Orbit Evolution of Apophis and its Transformation into the Earth's Satellite

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As a result of the analysis of publications it is established that uncertainty of trajectories of Apophis are caused by imperfection of methods of its determination. The differential equations of motion of Apophis, planets, the Moon and the Sun are integrated by new numerical method and the evolution of the asteroid orbit is investigated. The Apophis will pass by the Earth at a distance of 6.1 its radii on April 13th, 2029. It will be its closest approach with the Earth during next 1000 years. A possibility of transformation of Apophis orbit to an orbit of the Earth's satellite, which can be used for various tasks, is considered.

1. Introduction

The background behind the problem we treat in the present study was recently outlined in [1]. On June 19 – 20, 2004, asteroid Apophis was discovered by astronomers at the Kitt Peak Observatory [2], and on December 20, 2004 this asteroid was observed for the second time by astronomers from the Siding Spring Survey Observatory [3]. Since then, the new asteroid has command international attention. First gained data on identification of Apophis' orbital elements were employed to predict the Apophis path. Following the first estimates, it was reported in [4] that on April 13, 2029 Apophis will approach the Earth center to a minimum distance of 38000 km. As a result of the Earth gravity, the Apophis orbit will alter appreciably. Unfortunately, presently available methods for predicting the travel path of extraterrestrial objects lack sufficient accuracy, and some authors have therefore delivered an opinion that the Apophis trajectory will for long remain unknown, indeterministic, and even chaotic (see [1, 4, 5]). Different statistical predictions points to some probability of Apophis' collision with the Earth on April 13, 2036. It is this aspect, the impact risk, which has attracted primary attention of workers dealing with the problem.

L.V. Rykhlova et al. [4] have attempted an investigation into the possibility of an event that the Apophis will closely approach the Earth. They also tried to evaluate possible threats stemming from this event. Various means to resist the fall of the asteroid onto Earth were put forward, and proposals for tracking Apophis missions, made. Finally, the need for prognostication studies of the Apophis path accurate to a one-kilometer distance for a period till 2029 was pointed out.

Many points concerning the prospects for tracking the Apophis motion with ground- and space-based observing means were discussed in [1, 4 - 6]. Since the orbits of the asteroid and Earth pass close to each other, then over a considerable portion of the Apophis orbit the asteroid disc will only be partially shined or even hidden from view. That is why it seems highly desirable to identify those periods during which the asteroid will appear accessible for observations with ground means. In using space-based observation means, a most efficient orbital allocation of such means needs to be identified.

Prediction of an asteroid motion presents a most challenging problem in astrophysics. In [7], the differential equations for the perturbed motion of the asteroid were integrated by the Everhart method [8]; in those calculations, for the coordinates of perturbing bodies were used the JPL planetary ephemeris DE403 and DE405 issued by the Jet Propulsion Laboratory, USA. Sufficient attention was paid to resonance phenomena that might cause the hypothetical 2036 Earth impact.

L.E. Bykova et al. [9, 10] used 933 observations to improve the identification accuracy for initial Apophis orbital parameters. Yet, the routine analysis has showed that, as a result of the pass of the asteroid through several resonances with Earth and Mars, the motion of the asteroid will probably become chaotic. With the aim to evaluate the probability of an event that Apophis will impact the Earth in 2036, the authors of [10] have made about 10 thousand variations of initial conditions, 13 of which proved to inflict a fall of Apophis onto Earth.

E.A. Smirnov [11] has attempted a test of various integration methods for evaluating their capabilities in predicting the motion of an asteroid that might impact the Earth. The Everhart method, the Runge-Kutta method of fourth order, the Yoshida methods of sixth and eighth orders, the Hermit method of fourth and sixth orders, the Multistep Predictor-Corrector (MS-PC) method of sixth and eighth orders, and the Parker-Sochacki method were analyzed. The Everhart and MS-PC methods proved to be less appropriate than the other methods. E.A. Smirnov pointed to the fact that, in the problems with singular points, finite-difference methods normally fail to accurately approximate higher-order derivatives. This conclusion is quite significant since below we will report on an integration method for motion equations free of such deficiencies.

It should be emphasized that, at close Apophis-to-Earth distances E.A. Smirnov used, instead of the Everhart method, the Runge-Kutta method [11]. Since the Everhart method was widely used in integrating differential motion equations for solar-system bodies, it is this method whose inaccuracies in treating encounter problems must be analyzed first of all.

In [12], mathematical problems on asteroid orbit prediction and modification were considered. Possibilities offered by the impact-kinetic and thermonuclear methods in correcting the Apophis trajectory were evaluated.

An in-depth study of the asteroid was reported in [1]. A chronologically arranged outline of observational history was given, and the trend with progressively reduced uncertainty region for Apophis' orbit-element values was traced. Much attention was paid to discussing the orbit prediction accuracy and the bias of various factors affecting this accuracy. The influence of uncertainty in planet coordinates and in the physical characteristics of the asteroid, and also the perturbing action of other asteroids, were analyzed. The effects on integration accuracy of digital length, non-spherical shape of Earth and Moon, solar-radiation-induced perturbations, non-uniform thermal heating, and other factors, were examined.

The equations of perturbed motion of the asteroid were integrated with the help of the Standard Dynamic Model (SDM), with the coordinates of other bodies taken from the JPL planetary ephemeris DE405. It is a well-known fact that the DE405 ephemerid was compiled as an approximation to some hundred thousand observations that were made till 1998. Following the passage to the ephemeris DE414, that approximates observational data till 2006, the error in predicting the Apophis trajectory on 2036 has decreased by 140 000 km. According to [1], this error proved to be several tens times greater than the errors induced by all minor perturbations. Note that this result points to the necessity of employing a more accurate method for predicting the asteroid path.

In [1], prospects for further refinement of Apophis' trajectory were discussed at length. Time periods suitable for optical and radar measurements, and also observational programs for oppositions with Earth in 2021 and 2029 and spacecraft missions for 2018 and 2027 were scheduled. Future advances in error minimization for asteroid trajectory due to the above activities were evaluated.

It should be noted that the ephemerides generated as an approximation to observational data enable rather accurate determination of a body's coordinates in space within the approximation time interval. The prediction accuracy for the coordinates on a moment remote from this interval worsens, the worsening being the greater the more the moment is distant from the approximation interval. Therefore, the observations and the missions scheduled in [1] will be used in refining future ephemerides.

In view of the afore-said, in calculating the Apophis trajectory the equations of perturbed motion were integrated [1, 7, 12], while the coordinates of other bodies were borrowed from the ephemerid. Difference integration methods were employed, which for closely spaced bodies yield considerable inaccuracies in calculating higher-order derivatives. Addition of minor interactions to the basic Newtonian gravitational action complicates the problem and enlarges the uncertainty region in predicting the asteroid trajectory. Many of the weak interactions lack sufficient quantitative substantiation. Moreover, the physical characteristics of the asteroid and the interaction constants are known to some accuracy. That is why in making allowance for minor interactions expert judgments were used. And, which is most significant, the error in solving the problem on asteroid motion with Newtonian interaction is several orders greater than the corrections due to weak additional interactions.

As it was shown in [1], some additional activities are required, aimed at further refinement of Apophis' trajectory. In this connection, more accurate determination of Apophis' trajectory is of obvious interest since, following such a determination, the range of possible alternatives would diminish.

For integration of differential motion equations of solar-system bodies over an extended time interval [13], a program "Galactica" was developed. In this program, only the Newtonian gravity force was taken into account, and no differences for calculating derivatives were used. In the problems for the composite model of Earth rotation [14] and for the gravity maneuver near Venus [15], motion equations with small body-to-body distances, the order of planet radius, were integrated. Following the solution of those problems and subsequent numerous checks of numerical data, we have established that, with the program "Galactica", we were able to rather accurately predict the Apophis motion over its travel path prior to and after the approach to the Earth. In view of this, in the present study we have attempted an investigation into Apophis' orbit evolution; as a result of this investigation, some fresh prospects toward possible use of this asteroid have opened.

2. Problem Statement

For the asteroid, the Sun, the planets, and the Moon, all interacting with one another by the Newton law of gravity, the differential motion equations have the form [16]:

$$\frac{d^2\mathbf{r}_i}{dt^2} = -G\sum_{k\neq i}^{n}\frac{m_k\mathbf{r}_{ik}}{r_{ik}^{\ 3}}, \quad i = 1,2,\ldots,n, \qquad (1)$$

where $\mathbf{r}_i$ is radius-vector of a body with mass m_i relatively Solar System barycenter; G is gravitational constant; $\mathbf{r}_{ik}$ is vector $\mathbf{r}_i - \mathbf{r}_k$ and r_{ik} is its module; n = 12.

As a result of numerical experiments and their analysis we came to a conclusion, that finite-difference methods of integration do not provide necessary accuracy. For the integration of equations (1) we have developed algorithm and program "Galactica". The meaning of function at the following moment of time $t=t_0 + \Delta t$ is determined with the help of Tailor series, which, for example, for coordinate x looks like:

$$x = x_0 + \sum_{k=1}^{K}\frac{1}{k!}x_0^{(k)}(\Delta t)^k \qquad (2)$$

where $x_0^{(k)}$ is derivative of k order at the initial moment t_0.

The meaning of velocity x' is defined by the similar formula, and acceleration x_0'' by the formula (1). Higher derivatives $x_0^{(k)}$ are determined analytically as a result of differentiation of the equations (1). The calculation algorithm of the sixth order is now used, i.e. with K=6.

3. Preparation of Initial Data

We consider the problem of interest in the barycentric coordinate system on epoch J2000.0, Julian day JD_s = 2451545. Apophis' orbital elements, such as the eccentricity e, the semi-major axis a, the ecliptic obliquity i_e, the ascending node angle Ω, the ascending node-perihelion angle ω_e, etc., and Apophis' position elements, such as the mean anomaly M, were borrowed from the

JPL Small-Body Database [17] as specified on November 30.0, 2008. The data, represented to 16 decimal digits, are given in Table 1. In [17], the uncertainty regions for those data were also given. The relative uncertainty value δ is in the range from $2.4\cdot10^{-8}$ to $8\cdot10^{-7}$. The same data are cited in the asteroid database by Edward Bowell [18], although in [18] these data are represented only to 8 decimal digits, and they differ from the former data in the 7-th digit, i.e., within δ. In [1], the orbital elements of Apophis on epoch JD = 2453979.5 (September 01.0, 2006) were used, which, except for M, differ from the element values in Table 1 in the 4th or 5th digit. The latter difference is three or four orders greater than the uncertainty δ.

Element	Value	Units
e	.1912119299890948	
a	.9224221637574083	AU
q	.7460440415606373	AU
i_e	3.331425002325445	deg
Ω	204.4451349657969	deg
ω_e	126.4064496795719	deg
M	254.9635275775066	deg
t_p	2454894.912750123770 (2009-Mar-04.41275013)	JD d
P	323.5884570441701 0.89	d yr
n	1.112524233059586	deg/d
Q	1.098800285954179	AU

Table 1. Orbital elements of asteroid Apophis [17] on epoch JD_0 = 2454800.5 (November 30.0, 2008) in the heliocentric ecliptic coordinate system of 2000.0 with JD_S = 2451545.

The element values in Table 1 were used to calculate the Cartesian coordinates of Apophis and the Apophis velocity in the barycentric equatorial system by the following algorithm [13, 20].

From the Kepler equation

$$E - e\cdot\sin E = M \tag{3}$$

we calculate the eccentric anomaly E and, then, from E, the true anomaly φ_0:

$$\varphi_0 = 2\cdot\mathrm{arctg}[\sqrt{(1+e)/(1-e)}\cdot\mathrm{tg}(0.5\cdot E)] \tag{4}$$

In subsequent calculations, we used results for the two-body interaction (the Sun and the asteroid) [15, 20]. The trajectory equation of the body in a polar coordinate system with origin at the Sun has the form:

$$r = \frac{R_p}{(\alpha_1+1)\cos\varphi - \alpha_1} \tag{5}$$

where the polar angle φ , or, in astronomy, the true anomaly, is reckoned from the perihelion position $r = R_p$; $\alpha_1 = -1/(1+e)$ is the trajectory parameter; and $R_p = a\cdot(2\alpha_1+1)/\alpha_1$ is the perihelion radius. The expressions for the radial υ_r and transversal υ_t velocities are

$$\upsilon_r = \upsilon_p\sqrt{(\alpha_1+1)^2 - (\alpha_1 + 1/\bar{r})^2},$$

$$\text{for } \varphi > \pi \quad \upsilon_r < 0; \quad \upsilon_t = \upsilon_p/\bar{r} \tag{6}$$

where $\bar{r} = r/R_p$ is the dimensionless radius, and the velocity at perihelion is

$$\upsilon_p = \sqrt{G(m_S + m_{As})/(-\alpha_1)R_p)} \tag{7}$$

where $m_S = m_{11}$ is the Sun mass (the value of m_{11} is given in Table 2), and $m_{As}=m_{12}$ is the Apophis mass.

The time during which the body moves along an elliptic orbit from the point of perihelion to an orbital position with radius $\bar{r}$ is given by

$$t = \frac{R_p}{\upsilon_p}\left[\frac{\bar{r}\,|\,\bar{\upsilon}_r\,|}{2\alpha_1+1} - \frac{\alpha_1\left(\pi/2 + \arcsin\{[(2\alpha_1+1)\bar{r} - \alpha_1]/(-\alpha_1-1)\}\right)}{(-2\alpha_1-1)^{3/2}}\right] \tag{8}$$

where $\bar{\upsilon}_r = \upsilon_r/\upsilon_p$ is the dimensionless radial velocity.

At the initial time $t_0 = 0$, which corresponds to epoch JD_0 (see Table 1), the polar radius of the asteroid r_0 as dependent on the initial polar angle, or the true anomaly φ_0, can be calculated by formula (5). The initial radial and initial transversal velocities as functions of r_0 can be found using formulas (6).

The Cartesian coordinates and velocities in the orbit plane of the asteroid (the axis x_o goes through the perihelion) can be calculated by the formulas

$$x_o = r_0\cdot\cos\varphi_0; \qquad y_o = r_0\cdot\sin\varphi_0 \tag{9}$$

$$\upsilon_{xo} = \upsilon_r\cdot\cos\varphi_0 - \upsilon_t\cdot\sin\varphi_0; \quad \upsilon_{yo} = \upsilon_r\cdot\sin\varphi_0 + \upsilon_t\cdot\cos\varphi_0 \tag{10}$$

The coordinates of the asteroid in the heliocentric ecliptic coordinate system can be calculated as

$$x_e = x_o\cdot(\cos\omega_e\cdot\cos\Omega - \sin\omega_e\cdot\sin\Omega\cdot\cos i_e) - y_o\cdot(\sin\omega_e\cdot\cos\Omega + \cos\omega_e\cdot\sin\Omega\cdot\cos i_e) \tag{11}$$

$$y_e = x_o\cdot(\cos\omega_e\cdot\sin\Omega - \sin\omega_e\cdot\cos\Omega\cdot\cos i_e) - y_o\cdot(\sin\omega_e\cdot\sin\Omega - \cos\omega_e\cdot\cos\Omega\cdot\cos i_e) \tag{12}$$

$$z_e = x_o\sin\omega_e\cdot\sin i_e + y_o\cdot\cos\omega_e\cdot\sin i_e \tag{13}$$

The velocity components of the asteroid $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} in this coordinate system can be calculated by formulas analogous to (11) – (13).

Since equations (1) are considered in a motionless equatorial coordinate system, then elliptic coordinates (11) – (13) can be transformed into equatorial ones by the formulas

$$x = x_e; \quad y = y_e\cdot\cos\varepsilon_0 - z_e\cdot\sin\varepsilon_0; \quad z = y_e\cdot\sin\varepsilon_0 + z_e\cdot\sin\varepsilon_0 \tag{14}$$

where ε_0 is the angle between the ecliptic and the equator in epoch JD_S.

The velocity components $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} can be transformed into the equatorial ones υ_x, υ_y and υ_z by formulas analogous to (14).

In the calculations, six orbital elements from Table 1, namely, e, a i_e, Ω, ω_e, and M, were used. Other orbital elements were used for testing the calculated data. The perihelion radius R_p and the aphelion radius $R_a = -R_p/(2\alpha_1+1)$ were compared to q and Q, respectively. The orbital period was calculated by formula (8) as twice the time of motion from perihelion to aphelion ($r = R_a$). The same formula was used to calculate the moment at which the asteroid passes the perihelion ($r = r_0$). The calculated values of those quantities were compared to the values of P and t_p given in Table 1. The largest relative difference in terms of q and Q was within $1.9\cdot10^{-16}$, and in terms of P and t_p, within $8\cdot10^{-9}$.

The coordinates and velocities of the planets and the Moon on epoch JD_0 were calculated by the DE406/LE406 JPL-theory [21, 22]. The masses of those bodies were modified in [13], and the Apophis mass was calculated assuming the asteroid to be a ball of diameter d = 270 m and density ρ = 3000 kg/m³. The masses of all bodies and the initial conditions are given in Table 2.

The starting-data preparation and testing algorithm (3) - (14) was embodied as a MathCad worksheet (program Ast-Coor2.mcd).

Bodies, j	Bodies masses in *kg*, their coordinates in *m* and velocities in *m/s*			
	m_{bj}	x_j, v_{xj},	y_j, v_{yj}	z_j, v_{zj}
1	3.30187842779737E+23	-17405931955.9539	-60363374194.7243	-30439758390.4783
		37391.7107852059	-7234.98671125365	-7741.83625612424
2	4.86855338156022E+24	108403264168.357	-2376790191.8979	-7929035215.64079
		1566.99276862423	31791.7241663148	14204.3084779893
3	5.97369899544255E+24	55202505242.89	125531983622.895	54422116239.8628
		-28122.5041342966	10123.4145376039	4387.99294255716
4	6.4185444055007E+23	-73610014623.8562	-193252991786.298	-86651102485.4373
		23801.7499674501	-5108.24106287744	-2985.97021694235
5	1.89900429500553E+27	377656482631.376	-609966433011.489	-270644689692.231
		11218.8059775149	6590.8440254003	2551.89467211952
6	5.68604198798257E+26	-1350347198932.98	317157114908.705	189132963561.519
		-3037.18405985381	-8681.05223681593	-3454.56564456648
7	8.68410787490547E+25	2972478173505.71	-397521136876.741	-216133653111.407
		979.784896813787	5886.28982058747	2564.10192504801
8	1.02456980223201E+26	3605461581823.41	-2448747002812.46	-1092050644334.28
		3217.00932811768	4100.99137103454	1598.60907148943
9	1.65085753263927E+22	53511484421.7929	-4502082550790.57	-1421068197167.72
		5543.83894965145	-290.586427181992	-1757.70127979299
10	7.34767263035645E+22	55223150629.6233	125168933272.726	54240546975.7587
		-27156.1163326908	10140.7572420768	4468.97456956941
11	1.98891948976803E+30	0	0	0
		0	0	0
12	30917984100.3039	-133726467471.667	-60670683449.3631	-26002486763.62
		16908.9331065445	-21759.6060221801	-7660.90393288287

Table 2. The masses m_{bj} of the planets from Mercury to Pluto, the Moon, the Sun, and Apophis, and the initial condition on epoch JD_0 = 2454800.5 (November 30.0, 2008) in the heliocentric equatorial coor dinate system on epoch 2000.0, JD_S = 2451545. G = 6.67259E-11 m³/(s²·kg).

4. Apophis' Encounter with the Planets and the Moon

In the program "Galactica", a possibility to determine the minimum distance R_{min} to which the asteroid approaches a celestial body over a given interval ΔT was provided. Here, we integrated equations (1) with the initial conditions indicated in Table 2. The integration was performed on the NKS-160 supercomputer at the Computing Center SB RAS, Novosibirsk. In the program «Galactica», an extended digit length (34 decimal digits) was used, and for the time step a value $dT = 10^{-5}$ year was adopted. The computations were performed over three time intervals, 0 ÷ 100 years (Fig. 1, *a*), 0 ÷ -100 years (Fig. 1, *b*), and 0 ÷ 1000 years (Fig. 1, *c*).

In the graphs of Fig. 1 the points connected with the heavy line show the minimal distances R_{min} to which the asteroid approaches the bodies indicated by points embraced by the horizontal line. In other words, a point in the line denotes a minimal distance to which, over the time ΔT = 1 year, the asteroid will approach a body denoted by the point in the horizontal line at the same moment. It is seen from Fig. 1, *a* that, starting from November 30.0, 2008, over the period of 100 years there will be only one Apophis' approach to the Earth (point A) at the moment T_A = 0.203693547133403 century to a minimum distance R_{minA} = 38906.9 km. A next approach (point B) will be to the Earth as well, but at the moment T_B = 0.583679164042455 century to a minimum distance R_{minB} = 622231 km, which is 16 times greater than the minimum distance at the first approach. Among all the other bodies, a closest approach with be to the Moon (point D) (see Fig. 1, *b*) at T_D = -0.106280550824626 century to a minimum distance R_{minD} =3545163 km.

In the graphs of Figs. 1, *a* and *b* considered above, the closest approaches of the asteroid to the bodies over time intervals ΔT = 1 year are shown. In integrating equations (1) over the 1000-year interval (see Fig. 1, *c*), we considered the closest approaches of the asteroid to the bodies over time intervals ΔT = 10 years. Over those time intervals, no approaches to Mercury and Mars were identified; in other words, over the 10-year intervals the asteroid closes with other bodies. Like in Fig. 1, *a*, there is an approach to the Earth at the moment T_A. A second closest approach is also an approach to the Earth at the point E at T_E = 5.778503 century to a minimum distance R_{minE} = 74002.9 km. During the latter approach, the asteroid will pass the Earth at a minimum distance almost twice that at the moment T_A.

With the aim to check the results, equations (1) were integrated over a period of 100 years with double digit length (17 decimal digits) and the same time step, and also with extended digit length and a time step $dT = 10^{-6}$ year. The integration accuracy (see Table 3) is defined (see [13]) by the relative change of δM_z, the z-projection of the angular momentum of the whole solar system for the 100-year period. As it is seen from Table 3, the quantity δM_z varies from $-4.5\cdot 10^{-14}$ to $1.47\cdot 10^{-26}$, i.e., by 12 orders of magnitude. In the last two columns of Table 3, the difference between the moments at which the asteroid most closely approaches the Earth at point A (see Fig. 1, *a*) and the difference between the approach distances relative to solution 1 are indicated. In solution 2, obtained with the short digit length, the approach moment has not changed, whereas the minimum distance has reduced by 2.7 m. In solution 3, obtained with ten times reduced integration step, the approach moment has changed by $-2\cdot 10^{-6}$ year, or by -1.057'. This change being smaller than the step $dT = 1\cdot 10^{-5}$ for solution 1 and being equal twice the step for solution 3, the value of this change provides a refinement for the approach moment. Here, the refinement for the closest approach distance by -1.487 km is also obtained.

№ solution	L_{nb}	dT, yr	δM_z	T_{Ai}-T_A, yr	R_{minAi}-R_{minA}, km
1	34	$1\cdot 10^{-5}$	$1.47\cdot 10^{-21}$	0	0
2	17	$1\cdot 10^{-5}$	$-4.5\cdot 10^{-14}$	0	$-2.7\cdot 10^{-3}$
3	34	$1\cdot 10^{-6}$	$1.47\cdot 10^{-26}$	$-2\cdot 10^{-6}$	-1.487

Table 3. Comparison between the data on Apophis' encounter with the Earth obtained with different integration accuracies: L_{nb} is the digit number in decimal digits.

Since all test calculations were performed considering the parameters of solution 1, it follows from here that the data that will be presented below are accurate in terms of time within 1', and in terms of distance, within 1.5 km. We emphasize here that the graphical data of Fig. 1, *a* for solutions 1 and 3 are perfectly coincident. The slight differences of solution 2 from solutions 1 and 3 are observed for $T > 0.87$ century.

So, during the forthcoming one-hundred-year period the asteroid Apophis will most closely approach the Earth only. This event will occur at the time T_A counted from epoch JD_0. The approach refers to the Julian day JD_A = 2462240.406809 and calendar date April 13, 2029, 21 hour 45'47" GMT. The asteroid will pass at a minimum distance of 38907 km from the Earth center, i.e., at a distance of 6.1 of Earth radii. A next approach of Apophis to the Earth will be on the 578-th year from epoch JD_0; at that time, the asteroid will pass the Earth at an almost twice greater distance.

The calculated time at which Apophis will close with the Earth, April 13, 2029, coincides with the approach times that were obtained in other reported studies. For instance, in the recent publication [1] this moment is given accurate to one minute: 21 hour 45' UTC, and the geocentric distance was reported to be in the range from 5.62 to 6.3 Earth radii, the distance of 6.1 Earth radii falling into the latter range. The good agreement between the data obtained by different methods proves the obtained data to be quite reliable. On the other hand, the calculations were performed assuming slightly different values of Apophis' initial orbital elements, the difference being in the fourth-fifth digit. We therefore believe that subsequent refinements of orbit-element values will yield no substantial corrections for the 2029 approach data.

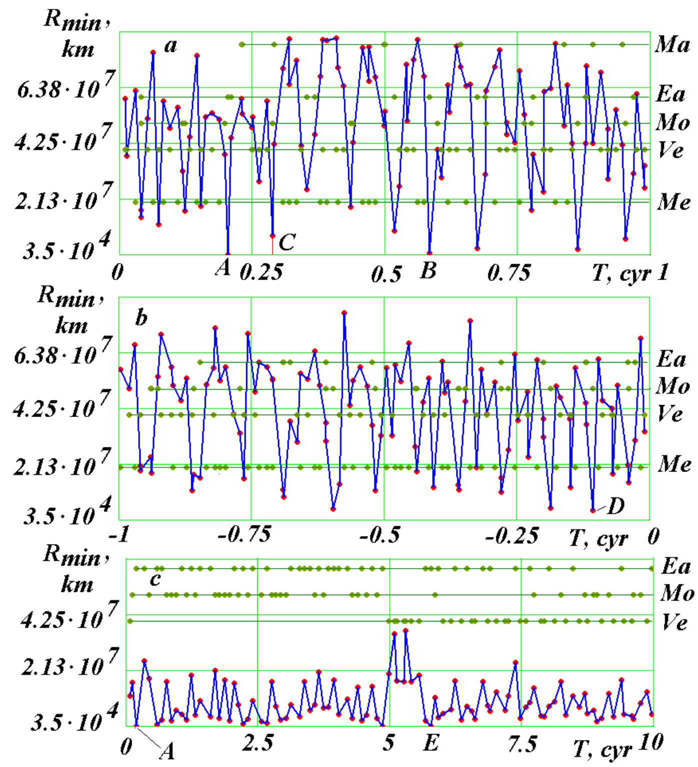

Fig. 1. Apophis' encounters with celestial bodies during the time ΔT to a minimum distance R_{min}, km: Mars (Ma), Earth (Ea), Moon (Mo), Venus (Ve) and Mercury (Me); *a*, *b* – ΔT = 1 year; *c* – ΔT = 10 years. *T*, cyr is the time in Julian centuries from epoch JD_0 (November 30.0, 2008).

As for the possible approach of Apophis to the Earth in 2036, there will be no such an approach (see Fig. 1, *a*). A time-closest Apophis' approach at the point *F* to a minimum distance of 7.26 million km will be to the Moon, September 5, 2037.

5. Apophis Orbit Evolution

In integrating motion equations (1) over the interval –1 century ≤ T ≤ 1 century the coordinates and velocities of the bodies after a lapse of each one year were recorded in a file, so that a total of 200 files for a one-year time interval were obtained. Then, the data contained in each file were used to integrate equations (1) again over a time interval equal to the orbital period of Apophis and, following this, the coordinates and velocities of the asteroid, and those of Sun, were also saved in a new file. These data were used in the program DefTra to determine the parameters of Apophis' orbit relative to the Sun in the equatorial coordinate system. Such calculations were performed hands off for each of the 200 files under the control of the program PaOrb. Afterwards, the angular orbit parameters were recalculated into the ecliptic coordinate system (see Fig. 2).

As it is seen from Fig. 2, the eccentricity e of the Apophis orbit varies non-uniformly. It shows jumps or breaks. A most pronounced break is observed at the moment T_A, at which Apophis most closely approaches the Earth. A second most pronounced break is observed when Apophis approaches the Earth at the moment T_B.

The longitude of ascending node Ω shows less breaks, exhibiting instead rather monotonic a decrease (see Fig. 2). Other orbital elements, namely, i_e, ω_e, a, and P, exhibit pronounced breaks at the moment of Apophis' closest pass near the Earth (at the moment T_A).

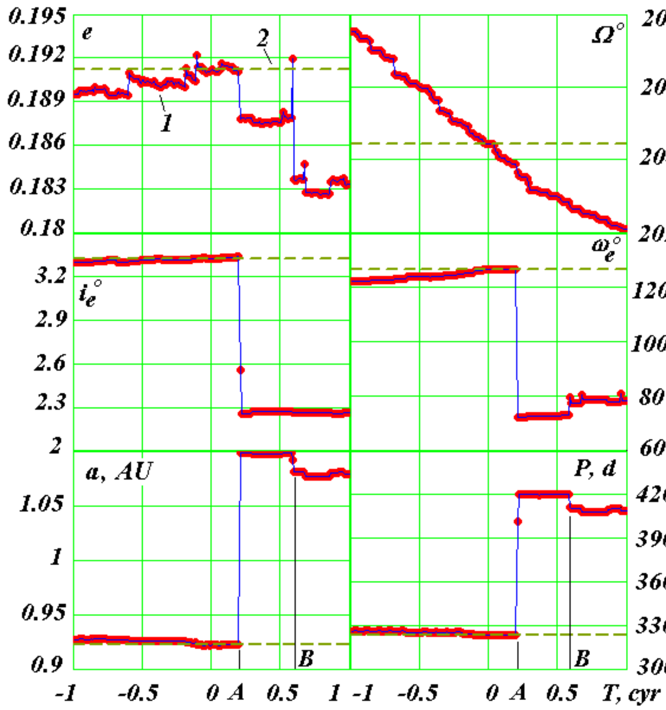

Fig. 2. Evolution of Apophis' orbital parameters under the action of the planets, the Moon and the Sun over the time interval -100 years ÷ +100 years from epoch November 30.0, 2008: *1* – as revealed through integration of motion equations (1); 2 – initial values according to Table 1. The angular quantities: Ω, i_e, and ω_e are given in degrees; the major semi-axis a in AU; and the orbital period P in days.

The dashed line in Fig. 2 indicates the orbit-element values at the initial time, also indicated in Table 1. As it is seen from the graphs, those values coincide with the values obtained by integration of equations (1), the relative difference of e, Ω, i_e, ω_e, a, and P from the initial values at the moment $T=0$ (see Table 1) being respectively $9.4 \cdot 10^{-6}$, $-1.1 \cdot 10^{-6}$, $3.7 \cdot 10^{-6}$, $-8.5 \cdot 10^{-6}$, $1.7 \cdot 10^{-5}$, and $3.1 \cdot 10^{-5}$. This coincidence testifies the reliability of computed data at all calculation stages, including the determination of initial conditions, integration of equations, determination of orbital parameters, and transformations between the different coordinate systems.

As it was mentioned in Introduction, apart from non-simplified differential equations (1) for the motion of celestial bodies, other equations were also used. It is a well-known fact that in perturbed-motion equations orbit-element values are used. For this reason, such equations will yield appreciable errors in determination of orbital-parameter breaks similar to those shown in Fig. 2. Also, other solution methods for differential equations exist, including those in which expansions with respect to orbital elements or difference quotients are used. As it was already mentioned in Introduction, these methods proved to be sensitive to various resonance phenomena and sudden orbit changes observed on the approaches between bodies. Equations (1) and method (2) used in the present study are free of such shortcomings. This suggests that the results reported in the present paper will receive no notable corrections in the future.

6. Examination of Apophis' Trajectory in the Vicinity of Earth

In order to examine the Apophis trajectory in the vicinity of Earth, we integrated equations (1) over a two-year period starting from T_1 = 0.19 century. Following each 50 integration steps, the coordinate and velocity values of Apophis and Earth were recorded in a file. The moment T_A at which Apophis will most closely approach the Earth falls into this two-year period. The ellipse E_0E_1 in Fig. 3 shows the projection of the two-year Earth's trajectory onto the equatorial plane xOy. Along this trajectory, starting from the point E_0, the Earth will make two turns. The two-year Apophis trajectory in the same coordinates is indicated by points denoted with the letters Ap. Starting from the point Ap_0, Apophis will travel the way $Ap_0Ap_1Ap_eAp_2Ap_0Ap_1$ to most closely approach the Earth at the point Ap_e at the time T_A. After that, the asteroid will follow another path, namely, the path $Ap_eAp_3Ap_f$.

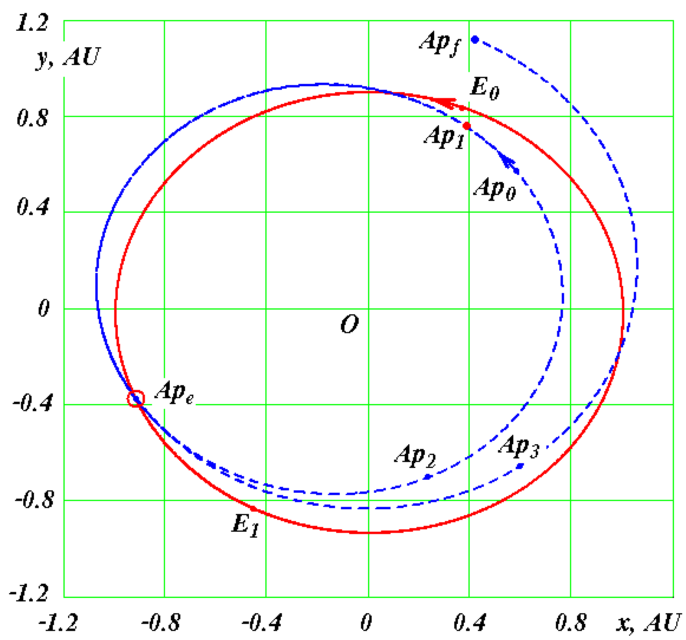

Fig. 3. The trajectories of Apophis (Ap) and Earth (E) in the barycentric equatorial coordinate system xOy over a two-year period: Ap_0 and E_0 are the initial position of Apophis and Earth; Ap_f is the end point of the Apophis trajectory; Ap_e is the point at which Apophis most closely approaches the Earth; the coordinates x and y are given in AU.

Figure 4, *a* shows the trajectory of Apophis relative to the Earth. Here, the relative coordinates are determined as the difference between the Apophis (Ap) and Earth (E) coordinates:

$$y_r = y_{Ap} - y_E; \qquad x_r = x_{Ap} - x_E. \tag{15}$$

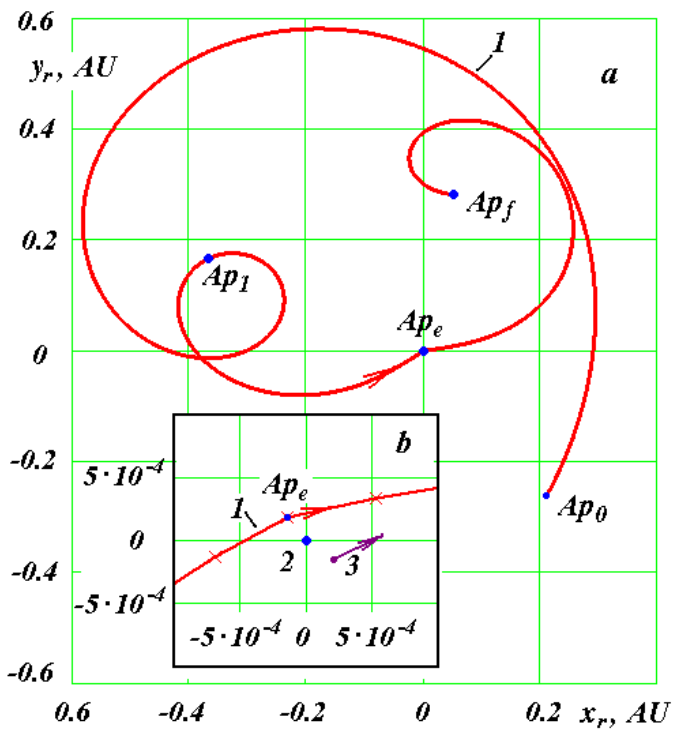

Fig. 4. Apophis' trajectory (1) in the geocentric equatorial coord i-nate system x_rOy_r: *a* – on the normal scale, *b* – on magnified scale on the moment of Apophis' closest approaches to the Earth (2); *3* – Apophis' position at the moment of its closest approach to the Earth following the correction of its trajectory with factor k = 0.9992 at the point Ap_1; the coordinates x_r and y_r are given in AU.

Along trajectory 1, starting from the point Ap_0, Apophis will travel to the Earth-closest point Ap_e, the trajectory ending at the point Ap_f. The loops in the Apophis trajectory represent a reverse motion of Apophis with respect to Earth. Such loops are made by all planets when observed from the Earth [20].

At the Earth-closest point Ap_e the Apophis trajectory shows a break. In Fig. 4, *b* this break is shown on a larger scale. Here, the Earth is located at the origin, point 2. The Sun (see Fig. 3) is located in the vicinity of the barycenter *O*, i.e., in the upper right quadrant of the Earth-closest point Ap_e. Hence, the Earth-closest point will be passed by Apophis as the latter will move in between the Earth and the Sun (see Fig. 4, *b*). As it will be shown below, this circumstance will present certain difficulties for possible use of the asteroid.

7. The Transformation of Asteroid Apophis into the Earth's Satellite

So, on April 13, 2029, we will become witnesses of a unique phenomenon, the pass of a body 31 million tons in mass near the Earth at a minimum distance of 6.1 Earth radii from the center of Earth. Over subsequent 1000 years, Apophis will never approach our planet closer.

Many space-travel scientists, for instance, K.E. Tsiolkovsky and Yu.A. Kondratyuk, believed that the near-Earth space will be explored using large manned orbital stations. Yet, delivering heavy masses from Earth into orbit presents a difficult engineering and ecological problem. For this reason, the lucky chance to turn the asteroid Apophis into an Earth bound satellite and, then, into a habited station presents obvious interest.

Among the possible applications of a satellite, the following two will be discussed here. First, a satellite can be used to create a space lift. It is known that a space lift consists of a cable tied with one of its ends to a point at the Earth equator and, with the other end, to a massive body turning round the Earth in the equatorial plane in a 24-hour period, P_d = 24·3600 sec. The radius of the satellite geostationary orbit is

$$R_{gs} = \sqrt[3]{P_d^2 G(m_A + m_E)/4\pi^2} = 42241 \text{ km} = 6.62\, R_{Ee} \quad (16)$$

In order to provide for a sufficient cable tension, the massive body needs to be spaced from the Earth center a distance greater than R_{gs}. The cable, or several such cables, can be used to convey various goods into space while other goods can be transported back to the Earth out of space.

If the mankind will become able to make Apophis an Earth bound satellite and, then, deflect the Apophis orbit into the equatorial plane, then the new satellite would suit the purpose of creating a space lift.

A second application of an asteroid implies its use as a "shuttle" for transporting goods to the Moon. Here, the asteroid is to have an elongated orbit with a perihelion radius close to that of a geostationary orbit and an apogee radius approaching the perigee radius of the lunar orbit. In the latter case, at the geostationary-orbit perigee goods would be transferred onto the satellite Apophis and then, at the apogee, those goods would arrive at the Moon.

The two applications will entail the necessity of solving many difficult problems which may finally prove unsolvable. On the other hand, none of those problems will be solved at all without making Apophis an Earth satellite. Consider now the possibilities available here.

The velocity of the asteroid relative to the Earth at the Earth-closest point Ap_e is v_{AE} =7.39 km/s. The velocity of an Earth bound satellite orbiting at a fixed distance R_{minA} from the Earth (circular orbit) is

$$v_{CE} = \sqrt{G(m_A + m_E)/R_{\min A}} = 3.2 \text{ km/s} \quad (17)$$

For the asteroid to be made an Earth-bound satellite, its velocity v_{AE} should be brought close to v_{CE}. We performed integration of equations (1) assuming the Apophis velocity at the moment T_A to be reduced by a factor of 1.9, i.e., the velocity v_{AE} =7.39 km/s at the moment T_A was decreased to 3.89 km/s. In the later case, Apophis becomes an Earth bound satellite with the following orbit characteristics: eccentricity e_{s1} = 0.476, equator-plane inclination angle i_{s1} = 39.2°, major semi-axis a_{s1} = 74540 km, and sidereal orbital period P_{s1} = 2.344 days.

We examined the path evolution of the satellite for a period of 100 years. In spite of more pronounced oscillations of the orbital elements of the satellite in comparison with those of planetary orbit elements, the satellite's major semi-axis and orbital period proved to fall close to the indicated values. For the relative variations of the two quantities, the following estimates were obtained: $|\delta a| < \pm 2.75\cdot10^{-4}$ and $|\delta P| < \pm 4.46\cdot10^{-4}$. Yet, the satellite orbits in a direction opposite both to the Earth rotation direction and the direction of Moon's orbital motion. That is why the two discussed applications of such a satellite turn to be impossible.

Thus, the satellite has to orbit in the same direction in which the Earth rotates. Provided that Apophis (see Fig. 4, *b*) will round the Earth from the night-side (see point *3*) and not from the day-side (see line *1*), then, on a decrease of its velocity the satellite will be made a satellite orbiting in the required direction.

For this matter to be clarified, we have integrated equations (1) assuming different values of the asteroid velocity at the point Ap_1 (see Fig. 3). This point, located at half the turn from the Earth-closest point Ap_e, will be passed by Apophis at the time T_{Ap1}=0.149263369488169 century. At the point Ap_1 the projections of the Apophis velocity in the barycentric equatorial coordinate system are v_{Ap1x} = -25.6136689 km/s, v_{Ap1y} = 17.75185451 km/s, and v_{Ap1z} = 5.95159206 km/s. In the numerical experiments, the component values of the satellite velocity were varied to one and the same proportion by multiplying all them by a single factor *k*, and then equations (1) were integrated to determine the trajectory of the asteroid. Figure 5 shows the minimum distance to which Apophis will approach the Earth versus the value of *k* by which the satellite velocity at the point Ap_1 was reduced.

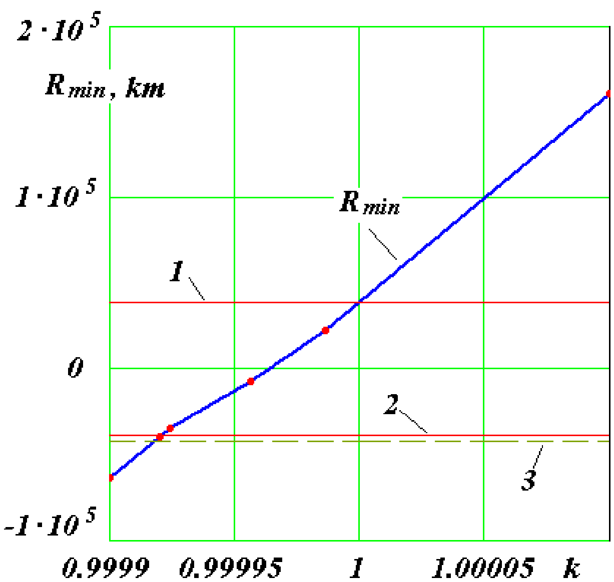

Fig. 5. The minimum distance R_{min} to which Apophis will approach the Earth center versus the value of *k* (*k* is the velocity reduction factor at the point A_{p1} (see Fig. 3)). The positive values of R_{min} refer to the day-side: the values of R_{min} are given in km; *1* – the minimum distance to which Apophis will approach the Earth center on April 13, 2029 (day -side); *2* – the minimum distance to which Apophis will approach the Earth center after the orbit correction (night-side); *3* – geostationary orbit radius R_{gs}.

We found that, on decreasing the value of *k* (see Fig. 5), the asteroid will more closely approach the Earth, and at *k* = 0.9999564 Apophis will collide with the Earth. On further decrease of asteroid velocity the asteroid will close with the Earth on the Sun-opposite side, and at *k* = 0.9992 the asteroid will approach the Earth center (point 3 in Fig. 4, *b*) to a minimum distance R_{min3} = 39157 km at the time T_3 = 0.2036882 century. This distance R_{min3} roughly equals the distance R_{minA} to which the asteroid was found to approach the Earth center while moving in between the Earth and the Sun.

In this case, the asteroid velocity relative to the Earth is also v_{AE} =7.39 km/s. On further decrease of this velocity by a factor of 1.9, i.e., down to 3.89 km/s Apophis will become an Earth bound satellite with the following orbit parameters: eccentricity e_{s2} = 0.486, equator plane inclination angle i_{s2} = 36°, major semi-axis a_{s2} = 76480 km, and sidereal period P_{s2} = 2.436 day. In addition, we investigated into the path evolution of the Earth bound satellite over a 100-year period. The orbit of the satellite proved to be stable, the satellite orbiting in the same direction as the Moon does.

Thus, for Apophis to be made a near-Earth satellite orbiting in the required direction, two decelerations of its velocity need to be implemented. The first deceleration is to be effected prior to the Apophis approach to the Earth, for instance, at the point Ap_1 (see Fig. 3), 0.443 year before the Apophis approach to the Earth. Here, the Apophis velocity needs to be decreased by 2.54 m/s. A second deceleration is to be effected at the moment the asteroid closes with the Earth. In the case under consideration, in which the asteroid moves in an elliptic orbit, the asteroid velocity needs to be decreased by 3.5 km/s.

Slowing down a body weighing 30 million tons by 3.5 km/s is presently a difficult scientific and engineering problem. For instance, in [4] imparting Apophis with a velocity of 10^{-6} m/s was believed to be a problem solvable with presently available engineering means. On the other hand, the authors of [4] consider increasing the velocity of such a body by about 1-2 cm/s a difficult problem. Yet, with Apophis being on its way to the Earth, we still have a twenty-year leeway. After the World War II, even more difficult a problem, that on injection of the first artificial satellite in near-Earth orbit and, later, the launch of manned space vehicles, was successfully solved in a period of ten years. That is why we believe that, with consolidated efforts of mankind, the objective under discussion will definitely be achieved.

It should be emphasized that the authors of [1] considered the possibility of modifying the Apophis orbit for organizing its impact onto asteroid (144898) 2004 VD17. There exists a small probability of the asteroid's impact onto the Earth in 2102. Yet, the problem on reaching a required degree of coordination between the motions of the two satellites presently seems to be hardly solvable. This and some other examples show that many workers share an opinion that substantial actions on the asteroid are necessary for making the solution of the various space tasks a realistic program.

Conclusions

1. Through an analysis of literature sources, deficiencies of the previous calculation methods for asteroid motion were revealed, including: (i) inaccuracies in calculating difference quotients, (ii) the use of ephemerides outside their approximation interval, (ii) effects induced by resonances in simplified differential motion equations, (iv) sensitivity of perturbed motion equations to orbit-element breaks.

2. A method free of such deficiencies was used to numerically integrate non-simplified motion equations of Apophis, the planets, the Moon, and the Sun over a 1000-year period.

3. In the course of test computations, through a comparison with results obtained by other authors, reliability of obtained data was confirmed.

4. On 21 hour 45' GMT, April 13, 2029 Apophis will pass close to the Earth, at a minimum distance of 6.1 Earth radii from Earth's center. This will be the closest pass of Apophis near the Earth in the forthcoming one thousand years.

5. Calculations on making Apophis an Earth bound satellite appropriate for solving various space exploration tasks were performed.

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