

**PALEOTEMPERATURES OF THE EARTH'S SURFACE.****1. DETERMINATION OF THE AVERAGE ANNUAL TEMPERATURE OF THE EARTH AND HEMISPHERES****I. I. Smul'skii**

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*The author has presented functional dependences to determine the distribution of the average annual temperature and annual insolation along an Earth latitude.*

**Keywords:** midlatitude temperature, annual insolation, paleoclimate.

**Introduction.** The Sun illuminates the Earth and, depending on the quantity of heat, its surface temperature sets in: in the equatorial latitudes, it is the highest, and at the polar latitudes, it is the lowest. Since the parameters of orbital and rotational motion of the Earth evolve, the distribution of solar heat on its surface changes with time. According to the new astronomical theory of climate [1], these changes are such that in some historical periods, at high latitudes, the quantity of heat received in the six summer months is greater than that at the equator now. Then a warm period begins, the glaciers of Greenland and Antarctica melt, and the sea level rises. In other times, at high latitudes (from 90 to 55°), there is less heat received in summer than it is the case at the poles now. The snow fallen in winter does not have time to melt and a glacier is formed, i.e., a glacial period sets in. Water accumulates in the polar glaciers, and the sea level drops. The distribution of heat along an Earth latitude in the said periods can be estimated accurately. Therefore, if we know the dependence of the Earth surface temperature on the quantity of solar heat, it can be used to calculate the Earth surface temperature in other periods too.

In the astronomic theory of climate change, consideration is given to the distributions of heat quantities along an Earth latitude during the six summer and winter months and for the whole year. They vary in time intervals of tens of thousands of years. In time intervals counted in years, the Earth insolation does not change, in practice. Temperatures of half-year periods and annual temperatures vary from year to year within fairly narrow limits. Therefore, their mean values for several decades are calculated. In this investigation, consideration is given to the distribution of the Earth average annual temperature along its latitude and the relation of this distribution to the distribution of the annual Earth insolation.

There are methods to estimate paleotemperature [2, 3] using various assumptions that do not have such a rigorous quantitative basis as insolation of the Earth. Therefore, a number of investigators have attempted to find the functional dependence of the temperature of the Earth on its insolation. M. Milankovich used the physical laws of radiative heat transfer as a basis for creating a theory that makes it possible to use Earth insolation to determine the distribution of the temperature of the Earth along its latitude during the summer and winter half-year periods and for the whole year [4]. He established that in the case of the atmosphere at rest and a homogeneous Earth surface, the Earth temperature varies from 32.8°C at the equator to –34.8°C at the poles. However, it is difficult to determine the temperature of the real Earth in such a way. Therefore, attempts have been made to establish a correlational dependence between arrays of data on the observed temperature of the Earth and its insolation [5].

The change in the average annual temperature of the Earth near-surface layer along its latitude is called meridional distribution of zonally averaged temperature, zonally averaged temperature, normal temperature, or midlatitude temperature. In this investigation, this temperature is considered as a mathematical function of the latitude  $\varphi$ ; therefore, it is mentioned as a latitude-distributed quantity or midlatitude temperature. Determination of the Earth temperature distribution along its latitude  $t(\varphi)$  is a complex problem complicated by many circumstances [5, 6].

During the past 170 years, investigations aimed at the analysis of temperature rows and determination of the midlatitude Earth temperature have been conducted by many investigators [6–23]. G. Kramm et al. plotted 15 profiles of

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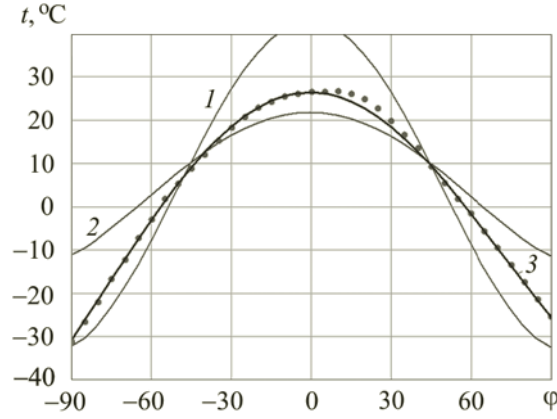


Fig. 1. Comparison of a generalized average annual temperature  $t_{oi}$  (dots) with temperatures determined by dependences (1) (1 and 2) and (2) (3): 1) solid surface of the Earth; 2) Earth surface covered with water.

midlatitude Earth temperature on the general graph [6] on the basis of which we have proposed a generalized average annual temperature of the Earth  $t_{oi}$  (Fig. 1). J. Forbes suggested a formula for the distribution of the temperature of the Earth along its latitude in the Northern hemisphere [8]:

$$t_F(\varphi) = A + B(\cos \varphi)^m + Cn \cos 2\varphi, \quad (1)$$

where  $A = -10.8^\circ\text{C}$ ,  $B = 32.9^\circ\text{C}$ ,  $C = 21.2^\circ\text{C}$ , and  $m = 1.25$ . In this case, the coefficient  $n = 1$ , if the Earth surface is dryland, and  $n = 0$  for the Earth fully covered by the ocean. Figure 1 shows that the variations of temperature of the dryland from the poles to the equator are two times higher than the changes in the temperature of the Earth covered by the ocean. At the latitude of  $45^\circ$ , the values of the generalized average annual temperature of the Earth  $t_{oi}$  coincide with similar values determined by formula (1). By analogy with dependence (1), we constructed a functional dependence made up of two harmonics,

$$t_{o,a} = k_1 + k_2 \sin(k_3\varphi + k_4) + k_5 \sin(k_6\varphi + k_7), \quad (2)$$

which approximates the average annual temperature  $t_{oi}$ , where  $k_1 = -48.841^\circ\text{C}$ ,  $k_2 = 47.840^\circ\text{C}$ ,  $k_3 = 0.12389$ ,  $k_4 = 0.81818$ ,  $k_5 = 40.602^\circ\text{C}$ ,  $k_6 = 1.2165$ , and  $k_7 = 1.6566$ . The average difference of  $t_{o,a}$  from the indexed annual temperature  $t_{oi}$  is  $0.44^\circ\text{C}$ , and the maximum one is  $2^\circ\text{C}$  at the latitude  $\varphi = 20^\circ$ . It follows from Fig. 1 that at the latitudes of the Northern hemisphere from  $10$  to  $30^\circ$ , the temperature is  $1$ – $2^\circ\text{C}$  higher compared to the same latitudes of the Southern hemisphere. Apparently, this is due to the fact that in the Northern hemisphere, dryland occupies most of the Earth surface compared to the Southern hemisphere. Further on, we will use indexed temperature  $t_{oi}$  as the average annual near-surface temperature of the Earth in the recent epoch. To verify various calculations, we will also use an approximating dependence (2).

**Average Annual Temperature of the Earth and Hemispheres.** Let there be quantity  $f$  distributed along the Earth latitude  $\varphi$ . We find its mean value across the entire Earth surface and over the hemispheres. We use a spherical Earth model with mean radius  $R_{E,m} = (R_{E,e} + R_{E,p})/2 = 6367.5$  km, where  $R_{E,e} = 6378.1$  km and  $R_{E,p} = 6356.8$  km are the equatorial and polar radii of the Earth. Let us consider a zone at the latitude  $\varphi$  with a width  $\Delta\varphi$ , a length  $2\pi R_{E,m} \cos \varphi$ , and an area

$$\Delta F = 2\pi R_{E,m} \cos \varphi R_{E,m} \Delta\varphi.$$

Introducing the notation

$$\Delta S = f \Delta F = 2\pi f R_{E,m}^2 \cos \varphi \Delta\varphi,$$

the mean value of the function  $f_E$  over the entire Earth surface  $4\pi R_{E,m}^2$  can be represented in the form

$$f_E = \left( \int_{-0.5\pi}^{0.5\pi} \Delta S d\varphi \right) / 4\pi R_{E,m}^2 = 0.5 \int_{-0.5\pi}^{0.5\pi} f \cos \varphi d\varphi , \quad (3)$$

where the angle  $\varphi$  varied from negative values to positive ones. In analogy with (3), we write the mean values of the quantity  $f$  for the Southern and Northern hemispheres:

$$f_{SH} = \int_{-0.5\pi}^0 f \cos \varphi d\varphi , \quad f_{NH} = \int_0^{0.5\pi} f \cos \varphi d\varphi . \quad (4)$$

We make use of formulas (3) and (4) to determine Earth-surface-average temperatures. For a temperature in the form of an indexed variable  $t_{oi}$ , use is made of summation at short intervals  $\Delta\varphi$  instead of integration on an infinitesimal increment of latitude  $d\varphi$ . In this case, the temperature  $t_{oi}$  and the latitude  $\varphi_i$  must relate to the center of the interval  $\Delta\varphi$ . Then, according to (3), the Earth-surface-average temperature will be determined as

$$t_{o,E} = 0.5 \sum_{i=1}^{I_{f,2}} t_{oi} \cos \varphi_i \Delta\varphi_i , \quad (5)$$

where  $i = 1, 2, \dots, I_{f,2}$ ,  $I_{f,2}$  is a whole (integer) number of intervals  $\Delta\varphi$ ,  $\Delta\varphi = 180^\circ/I_{f,2}$ ,  $\varphi_1 = -0.5 \cdot 180^\circ + 0.5\Delta\varphi$ ,  $\varphi_{i+1} = \varphi_i + \Delta\varphi$ . At the said value  $\varphi_1$ , the latitude  $\varphi_i$  and the value of the temperature  $t_{oi}$  are calculated to the center of the latitude dependence  $\Delta\varphi$ . In analogy with (4) and (5), we write the surface-average temperatures of the hemispheres:

$$t_{o,SH} = \sum_{i=1}^{0.5I_{f,2}} t_{oi} \cos \varphi_i \Delta\varphi_i , \quad t_{o,NH} = \sum_{0.5I_{f,2}+1}^{I_{f,2}} t_{oi} \cos \varphi_i \Delta\varphi_i . \quad (6)$$

For the approximated dependence  $t_{o,a}$  (2), the average temperature of the Earth will be determined by integrating expression (3). After substituting (2) into (3), we have

$$t_{o,E,a} = 0.5 \int_{-0.5\pi}^{0.5\pi} k_1 \cos \varphi d\varphi + 0.5k_2J_1 + 0.5k_5J_2 , \quad (7)$$

$$J_1 = \int_{-0.5\pi}^{0.5\pi} \sin(k_3\varphi + k_4) \cos \varphi d\varphi , \quad J_2 = \int_{-0.5\pi}^{0.5\pi} \sin(k_6\varphi + k_7) \cos \varphi d\varphi .$$

The integrals  $J_1$  and  $J_2$  can be taken twice in parts. After their integration, the Earth temperature by approximation dependence (3) is as follows

$$t_{o,E,a} = k_1 + 0.5k_2 \frac{\sin(0.5\pi k_3 + k_4) + \sin(-0.5\pi k_3 + k_4)}{1 - k_3^2} + 0.5k_5 \frac{\sin(0.5\pi k_6 + k_7) + \sin(-0.5\pi k_6 + k_7)}{1 - k_6^2} . \quad (8)$$

Similarly, the average annual temperature of the hemispheres can be obtained.

Expressions (5) and (6) were used to calculate the average temperatures of the Earth and hemispheres in terms of integrated temperature  $t_{oi}$ . This calculation is performed for  $I_{f,2} = 90$ , i.e., with an interval of  $2^\circ$  of the latitude. Initially, the temperature  $t_{oi}$  was determined at an interval of  $5^\circ$ , therefore its value in intermediate points was determined by parabolic interpolation. The following average temperature values were obtained:

$$t_{o,E} = 14.41^\circ\text{C} , \quad t_{o,NH} = 15.10^\circ\text{C} , \quad t_{o,SH} = 13.73^\circ\text{C} . \quad (9)$$

The average Earth temperature determined by approximation dependence (8)  $t_{0,E,a} = 14.07^\circ\text{C}$ , i.e., it is a little lower than  $t_{0,E}$ . This is due to the fact that the approximation dependence (2) (line 3 in Fig. 1) does not account for the excess of temperature  $t_{0,i}$  at the latitudes from  $10$  to  $30^\circ$  of the Northern hemisphere. The temperature values (9) do not differ, in practice, from the average temperature of the Earth and hemispheres determined in the period from 1991 to 2018 [6, 25–27], therefore, we can consider that the integrated distribution of temperature along the Earth latitude  $t_{0,i}$  based on all existing investigations reflects reliably the latitude distribution of temperature in the modern epoch.

**Earth Insolation.** Earth insolation is understood to be the quantity of heat coming from the sun per  $1 \text{ m}^2$  of the Earth surface during a certain period of time, for example, during the six months of summer ( $Q_s$ ) or winter ( $Q_w$ ) or the whole year ( $Q_t$ ). We will consider the annual insolation  $Q_t$ . The first version of the astronomic theory of climate change created almost 100 years ago by M. Milankovich [4] did not account for the evolution of the rotational motion of the Earth. Therefore, in this version, changes in the Earth insolation could not explain changes in paleoclimate [28, 29]. In the new astronomic theory [1, 30, 31], changes in insolation were determined with account for the evolution of the rotational motion of the Earth [32]. The insolation fluctuations obtained in it in terms of both amplitudes and the moments of occurrence of warmings and coolings coincide with corresponding changes in the paleoclimate [30].

In [1, 31] it was established that in different epochs, the annual insolation  $Q_t$  changed significantly along the latitude. However, at the latitude of  $45^\circ$  it does not change, in practice, and is equal to  $Q_{t0}^{45} = 9.932 \cdot 10^6 \text{ kJ/m}^2$  with an accuracy to four decimal places. Therefore, the quantity  $Q_{t0}^{45}$  can be taken as a characteristic of the Earth's irradiation by the Sun. It should be noted that this value of  $Q_{t0}^{45}$  was obtained for the solar constant  $J_0 = 2 \text{ cal/(cm}^2 \cdot \text{min)} = 1.3956 \text{ kW/m}^2$  which was used by M. Milankovich. In this investigation, use is also made of this value for comparison of the obtained results. Note that the value of the solar constant  $J_0 = 1.36622 \text{ kW/m}^2$  [33] is taken as a space absolute radiometric standard (reference) (SARR).

Further on, we will consider the annual insolation in (nondimensional) units  $Q_{t,nd} = Q_t/Q_{t0}^{45}$ . Figure 2a shows the nondimensional annual insolation  $Q_{t,nd}$  for the modern epoch ( $T = 0 \text{ kyr}$ ) and the epochs  $T = -15.88, -13.28$ , and  $46.44 \text{ kyr}$ , where  $1 \text{ kyr}$  is 1 thousand years. The recent epochs are characterized by their having substantial changes in insolation [30]. As follows from Fig. 2a, the quantity  $Q_{t,nd}$  in high latitudes, in particular, at the poles varies from 0.361 for the very cold period 46.33 thousand years ago (t.y.a) to 0.748 for the very warm period 31.28 thousand years ago, i.e., more than twice. At the equator, the changes in the quantity of heat during the year are significantly smaller: from  $Q_{t,nd} = 1.306$  in the warm epoch 31.28 thousand years ago to  $Q_{t,nd} = 1.388$  in the ice age 46.44 thousand years ago. This change has an opposite direction, i.e., it becomes warmer in cold epochs at the equatorial latitudes. As we can see, the main changes in heat occur at high latitudes.

M. Milankovich introduced summer insolation  $I$  at the equivalent latitudes [4]. The same insulations at the equivalent latitudes during the year  $I_t$  and for the six winter months  $I_w$  were introduced in [34]. Figure 2b shows annual insulations at the equivalent latitudes  $I_t$  for the same four epochs also shown in Fig. 2a. Degrees of the latitude  $I_t$  are plotted along the vertical axis in the inverse order:  $0^\circ$  at the top and  $90^\circ$  at the bottom, i.e., the upper values relate to the warm periods, and the lower ones refer to the cold ones. As seen from Fig. 2b, in the modern epoch ( $T = 0 \text{ kyr}$ ), the insolation  $I_t$  is represented by the straight lines following the diagonal. The insulations  $I_t$  in other epochs located under these lines are indicative of a colder climate, and those above them indicate a warmer climate. For example, in the epoch 31.28 thousand years ago, the annual insolation  $I_t$  at the latitudes of  $90-70^\circ$  is near  $60^\circ$ , i.e., in this epoch, at the polar latitudes, the climate was identical to the current climate at the latitude of  $60^\circ$ . Insolation at equivalent latitudes is determined by the quantity of heat  $Q$ . The algorithm of its calculation for the summer period is given in [35, 36], and for the six winter months and for a year, it is cited in [34].

The annual insolation  $Q_{t,nd}(\varphi)$  is the specific quantity of solar heat during the year per unit area ( $1 \text{ m}^2$ ) of the Earth at the latitude  $\varphi$ . We determine the specific quantity of heat for the entire Earth ( $Q_{t,nd,E}$ ) and for the Northern ( $Q_{t,nd,NH}$ ) and Southern ( $Q_{t,nd,SH}$ ) hemispheres. These values were calculated by formulas (5) and (6), in which, instead of  $t_{0,i}$ , use is made of  $Q_{t,nd}$ . The calculation results are shown in Table 1. As is seen from this table, first, the specific annual insolation of the Earth is identical, in practice, for all epochs. In the modern epoch ( $T = 0 \text{ kyr}$ ), it is equal to  $Q_{t,nd,E,0} = 1.1087$  with an accuracy to four decimal places. Changes of the quantity  $Q_{t,nd,E}$  in other epochs do not exceed 0.01%. Furthermore, in the coldest epoch ( $T = -46.44 \text{ kyr}$ ), it is lower than  $Q_{t,nd,E,0}$  by 0.0001. Second, the nondimensional specific annual insolation of the Earth  $Q_{t,nd,E,0}$  is 1.1087 insolation at the latitude of  $45^\circ$ , i.e., the dimensional specific annual insolation is equal to

$$Q_{t,E,d} = 1.1087 Q_{t0}^{45} . \quad (10)$$

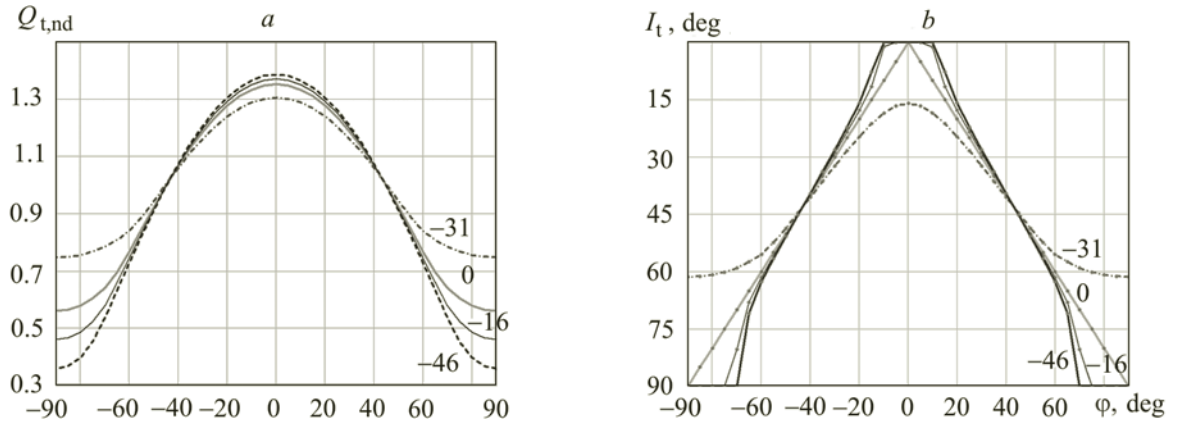


Fig. 2. Distribution of annual insolation  $Q_{t,nd}$  (a) and  $I_t$  (b) along the Earth latitude in the modern epoch  $T = 0$  kyr (0) and in three extreme epochs  $T = -15.88$  (-16),  $-31.28$  (-31), and  $-46.44$  kyr (-46).

TABLE 1. Specific Annual Insolation of the Earth and Hemispheres in Four Epochs at  $Q_{t0}^{45} = 9.932 \cdot 10^6$  kJ/m<sup>2</sup>

| $T$ , kyr | $Q_{t,nd,E}$ | $Q_{t,nd,NH}$ | $Q_{t,nd,SH}$ |
|-----------|--------------|---------------|---------------|
| 0         | 1.108676     | 1.108627      | 1.108725      |
| -15.88    | 1.108723     | 1.108670      | 1.108775      |
| -31.28    | 1.108677     | 1.108634      | 1.108720      |
| -46.44    | 1.108601     | 1.108546      | 1.108655      |

Third, the specific annual insolation across the hemispheres, in practice, does not differ from  $Q_{t,nd,E,0}$  for all epochs, excluding the Northern hemisphere of the coldest epoch 46.44 thousand years ago where it is 0.0002 lower. It should be noted that for the Southern hemisphere, in the four epochs under consideration,  $Q_{t,nd,SH}$  is 0.0001 higher than  $Q_{t,nd,NH}$  for the Northern hemisphere. However, this result has been obtained for four epochs, three of which are extreme, therefore, it cannot be extended to all epochs.

The total quantity of solar heat per year in a dimensional form across the entire Earth surface  $4\pi R_{E,m}^2$  is equal to

$$Q_{t,E,d} = 4\pi R_{E,m}^2 Q_{t,nd,E,0} Q_{t0}^{45} = 5.6105 \cdot 10^{21} \text{ kJ/year} . \quad (11)$$

The values  $Q_{t,nd,E,0}$  and  $Q_{t0}^{45}$  included in (11) have been determined as a result of summing of daily insolation during one revolution of the Earth on the orbit with respect to the Sun, i.e., during a tropical year  $P_{tr} = 365.24219879$  days. The daily insolation is determined with account for the distance of the Earth from the Sun, the angle of incidence by solar rays onto an area of the Earth surface at the latitude  $\phi$  and the duration of a light day.

The total quantity of heat per year can also be calculated in a different way. The radiation flux from the Sun falls on the diametric section of the Earth  $\pi R_{E,m}^2$ . Therefore, during the year  $P_{tr}$ , it will get the quantity of heat

$$Q_{t,E,d,D} = J_0 P_{tr,sc} \pi R_{E,m}^2 , \quad (12)$$

where  $P_{tr,sc} = P_{tr}(24 \cdot 3600)$  is a tropical year in seconds.

We determine the ratio between the influxes of heat onto the spherical surface of the Earth (11) and across its diametric section (12):

$$k_{sD} = \frac{Q_{t,E,d}}{Q_{t,E,d,D}} = \frac{4Q_{t,E,0} Q_{t0}^{45}}{J_0 P_{tr,sc}} = 1.00013 . \quad (13)$$

TABLE 2. Distribution of the Average Near-Surface Temperature of the Earth  $t_{oi}$ , Along the Earth Latitude  $\varphi$  in the Modern Epoch (30.12.1949, i.e., December 30, 1949), the Relative Heat Capacity  $C_{\varphi,r,i}$  and the Nonsimentional Annual Insulations  $Q_{t,nd}$  in the Five Epochs  $T$  at  $Q_{t0}^{45} = 9.932 \cdot 10^6$  GJ/m<sup>2</sup> and  $J_0 = 1.3956$  kW/m<sup>2</sup> = 2 cal/(cm<sup>2</sup>·min); the Time  $T$  is Counted from December 30, 1949

| $i$ | $\varphi_i^\circ$ | $t_{oi}, ^\circ\text{C}$ | $C_{\varphi,r,i}, 1/^\circ\text{C}$ | Nondimensional insolation $Q_{t,nd}$ in the epochs $T$ , kyr |          |          |          |          |
|-----|-------------------|--------------------------|-------------------------------------|--------------------------------------------------------------|----------|----------|----------|----------|
|     |                   |                          |                                     | 0                                                            | −4.16    | −15.88   | −31.28   | −46.44   |
| 1   | 90                | −25.3                    | 0.0082387                           | 0.561598                                                     | 0.565376 | 0.461674 | 0.748198 | 0.360717 |
| 2   | 85                | −21.30                   | 0.0098134                           | 0.566225                                                     | 0.569951 | 0.468165 | 0.750422 | 0.369929 |
| 3   | 80                | −17.42                   | 0.0115919                           | 0.580347                                                     | 0.583889 | 0.488071 | 0.757192 | 0.398582 |
| 4   | 75                | −13.45                   | 0.0137756                           | 0.604734                                                     | 0.607949 | 0.523033 | 0.768796 | 0.451937 |
| 5   | 70                | −9.44                    | 0.0167250                           | 0.641215                                                     | 0.643904 | 0.579104 | 0.785794 | 0.538128 |
| 6   | 65                | −5.59                    | 0.0180904                           | 0.696186                                                     | 0.697851 | 0.659872 | 0.809206 | 0.633598 |
| 7   | 60                | −1.48                    | 0.0196409                           | 0.769922                                                     | 0.770953 | 0.747047 | 0.841166 | 0.730021 |
| 8   | 55                | 1.87                     | 0.0206529                           | 0.848057                                                     | 0.848684 | 0.834373 | 0.888389 | 0.824123 |
| 9   | 50                | 5.52                     | 0.0210592                           | 0.925603                                                     | 0.925927 | 0.918946 | 0.945931 | 0.914011 |
| 10  | 45                | 9.33                     | 0.0211920                           | 0.999987                                                     | 1.00007  | 0.998978 | 1.00500  | 0.99837  |
| 11  | 40                | 13.54                    | 0.0211837                           | 1.06956                                                      | 1.06944  | 1.07320  | 1.06223  | 1.07619  |
| 12  | 35                | 16.62                    | 0.0211034                           | 1.13313                                                      | 1.13285  | 1.14065  | 1.11567  | 1.14664  |
| 13  | 30                | 19.85                    | 0.0209811                           | 1.18980                                                      | 1.18939  | 1.20053  | 1.16399  | 1.20902  |
| 14  | 25                | 22.82                    | 0.0208439                           | 1.23885                                                      | 1.23832  | 1.25222  | 1.20622  | 1.26277  |
| 15  | 20                | 24.92                    | 0.0206845                           | 1.27971                                                      | 1.27910  | 1.29520  | 1.24165  | 1.30740  |
| 16  | 15                | 26.19                    | 0.0205113                           | 1.31194                                                      | 1.31126  | 1.32905  | 1.26972  | 1.34251  |
| 17  | 10                | 26.71                    | 0.0202879                           | 1.33519                                                      | 1.33447  | 1.35345  | 1.29004  | 1.36781  |
| 18  | 5                 | 26.63                    | 0.0198350                           | 1.34924                                                      | 1.34849  | 1.36818  | 1.30235  | 1.38307  |
| 19  | 0                 | 26.50                    | 0.0204487                           | 1.35394                                                      | 1.35317  | 1.37310  | 1.30646  | 1.38817  |
| 20  | −5                | 26.16                    | 0.0210625                           | 1.34924                                                      | 1.34849  | 1.36818  | 1.30235  | 1.38307  |
| 21  | −10               | 25.47                    | 0.0206186                           | 1.33519                                                      | 1.33447  | 1.35345  | 1.29005  | 1.36781  |
| 22  | −15               | 24.30                    | 0.0204343                           | 1.31194                                                      | 1.31126  | 1.32905  | 1.26972  | 1.34251  |
| 23  | −20               | 22.92                    | 0.0203026                           | 1.27971                                                      | 1.27910  | 1.29520  | 1.24165  | 1.30740  |
| 24  | −25               | 20.82                    | 0.0201992                           | 1.23885                                                      | 1.23833  | 1.25223  | 1.20623  | 1.26278  |
| 25  | −30               | 18.27                    | 0.0200953                           | 1.18980                                                      | 1.18939  | 1.20054  | 1.16400  | 1.20903  |
| 26  | −35               | 15.33                    | 0.0199759                           | 1.13314                                                      | 1.13286  | 1.14065  | 1.11568  | 1.14664  |
| 27  | −40               | 12.03                    | 0.0198210                           | 1.06957                                                      | 1.06945  | 1.07321  | 1.06224  | 1.07619  |
| 28  | −45               | 8.91                     | 0.0195934                           | 0.999996                                                     | 1.00008  | 0.998986 | 1.00501  | 0.998376 |
| 29  | −50               | 5.38                     | 0.0192368                           | 0.925613                                                     | 0.925937 | 0.918955 | 0.945943 | 0.914017 |
| 30  | −55               | 1.79                     | 0.0186257                           | 0.848068                                                     | 0.848694 | 0.834383 | 0.888402 | 0.824130 |
| 31  | −60               | −2.85                    | 0.0174748                           | 0.769933                                                     | 0.770964 | 0.747057 | 0.841180 | 0.730029 |
| 32  | −65               | −7.20                    | 0.0157645                           | 0.696198                                                     | 0.697863 | 0.659882 | 0.809221 | 0.633605 |
| 33  | −70               | −12.23                   | 0.0139977                           | 0.641227                                                     | 0.643916 | 0.579115 | 0.785809 | 0.538135 |
| 34  | −75               | −16.54                   | 0.0120398                           | 0.604746                                                     | 0.607962 | 0.523045 | 0.768812 | 0.451945 |
| 35  | −80               | −22.00                   | 0.0095973                           | 0.580360                                                     | 0.583902 | 0.488082 | 0.757208 | 0.398590 |
| 36  | −85               | −26.49                   | 0.0081760                           | 0.566238                                                     | 0.569964 | 0.468176 | 0.750438 | 0.369937 |
| 37  | −90               | −31.12                   | 0.0069031                           | 0.561611                                                     | 0.565389 | 0.461686 | 0.748214 | 0.360725 |

As we can see from (13), the value  $k_{SD}$ , in practice, does not differ from 1. Therefore, the total quantity of heat supplied to the Earth from the Sun can be determined by a simple dependence (12) with an accuracy to 0.01%. The latitude distribution of the temperature  $t_{oi}$  and of the dimensional insulations  $Q_{t,nd}$  is shown in Table 2.

**Conclusions.** A generalized profile of the midlatitude annual temperature in the modern epoch has been proposed. The average temperatures of the Earth and hemispheres have been calculated. The profiles of the annual insolation in the modern and three extreme epochs of the past 50 thousand years have been analyzed. Annual insulations of the Earth and hemispheres in different epochs have been calculated, and their time invariance has been shown.

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## NOTATION

$D$ , diametric section of the Earth, km;  $J_0$ , solar constant, kW/m<sup>2</sup>;  $Q_t$ ,  $Q_s$ , and  $Q_w$ , insolation during the year and in the summer and winter periods, kW/m<sup>2</sup>;  $R_E$ , Earth radius, km;  $T$ , time, thousand years;  $t$ , temperature, °C;  $t_{oi}$ , generalized temperature distribution at the  $i$ th latitude according to observation data;  $\phi$ , latitude. Subscripts: a, approximation; d, dimensional; E, Earth; e, equatorial; f, finite; m, mean; nd, nondimensional; NH and SH, Northern and Southern hemispheres; o, observation; p, polar; r, relative; s, summer; t, total; tr, tropical; w, winter.

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