	0.4903	0.988	0.5274	0.005223	0.5328
1.121	0.1093	0.497	0.9854	0.5345	0.006687	0.5411
1.122	0.00311	0.5126	0.9779	0.5503	0.01557	0.5607

2.4.4. $\alpha_1^0 = -4$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.4$ $\alpha_1 = -3.6$ $\alpha = -7.2$

1	0.002108	0	1	0	0	0
1.011	0.1479	0.1483	1	0.1494	0.1463	0.1494
1.022	0.2074	0.2088	1	0.2119	0.0605	0.2119
1.033	0.2519	0.2546	1	0.2603	0.04579	0.2603
1.044	0.2886	0.2927	1	0.3014	0.03809	0.3014
1.056	0.3201	0.3258	1	0.3379	0.03311	0.3379
1.067	0.3475	0.3554	1	0.3711	0.02956	0.3711
1.078	0.3711	0.3823	1	0.402	0.02689	0.402
1.089	0.3897	0.4071	0.9999	0.4312	0.02486	0.4312
1.1	0.4008	0.4306	0.9996	0.4591	0.02343	0.4593
1.111	0.4	0.4532	0.999	0.4865	0.02263	0.4869
1.115	0.3959	0.4604	0.9987	0.4953	0.007421	0.4962
1.118	0.3893	0.4679	0.9982	0.5044	0.007473	0.5055
1.122	0.3796	0.4755	0.9976	0.5136	0.007581	0.515
1.126	0.3662	0.4832	0.9968	0.5231	0.007763	0.5248
1.129	0.3482	0.4913	0.9958	0.5328	0.00805	0.5351
1.133	0.3243	0.4998	0.9945	0.543	0.008495	0.5459
1.137	0.2922	0.509	0.9926	0.5539	0.009203	0.5578
1.14	0.2482	0.5194	0.99	0.5661	0.01042	0.5713
1.144	0.1824	0.5324	0.9857	0.5807	0.01297	0.5882
1.148	0.0005284	0.5622	0.971	0.6118	0.02986	0.6275

2.4.5. $\alpha_1^0 = -4$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.4359$ $\alpha_1 = -3.6$ $\alpha = -7.2$

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3011	0.03808	0.3011
1.056	0.3202	0.3256	1	0.3376	0.03311	0.3376
1.067	0.348	0.3551	1	0.3709	0.02954	0.3709
1.078	0.373	0.3819	1	0.4017	0.02681	0.4017
1.089	0.3957	0.4066	1	0.4306	0.02463	0.4306
1.1	0.4166	0.4294	1	0.458	0.02284	0.458
1.111	0.4359	0.4507	1	0.484	0.02133	0.484
2.111	0.8688	1.079	0.9973	1.861	0.6278	1.864
3.111	0.8999	1.25	0.9816	2.952	0.171	2.985
4.111	0.8812	1.337	0.9511	4	0.08754	4.106
5.111	0.8509	1.392	0.9079	5.03	0.05487	5.26
6.111	0.8187	1.431	0.8544	6.051	0.03831	6.458
7.111	0.7878	1.459	0.7923	7.067	0.02863	7.703
8.111	0.7589	1.482	0.7228	8.079	0.02241	8.997
9.111	0.7324	1.5	0.6472	9.088	0.01814	10.34
10.11	0.708	1.515	0.5662	10.1	0.01507	11.73
11.11	0.6857	1.528	0.4805	11.1	0.01277	13.16

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
112.1	0.2493	1.693	-13.68	111.3	0.165	309.1
223.1	0.1781	1.714	-31.82	220.8	0.02075	848.3
334.1	0.146	1.723	-50.68	330.2	0.009186	1542
445.1	0.1267	1.729	-69.93	439.6	0.005475	2362
556.1	0.1134	1.732	-89.42	548.9	0.003736	3291
667.1	0.1036	1.735	-109.1	658.1	0.002758	4316
778.1	0.09596	1.737	-128.9	767.4	0.002143	5431
889.1	0.0898	1.739	-148.8	876.6	0.001727	6628
1000	0.08469	1.74	-168.8	985.8	0.001431	7902
1111	0.08036	1.742	-188.8	1095	0.00121	9248

3.1.1. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5841	0.6227	1.002	0.7193	0.6197	0.7204
1.467	0.7161	0.8197	1.001	1.072	0.1971	1.075
1.7	0.76	0.9459	0.9945	1.379	0.1262	1.389
1.933	0.7567	1.039	0.9802	1.666	0.09323	1.695
2.167	0.7227	1.114	0.9554	1.945	0.07503	2.01
2.4	0.6652	1.179	0.9174	2.218	0.06444	2.346
2.633	0.5868	1.237	0.8619	2.488	0.05877	2.718
2.867	0.4845	1.295	0.7811	2.758	0.05748	3.153
3.1	0.3447	1.358	0.6552	3.03	0.06298	3.715
3.333	0	1.484	0.2894	3.321	0.126	5.048

3.1.2. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.1$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5841	0.6227	1.002	0.7193	0.6197	0.7204
1.467	0.7162	0.8197	1.001	1.072	0.197	1.075
1.7	0.7605	0.9458	0.9947	1.379	0.1261	1.389
1.933	0.7579	1.039	0.9805	1.666	0.09313	1.695
2.167	0.7247	1.114	0.956	1.944	0.07488	2.009
2.4	0.6684	1.178	0.9186	2.217	0.06421	2.343
2.633	0.5915	1.236	0.8642	2.487	0.05841	2.713
2.867	0.4917	1.293	0.7854	2.757	0.05685	3.143
3.1	0.3569	1.355	0.6643	3.028	0.06156	3.693
3.333	0.1	1.452	0.3943	3.31	0.09739	4.713
3.335	0.09487	1.454	0.3892	3.313	0.001825	4.733
3.337	0.08944	1.456	0.3831	3.315	0.001927	4.755
3.339	0.08367	1.458	0.3765	3.318	0.002049	4.778
3.341	0.07746	1.46	0.3694	3.321	0.002199	4.802
3.343	0.07071	1.462	0.3617	3.324	0.002388	4.829
3.345	0.06325	1.465	0.3532	3.326	0.002638	4.858
3.347	0.05478	1.468	0.3434	3.33	0.002991	4.892
3.349	0.04473	1.472	0.3318	3.333	0.003544	4.931
3.351	0.03163	1.476	0.3166	3.336	0.004613	4.983
3.353	0.0004061	1.487	0.2794	3.341	0.01119	5.109

3.1.3. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.3$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5842	0.6227	1.002	0.7193	0.6197	0.7204
1.467	0.7176	0.8195	1.001	1.072	0.1969	1.074
1.7	0.7649	0.9452	0.9955	1.378	0.1257	1.387
1.933	0.7671	1.038	0.9828	1.665	0.09233	1.691
2.167	0.7406	1.111	0.9611	1.942	0.07366	1.999
2.4	0.6932	1.174	0.9284	2.213	0.06241	2.324
2.633	0.6284	1.229	0.8819	2.481	0.05571	2.677

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.867	0.5462	1.282	0.817	2.748	0.05247	3.074
3.1	0.4424	1.335	0.7251	3.014	0.05293	3.546
3.333	0.3	1.395	0.5822	3.282	0.06052	4.174
3.335	0.2985	1.396	0.5814	3.284	0.0005924	4.181
3.337	0.297	1.396	0.5798	3.287	0.0005947	4.187
3.339	0.2955	1.397	0.5781	3.289	0.000597	4.194
3.341	0.2939	1.397	0.5765	3.291	0.0005994	4.201
3.343	0.2924	1.398	0.5749	3.293	0.0006018	4.207
3.345	0.2909	1.399	0.5732	3.296	0.0006042	4.214
3.347	0.2893	1.399	0.5716	3.298	0.0006067	4.221
3.349	0.2878	1.4	0.5699	3.3	0.0006093	4.228
3.351	0.2862	1.4	0.5682	3.303	0.0006119	4.235
3.353	0.2846	1.401	0.5665	3.305	0.0006145	4.241

3.1.4. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.5$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5844	0.6226	1.002	0.7193	0.6196	0.7204
1.467	0.7203	0.8192	1.001	1.072	0.1966	1.074
1.7	0.7737	0.944	0.9972	1.377	0.1248	1.384
1.933	0.7852	1.035	0.9874	1.662	0.09081	1.683
2.167	0.7713	1.106	0.9708	1.937	0.07139	1.982
2.4	0.7403	1.165	0.9465	2.205	0.05924	2.29
2.633	0.6963	1.217	0.913	2.47	0.0513	2.615
2.867	0.6415	1.263	0.8689	2.732	0.04612	2.963
3.1	0.5765	1.306	0.8117	2.992	0.04303	3.346
3.333	0.5	1.348	0.7374	3.251	0.04187	3.78
3.404	0.4743	1.361	0.7098	3.329	0.0127	3.924
3.474	0.447	1.374	0.6806	3.406	0.01289	4.076
3.544	0.4179	1.387	0.6485	3.484	0.01318	4.239
3.614	0.3866	1.4	0.6129	3.562	0.01362	4.413
3.684	0.3526	1.415	0.573	3.639	0.01426	4.603
3.754	0.3151	1.43	0.5275	3.717	0.01519	4.813
3.825	0.2726	1.446	0.4743	3.795	0.01663	5.052
3.895	0.2223	1.466	0.4093	3.873	0.01903	5.336
3.965	0.1569	1.489	0.3222	3.952	0.02395	5.706
4.035	0.0003049	1.545	0.1032	4.034	0.05578	6.601

3.1.5. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.7$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5847	0.6226	1.002	0.7192	0.6196	0.7203
1.467	0.7244	0.8187	1.002	1.071	0.1961	1.073
1.7	0.7866	0.9422	0.9996	1.375	0.1235	1.38
1.933	0.8116	1.031	0.9939	1.658	0.08866	1.671
2.167	0.8153	1.099	0.9843	1.93	0.06837	1.958
2.4	0.8057	1.155	0.9705	2.195	0.05529	2.245
2.633	0.7872	1.201	0.9522	2.455	0.0463	2.538
2.867	0.7624	1.241	0.9293	2.712	0.03986	2.839
3.1	0.733	1.276	0.9014	2.966	0.03509	3.151
3.333	0.7	1.307	0.8684	3.218	0.03149	3.477
3.58	0.6619	1.337	0.8282	3.483	0.03037	3.84
3.827	0.6209	1.365	0.7804	3.747	0.02809	4.225
4.074	0.5771	1.392	0.7251	4.009	0.02643	4.637
4.321	0.5304	1.417	0.6611	4.271	0.02532	5.083
4.568	0.4803	1.442	0.587	4.531	0.02474	5.572
4.815	0.426	1.467	0.5003	4.789	0.02476	6.118
5.062	0.3656	1.492	0.3971	5.047	0.02558	6.742

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
5.309	0.2958	1.52	0.2694	5.303	0.02776	7.489
5.557	0.2072	1.553	0.09735	5.556	0.03323	8.472
5.804	0.0008282	1.627	-0.3233	5.795	0.07325	10.86

3.1.6. $\alpha_1^0 = -10$ $\beta_{t0} = 0.3$ $\beta_{r0} = 0.8709$ $\alpha_1 = -3$ $\alpha = -6$

1	0.005773	0	1	0	0	0
1.233	0.5851	0.6226	1.002	0.7192	0.6196	0.7203
1.467	0.729	0.8181	1.003	1.07	0.1956	1.072
1.7	0.8008	0.9403	1.002	1.373	0.1221	1.376
1.933	0.8401	1.027	1.001	1.654	0.08643	1.659
2.167	0.8619	1.092	0.998	1.923	0.0654	1.933
2.4	0.8732	1.144	0.994	2.185	0.05169	2.202
2.633	0.8778	1.186	0.9886	2.441	0.04215	2.468
2.867	0.8781	1.221	0.9821	2.693	0.03519	2.734
3.1	0.8755	1.251	0.9744	2.943	0.02994	3
3.333	0.8709	1.277	0.9655	3.19	0.02585	3.267
6.333	0.7694	1.448	0.7766	6.286	0.1709	6.923
9.333	0.6817	1.518	0.4966	9.32	0.0697	11.07
12.33	0.6161	1.558	0.1619	12.33	0.0401	15.71
15.33	0.5658	1.584	-0.21	15.33	0.02682	20.8
18.33	0.5258	1.604	-0.6092	18.32	0.01954	26.31
21.33	0.4931	1.619	-1.03	21.31	0.01505	32.2
24.33	0.4658	1.631	-1.467	24.29	0.01205	38.47
27.33	0.4426	1.641	-1.919	27.27	0.009929	45.08
30.33	0.4225	1.649	-2.383	30.24	0.008365	52.02
33.33	0.4048	1.657	-2.857	33.21	0.007172	59.28
336.3	0.1329	1.756	-61.83	330.6	0.099	1702
669.3	0.09446	1.769	-131.6	656.3	0.01301	4746
1002	0.07725	1.774	-202.7	981.6	0.005781	8677
1335	0.06695	1.778	-274.6	1307	0.003451	13330
1668	0.05991	1.78	-346.9	1632	0.002356	18600
2001	0.05471	1.782	-419.6	1957	0.00174	24420
2334	0.05066	1.783	-492.5	2282	0.001353	30760
2667	0.0474	1.784	-565.6	2607	0.001091	37560
3000	0.0447	1.785	-638.9	2932	0.0009036	44800
3333	0.04241	1.786	-712.3	3256	0.0007645	52450

3.2.1. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5527	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.6379	0.6918	1.001	0.8293	0.1075	0.8297
1.4	0.6935	0.7742	1.001	0.9788	0.08241	0.9796
1.5	0.7239	0.8412	0.9998	1.118	0.06704	1.12
1.6	0.7273	0.8985	0.9965	1.252	0.05723	1.258
1.7	0.6976	0.9498	0.9891	1.383	0.05136	1.397
1.8	0.6236	0.999	0.9741	1.514	0.04914	1.548
1.9	0.4779	1.051	0.943	1.649	0.05249	1.728
2	0	1.153	0.8108	1.828	0.1019	2.121

3.2.2. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.1$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5527	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.6379	0.6918	1.001	0.8293	0.1075	0.8297
1.4	0.6935	0.7742	1.001	0.9788	0.0824	0.9796

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.5	0.7242	0.8412	0.9999	1.118	0.06702	1.12
1.6	0.728	0.8984	0.9966	1.252	0.05719	1.257
1.7	0.6992	0.9497	0.9893	1.382	0.05128	1.397
1.8	0.6268	0.9986	0.9746	1.513	0.04897	1.547
1.9	0.4848	1.051	0.9443	1.649	0.05204	1.725
2	0.1	1.137	0.8406	1.815	0.0862	2.057
2	0.09489	1.138	0.839	1.816	0.000982	2.061
2.001	0.08949	1.139	0.8373	1.817	0.001038	2.065
2.001	0.08373	1.14	0.8354	1.818	0.001104	2.07
2.002	0.07754	1.141	0.8334	1.82	0.001185	2.074
2.002	0.0708	1.143	0.8313	1.821	0.001288	2.079
2.002	0.06335	1.144	0.8288	1.823	0.001424	2.085
2.003	0.05487	1.146	0.826	1.824	0.001615	2.092
2.003	0.04482	1.148	0.8227	1.826	0.001915	2.099
2.003	0.0317	1.15	0.8183	1.829	0.002493	2.109
2.004	0.0006991	1.156	0.8075	1.834	0.005996	2.133

3.2.3. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.3$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5527	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.638	0.6918	1.001	0.8293	0.1075	0.8297
1.4	0.6942	0.7741	1.001	0.9787	0.08236	0.9795
1.5	0.7265	0.841	1	1.118	0.06689	1.12
1.6	0.7339	0.8979	0.9972	1.251	0.05689	1.256
1.7	0.7119	0.9486	0.9908	1.381	0.05066	1.394
1.8	0.652	0.9962	0.9783	1.511	0.04765	1.54
1.9	0.5364	1.045	0.9536	1.643	0.04879	1.707
2	0.3	1.107	0.8952	1.788	0.06172	1.943
2.004	0.2853	1.11	0.8907	1.795	0.003185	1.956
2.007	0.2697	1.114	0.8863	1.801	0.003347	1.969
2.011	0.2529	1.117	0.8815	1.808	0.003541	1.984
2.015	0.2347	1.121	0.8763	1.814	0.003781	1.999
2.019	0.2147	1.125	0.8705	1.821	0.004087	2.015
2.022	0.1925	1.129	0.8639	1.829	0.004493	2.034
2.026	0.1671	1.135	0.8562	1.836	0.005068	2.055
2.03	0.1368	1.14	0.8468	1.845	0.005975	2.079
2.034	0.09696	1.148	0.834	1.855	0.007739	2.111
2.037	0.0006129	1.167	0.8008	1.873	0.01863	2.188

3.2.4. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.5$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5527	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.6382	0.6918	1.001	0.8293	0.1075	0.8297
1.4	0.6956	0.774	1.001	0.9786	0.08228	0.9793
1.5	0.7311	0.8407	1	1.118	0.06664	1.119
1.6	0.7455	0.897	0.9984	1.25	0.05631	1.254
1.7	0.7365	0.9465	0.9938	1.379	0.04947	1.389
1.8	0.6998	0.9918	0.985	1.507	0.04533	1.528
1.9	0.6271	1.036	0.969	1.634	0.04384	1.678
2	0.5	1.082	0.9392	1.766	0.04632	1.854
2.013	0.478	1.088	0.9337	1.783	0.006395	1.88
2.025	0.4542	1.095	0.9276	1.8	0.006625	1.907
2.038	0.4281	1.102	0.9208	1.818	0.006914	1.935
2.05	0.3993	1.109	0.9132	1.836	0.007282	1.966

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
2.063	0.3672	1.117	0.9044	1.854	0.007764	1.999
2.076	0.3308	1.125	0.8942	1.873	0.008421	2.035
2.088	0.2886	1.135	0.8819	1.893	0.009375	2.075
2.101	0.2373	1.146	0.8664	1.914	0.01091	2.123
2.113	0.169	1.16	0.8446	1.937	0.01394	2.185
2.126	0.0005514	1.193	0.7849	1.976	0.033	2.334

3.2.5. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.7$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5528	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.6386	0.6917	1.001	0.8292	0.1075	0.8297
1.4	0.6976	0.7739	1.001	0.9785	0.08216	0.979
1.5	0.738	0.8402	1.001	1.117	0.06627	1.118
1.6	0.7626	0.8956	1	1.249	0.05546	1.251
1.7	0.772	0.9435	0.9979	1.376	0.04784	1.381
1.8	0.7658	0.9859	0.9938	1.501	0.04242	1.511
1.9	0.7427	1.025	0.987	1.624	0.03868	1.643
2	0.7	1.061	0.9761	1.746	0.03639	1.782
2.038	0.6776	1.075	0.9703	1.793	0.01368	1.838
2.077	0.6513	1.088	0.9636	1.84	0.01366	1.896
2.115	0.6207	1.102	0.9556	1.887	0.01375	1.956
2.154	0.585	1.116	0.946	1.935	0.01399	2.02
2.192	0.5433	1.13	0.9343	1.983	0.01442	2.088
2.231	0.494	1.146	0.9201	2.032	0.01514	2.162
2.269	0.4347	1.162	0.902	2.082	0.01634	2.245
2.308	0.3604	1.18	0.8781	2.134	0.01844	2.341
2.346	0.2586	1.203	0.8429	2.189	0.02288	2.465
2.384	0.000399	1.256	0.7389	2.267	0.05242	2.76

3.2.6. $\alpha_1^0 = -10$ $\beta_{t0} = 0.5$ $\beta_{r0} = 0.8642$ $\alpha_1 = -5$ $\alpha = -10$

1	0.004472	0	1	0	0	0
1.1	0.4166	0.4283	1.001	0.4568	0.4253	0.4572
1.2	0.5528	0.5842	1.001	0.6619	0.156	0.6623
1.3	0.639	0.6917	1.001	0.8292	0.1075	0.8296
1.4	0.6998	0.7737	1.001	0.9783	0.08203	0.9788
1.5	0.7453	0.8396	1.002	1.117	0.06588	1.117
1.6	0.7804	0.8942	1.002	1.248	0.05461	1.248
1.7	0.8083	0.9405	1.002	1.373	0.04627	1.374
1.8	0.8307	0.9804	1.002	1.495	0.03987	1.496
1.9	0.8491	1.015	1.002	1.614	0.03481	1.615
2	0.8642	1.046	1.002	1.731	0.03072	1.732
3.8	0.9303	1.307	0.9926	3.668	0.2605	3.697
5.6	0.8974	1.399	0.9594	5.517	0.09212	5.662
7.4	0.8521	1.448	0.9051	7.344	0.04954	7.72
9.2	0.8082	1.48	0.8344	9.162	0.03181	9.89
11	0.7684	1.503	0.7504	10.97	0.02255	12.17
12.8	0.7329	1.52	0.6557	12.78	0.01702	14.57
14.6	0.7014	1.533	0.5521	14.59	0.01343	17.09
16.4	0.6734	1.544	0.4409	16.39	0.01093	19.71
18.2	0.6482	1.553	0.3233	18.2	0.009124	22.43
20	0.6256	1.561	0.2001	20	0.007763	25.26
201.8	0.2198	1.664	-18.7	200.9	0.103	626.6
401.6	0.1568	1.677	-42.44	399.4	0.01308	1728
601.4	0.1284	1.682	-67.03	597.7	0.0058	3148
801.2	0.1114	1.686	-92.05	795.9	0.003459	4826

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1001	0.0997	1.688	-117.4	994.1	0.002361	6727
1201	0.09106	1.69	-142.9	1192	0.001743	8828
1401	0.08434	1.691	-168.5	1390	0.001355	11110
1600	0.07892	1.692	-194.3	1589	0.001092	13560
1800	0.07443	1.693	-220.1	1787	0.0009045	16170
2000	0.07062	1.694	-246.1	1985	0.0007652	18930

3.3.1. α_1^0 = -10 β_{t0} = 0.7 β_{r0} = 0 α_1 = -7 α = -14

1	0.00378	0	1	0	0	0
1.043	0.2838	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3894	0.3993	1	0.4221	0.1123	0.4222
1.129	0.4635	0.4812	1	0.5224	0.08196	0.5225
1.171	0.5208	0.5471	1	0.6094	0.06584	0.6095
1.214	0.5667	0.6025	1.001	0.6881	0.05539	0.6882
1.257	0.602	0.6505	1	0.7613	0.048	0.7614
1.3	0.6213	0.6932	1	0.8307	0.04276	0.8313
1.343	0.6075	0.7329	0.9981	0.8984	0.0397	0.9006
1.386	0.5157	0.7733	0.9917	0.9679	0.04035	0.9758
1.429	0	0.8458	0.9473	1.069	0.07254	1.121

3.3.2. α_1^0 = -10 β_{t0} = 0.7 β_{r0} = 0.1 α_1 = -7 α = -14

1	0.00378	0	1	0	0	0
1.043	0.2838	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3894	0.3993	1	0.4221	0.1123	0.4222
1.129	0.4635	0.4812	1	0.5224	0.08196	0.5225
1.171	0.5208	0.5471	1	0.6094	0.06584	0.6095
1.214	0.5667	0.6025	1.001	0.6881	0.05539	0.6882
1.257	0.6021	0.6504	1	0.7613	0.04799	0.7614
1.3	0.6217	0.6932	1	0.8307	0.04275	0.8313
1.343	0.6087	0.7328	0.9981	0.8984	0.03965	0.9005
1.386	0.5197	0.773	0.9919	0.9677	0.04018	0.9754
1.429	0.1	0.8369	0.9568	1.061	0.06386	1.103
1.429	0.09491	0.8376	0.9562	1.062	0.0005703	1.104
1.429	0.08953	0.8382	0.9556	1.062	0.0006025	1.105
1.429	0.08379	0.8388	0.955	1.063	0.0006411	1.107
1.429	0.07761	0.8395	0.9544	1.064	0.0006883	1.108
1.429	0.07088	0.8403	0.9536	1.064	0.000748	1.11
1.429	0.06343	0.8411	0.9528	1.065	0.0008269	1.111
1.429	0.05496	0.842	0.9519	1.066	0.0009379	1.113
1.429	0.0449	0.8431	0.9508	1.067	0.001112	1.116
1.43	0.03177	0.8446	0.9493	1.069	0.001448	1.118
1.43	0.001076	0.8482	0.9455	1.072	0.003596	1.126

3.3.3. α_1^0 = -10 β_{t0} = 0.7 β_{r0} = 0.3 α_1 = -7 α = -14

1	0.00378	0	1	0	0	0
1.043	0.2838	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3894	0.3993	1	0.4221	0.1123	0.4222
1.129	0.4635	0.4812	1	0.5224	0.08196	0.5225
1.171	0.5208	0.5471	1	0.6094	0.06584	0.6095
1.214	0.5668	0.6025	1.001	0.6881	0.05538	0.6882
1.257	0.6027	0.6504	1	0.7612	0.04797	0.7614
1.3	0.6245	0.6931	1	0.8306	0.04264	0.8311
1.343	0.6185	0.7324	0.9985	0.8979	0.0393	0.8997
1.386	0.551	0.7713	0.9936	0.9659	0.0389	0.9721
1.429	0.3	0.8201	0.9745	1.045	0.04879	1.069
1.43	0.2859	0.8219	0.9734	1.047	0.001891	1.073

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.431	0.2708	0.8239	0.9721	1.05	0.001987	1.077
1.432	0.2545	0.826	0.9706	1.053	0.002102	1.081
1.433	0.2367	0.8282	0.969	1.056	0.002245	1.086
1.434	0.217	0.8307	0.9672	1.059	0.002426	1.091
1.435	0.195	0.8333	0.9652	1.062	0.002667	1.096
1.436	0.1697	0.8363	0.9627	1.066	0.003009	1.103
1.438	0.1392	0.8399	0.9597	1.07	0.003547	1.11
1.439	0.09886	0.8445	0.9555	1.076	0.004594	1.119
1.44	0.0007959	0.8556	0.9442	1.087	0.01112	1.142

3.3.4. α_1^0 = -10 β_{t0} = 0.7 β_{r0} = 0.5 α_1 = -7 α = -14

1	0.00378	0	1	0	0	0
1.043	0.2838	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3894	0.3993	1	0.4221	0.1123	0.4222
1.129	0.4635	0.4812	1	0.5224	0.08196	0.5225
1.171	0.5208	0.5471	1	0.6094	0.06584	0.6095
1.214	0.567	0.6024	1.001	0.6881	0.05537	0.6882
1.257	0.604	0.6504	1.001	0.7612	0.04792	0.7613
1.3	0.63	0.6928	1	0.8303	0.04244	0.8307
1.343	0.6376	0.7314	0.9994	0.8969	0.03861	0.8981
1.386	0.6087	0.7681	0.9966	0.9628	0.03671	0.9664
1.429	0.5	0.8066	0.9885	1.031	0.03845	1.043
1.433	0.4816	0.8111	0.9867	1.039	0.004101	1.051
1.437	0.4611	0.8153	0.9851	1.046	0.004246	1.06
1.441	0.4379	0.8198	0.9833	1.053	0.004427	1.069
1.445	0.4116	0.8244	0.9812	1.061	0.004657	1.079
1.449	0.3815	0.8294	0.9787	1.069	0.00496	1.089
1.453	0.3465	0.8348	0.9757	1.077	0.005372	1.101
1.457	0.3046	0.8407	0.972	1.086	0.005972	1.113
1.462	0.2525	0.8477	0.9671	1.096	0.006938	1.128
1.466	0.1812	0.8565	0.9601	1.107	0.008855	1.147
1.47	0.0007395	0.8775	0.9393	1.13	0.02098	1.192

3.3.5. α_1^0 = -10 β_{t0} = 0.7 β_{r0} = 0.7 α_1 = -7 α = -14

1	0.00378	0	1	0	0	0
1.043	0.2838	0.2869	1	0.2951	0.2839	0.2952
1.086	0.3894	0.3993	1	0.4221	0.1123	0.4222
1.129	0.4635	0.4812	1	0.5224	0.08196	0.5225
1.171	0.5208	0.5471	1	0.6094	0.06584	0.6095
1.214	0.5673	0.6024	1.001	0.6881	0.05536	0.6882
1.257	0.6059	0.6503	1.001	0.7611	0.04785	0.7612
1.3	0.6383	0.6924	1.001	0.8299	0.04214	0.83
1.343	0.6652	0.7301	1.001	0.8956	0.03766	0.8958
1.386	0.6862	0.7642	1	0.9588	0.03407	0.9592
1.429	0.7	0.7954	1	1.02	0.03121	1.021
1.455	0.7039	0.8132	0.9999	1.057	0.01818	1.059
1.482	0.7032	0.8307	0.9992	1.094	0.01749	1.096
1.508	0.6967	0.8476	0.9981	1.131	0.01696	1.134
1.535	0.6828	0.8642	0.9964	1.167	0.01661	1.173
1.561	0.6595	0.8807	0.9939	1.204	0.01648	1.212
1.588	0.6238	0.8974	0.9903	1.241	0.01665	1.254
1.614	0.5708	0.9147	0.9849	1.279	0.01729	1.298
1.641	0.492	0.9334	0.9764	1.319	0.01878	1.348
1.667	0.367	0.9559	0.9619	1.362	0.02244	1.409
1.694	0.000562	1.005	0.9079	1.43	0.04928	1.549

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p

3.3.6. $\alpha_1^0 = -10$ $\beta_{t0} = 0.7$ $\beta_{r0} = 0.714$ $\alpha_1 = -7$ $\alpha = -14$

1	0.005345	0	1	0	0	0
1.043	0.2838	0.2874	1	0.2956	0.2824	0.2957
1.086	0.3895	0.3997	1	0.4225	0.1123	0.4227
1.129	0.4636	0.4817	1	0.5228	0.08196	0.523
1.171	0.5208	0.5475	1	0.6098	0.06584	0.6099
1.214	0.5673	0.6029	1	0.6885	0.05536	0.6887
1.257	0.606	0.6507	1	0.7615	0.04785	0.7617
1.3	0.639	0.6928	1	0.8303	0.04212	0.8305
1.343	0.6674	0.7304	1	0.8959	0.03758	0.8961
1.386	0.6922	0.7643	1	0.959	0.03388	0.9591
1.429	0.714	0.7951	1	1.02	0.03079	1.02
2.714	0.9276	1.193	1.001	2.523	0.3982	2.524
4	0.9542	1.319	0.9978	3.874	0.1255	3.884
5.286	0.947	1.383	0.9892	5.192	0.06384	5.234
6.571	0.9279	1.422	0.9744	6.499	0.03943	6.605
7.857	0.9044	1.449	0.9537	7.799	0.02716	8.008
9.143	0.8796	1.469	0.9275	9.096	0.02005	9.449
10.43	0.8551	1.485	0.8966	10.39	0.01554	10.93
11.71	0.8315	1.497	0.8615	11.68	0.01248	12.46
13	0.809	1.507	0.8226	12.97	0.01029	14.02
14.29	0.7878	1.516	0.7803	14.26	0.008669	15.64
144.1	0.3042	1.625	-7.754	143.9	0.109	330.1
286.9	0.2182	1.638	-19.21	286.2	0.0132	896.8
429.6	0.1791	1.644	-31.27	428.4	0.005829	1625
572.3	0.1554	1.647	-43.63	570.6	0.003471	2484
715	0.1392	1.649	-56.2	712.8	0.002367	3456
857.7	0.1272	1.651	-68.91	854.9	0.001747	4530
1000	0.1179	1.653	-81.73	997.1	0.001357	5697
1143	0.1103	1.654	-94.64	1139	0.001094	6949
1286	0.1041	1.655	-107.6	1281	0.0009058	8282
1429	0.09875	1.655	-120.6	1423	0.0007662	9691

3.4.1. $\alpha_1^0 = -10$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0$ $\alpha_1 = -9$ $\alpha = -18$

1	0.002108	0	1	0	0	0
1.011	0.1479	0.1483	1	0.1494	0.1463	0.1494
1.022	0.2074	0.2088	1	0.2119	0.0605	0.2119
1.033	0.2519	0.2546	1	0.2603	0.04579	0.2603
1.044	0.2886	0.2927	1	0.3014	0.03808	0.3014
1.056	0.3201	0.3258	1	0.3379	0.03311	0.3379
1.067	0.348	0.3554	1	0.3711	0.02954	0.3711
1.078	0.3729	0.3822	1	0.4019	0.02681	0.4019
1.089	0.3933	0.4068	1	0.4309	0.02468	0.4309
1.1	0.3881	0.4303	0.9997	0.4589	0.02346	0.459
1.111	0	0.4643	0.9935	0.4976	0.03403	0.5007

3.4.2. $\alpha_1^0 = -10$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.1$ $\alpha_1 = -9$ $\alpha = -18$

1	0.002108	0	1	0	0	0
1.011	0.1479	0.1483	1	0.1494	0.1463	0.1494
1.022	0.2074	0.2088	1	0.2119	0.0605	0.2119
1.033	0.2519	0.2546	1	0.2603	0.04579	0.2603
1.044	0.2886	0.2927	1	0.3014	0.03808	0.3014
1.056	0.3201	0.3258	1	0.3379	0.03311	0.3379
1.067	0.348	0.3554	1	0.3711	0.02954	0.3711
1.078	0.3729	0.3822	1	0.4019	0.02681	0.4019

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.089	0.3935	0.4068	1	0.4309	0.02468	0.4309
1.1	0.3896	0.4303	0.9997	0.4588	0.02342	0.459
1.111	0.1	0.4614	0.9949	0.4947	0.03114	0.4971
1.111	0.095	0.4613	0.995	0.4946	0.0002673	0.4973
1.111	0.08969	0.4615	0.9949	0.4948	0.0002822	0.4977
1.111	0.08401	0.4618	0.9948	0.4952	0.0003001	0.498
1.111	0.07788	0.4622	0.9947	0.4955	0.0003219	0.4984
1.111	0.0712	0.4625	0.9945	0.4959	0.0003496	0.4989
1.111	0.06377	0.4629	0.9943	0.4963	0.0003861	0.4994
1.111	0.0553	0.4633	0.9942	0.4967	0.0004376	0.4999
1.111	0.04522	0.4639	0.9939	0.4972	0.0005183	0.5005
1.111	0.03203	0.4645	0.9936	0.4979	0.0006744	0.5014
1.111	0.001027	0.4662	0.9928	0.4996	0.001687	0.5035

3.4.3. $\alpha_1^0 = -10$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.3$ $\alpha_1 = -9$ $\alpha = -18$

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3011	0.03808	0.3011
1.056	0.3202	0.3256	1	0.3376	0.03311	0.3376
1.067	0.348	0.3551	1	0.3709	0.02954	0.3709
1.078	0.3729	0.3819	1	0.4017	0.02681	0.4017
1.089	0.3945	0.4066	1	0.4306	0.02466	0.4306
1.1	0.4018	0.4297	1	0.4583	0.02315	0.4584
1.111	0.3	0.4543	0.9984	0.4876	0.02458	0.4884
1.112	0.2889	0.4551	0.9984	0.4885	0.001076	0.4893
1.112	0.2765	0.4562	0.9982	0.4898	0.001119	0.4907
1.112	0.2626	0.4574	0.998	0.4912	0.001173	0.4922
1.113	0.2468	0.4586	0.9977	0.4926	0.001241	0.4937
1.113	0.2288	0.4599	0.9974	0.4941	0.001328	0.4953
1.113	0.2078	0.4614	0.997	0.4957	0.001445	0.4971
1.114	0.1828	0.463	0.9966	0.4975	0.001614	0.4991
1.114	0.1516	0.4649	0.996	0.4995	0.001883	0.5015
1.115	0.1089	0.4673	0.9951	0.5021	0.002414	0.5045
1.115	0.001178	0.4731	0.9925	0.5081	0.005815	0.5117

3.4.4. $\alpha_1^0 = -10$ $\beta_{t0} = 0.9$ $\beta_{r0} = 0.4$ $\alpha_1 = -9$ $\alpha = -18$

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3011	0.03808	0.3011
1.056	0.3202	0.3256	1	0.3376	0.03311	0.3376
1.067	0.348	0.3551	1	0.3709	0.02954	0.3709
1.078	0.373	0.3819	1	0.4017	0.02681	0.4017
1.089	0.3954	0.4066	1	0.4306	0.02464	0.4306
1.1	0.4122	0.4295	1	0.4581	0.02293	0.4581
1.111	0.4	0.4516	0.9997	0.4849	0.02211	0.4851
1.112	0.3935	0.4544	0.9994	0.4882	0.002411	0.488
1.113	0.385	0.4569	0.9993	0.4912	0.002452	0.491
1.115	0.3741	0.4594	0.9991	0.4942	0.002509	0.4941
1.116	0.36	0.462	0.9989	0.4973	0.002589	0.4974
1.117	0.3418	0.4647	0.9986	0.5006	0.002701	0.5007
1.118	0.3183	0.4675	0.9982	0.504	0.002865	0.5043
1.119	0.2872	0.4706	0.9977	0.5076	0.003116	0.5082

The ending enc. 4

R/R_p	β_r	φ	x/R_p	y/R_p	$\Delta\varphi$	tc_1/R_p
1.121	0.2445	0.4742	0.9969	0.5117	0.003538	0.5126
1.122	0.1803	0.4786	0.9957	0.5166	0.004409	0.5182
1.123	0.00122	0.4889	0.9914	0.5274	0.0103	0.5312

3.4.5. α_1^0 = -10 β_{t0} = 0.9 β_{r0} = 0.4359 α_1 = -9 α = -18

1	0.003333	0	1	0	0	0
1.011	0.1479	0.1481	1	0.1492	0.1451	0.1492
1.022	0.2074	0.2086	1	0.2117	0.0605	0.2117
1.033	0.252	0.2544	1	0.2601	0.04579	0.2601
1.044	0.2886	0.2925	1	0.3011	0.03808	0.3011
1.056	0.3202	0.3256	1	0.3376	0.03311	0.3376
1.067	0.348	0.3551	1	0.3709	0.02954	0.3709
1.078	0.373	0.3819	1	0.4017	0.02681	0.4017
1.089	0.3957	0.4066	1	0.4306	0.02463	0.4306
1.1	0.4166	0.4294	1	0.458	0.02284	0.458
1.111	0.4359	0.4507	1	0.484	0.02133	0.484
2.111	0.8807	1.077	1	1.859	0.6263	1.859
3.111	0.9469	1.244	1	2.946	0.1662	2.946
4.111	0.97	1.325	1	3.988	0.08153	3.987
5.111	0.9807	1.374	1	5.012	0.04879	5.012
6.111	0.9865	1.406	1	6.029	0.03255	6.028
7.111	0.9901	1.43	1	7.04	0.02328	7.04
8.111	0.9924	1.447	1	8.049	0.01749	8.049
9.111	0.994	1.461	1	9.056	0.01362	9.056
10.11	0.9951	1.472	1	10.06	0.01091	10.06
11.11	0.9959	1.481	1	11.07	0.008941	11.07
112.1	1	1.562	1.003	112.1	0.081	112.1
223.1	1	1.566	1.006	223.1	0.004438	223.1
334.1	1	1.568	1.009	334.1	0.001489	334.1
445.1	1	1.569	1.012	445.1	0.0007464	445.1
556.1	1	1.569	1.014	556.1	0.0004484	556.1
667.1	1	1.569	1.017	667.1	0.0002992	667.1
778.1	1	1.569	1.02	778.1	0.0002138	778.1
889.1	1	1.57	1.023	889.1	0.0001604	889.1
1000	1	1.57	1.026	1000	0.0001248	1000
1111	1	1.57	1.029	1111	9.989e-005	1111

Appendix 5

NORMALISED PERIODS OF ORBITS AND THEIR RADIUSES OF APOCENTRES

α_1	β_p	$\overline{R}_a$	φ_a/π	$\bar{t}_a$
-.5000E+00	.1000E+00	.2300E+06	1.001	.1429E+09
-.5000E+00	.3000E+00	.2570E+04	1.012	.1802E+06
-.5000E+00	.5000E+00	.2500E+03	1.038	.5842E+04
-.5000E+00	.7000E+00	.3728E+02	1.115	.3700E+03
-.5000E+00	.8000E+00	.1316E+02	1.246	.8627E+02
-.6000E+00	.1000E+00	.4969E+01	1.001	.2101E+02
-.6000E+00	.3000E+00	.4712E+01	1.018	.1968E+02
-.6000E+00	.5000E+00	.4100E+01	1.063	.1694E+02
-.6000E+00	.7000E+00	.2867E+01	1.228	.1205E+02
-.6700E+00	.7400E+00	.1221E+01	2.879	.1064E+02
-.7000E+00	.1000E+00	.2482E+01	1.002	.8632E+01
-.7000E+00	.3000E+00	.2334E+01	1.025	.8172E+01
-.7000E+00	.5000E+00	.1991E+01	1.097	.7203E+01
-.7000E+00	.7000E+00	.1220E+01	1.823	.6914E+01
-.7300E+00	.6000E+00	.1442E+01	1.246	.5680E+01
-.7300E+00	.6600E+00	.1194E+01	1.562	.5841E+01
-.8000E+00	.1000E+00	.1653E+01	1.003	.5373E+01
-.8000E+00	.3000E+00	.1540E+01	1.032	.5112E+01
-.8000E+00	.5000E+00	.1270E+01	1.146	.4616E+01
-.8000E+00	.5990E+00	.1004E+01	1.390	.4362E+01
-.9000E+00	.1000E+00	.1239E+01	1.003	.3929E+01
-.9000E+00	.3000E+00	.1142E+01	1.044	.3764E+01
-.9000E+00	.4000E+00	.1046E+01	1.086	.3565E+01

Scientific issue

Smulsky Joseph J.

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- What is time
- How the bodies interact
- How the elementary particles move
- Whether A. Einstein is right
- How you can calculate the accelerator of the particles
- Whether there is an ether
- What moving star we can see
- How to enter a "black hole" and to leave it
- Whether it is possible to move with a superluminal velocity
- How to reach stars and to return on the Earth for 5 years
- What is the velocity of gravitation
- According what laws the Mercury moves
- How to calculate the movement of many bodies precisely

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"Prof. Smulsky is arguably the most outspoken and distinguished opponent of Special Theory of Relativity; his contribution honours for Special. Rel. Let ".

G.Walton, SRL - 1997.-V.1, N.3.-P.61.

$$\vec{F}_{12} = \frac{q_1 q_2}{\varepsilon} \frac{\vec{R}_{12}\left(1-\beta^2\right)}{\left\{R_{12}^2 - \left[\vec{\beta}\times\vec{R}_{12}\right]^2\right\}^{3/2}}$$