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Chapter 3

ASTEROIDS APOPHIS AND 1950 DA: 1000 YEARS ORBIT EVOLUTION AND PSSIBLE USE

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Abstract

The evolution of movement and possible use two asteroids is examined: Apophis and 1950 DA. As a result of the analysis of publications it is established that uncertainty of trajectories of Apophis are caused by imperfection of methods of its determination. The differential equations of motion of Apophis, planets, the Moon and the Sun are integrated by new numerical method and the evolution of the asteroid orbit is investigated. The Apophis will pass by the Earth at a distance of 6.1 its radii on April 13th, 2029. It will be its closest approach with the Earth during next 1000 years. A possibility of transformation of Apophis orbit to an orbit of the Earth's satellite, which can be used for various tasks, is considered. The similar researches have been executed for asteroid 1950 DA. The asteroid will twice approach the Earth to a minimal distance of 2.25 million km, in 2641 and in 2962. It can be made an Earth-bound satellite by increasing its aphelion velocity by ~ 1 km s-1 and by decreasing its perihelion velocity by ~ 2.5 km s-1.

Key Words: Near-Earth Objects; Asteroids, dynamics; Satellites, dynamics.

1. INTRODUCTION

Over the past decade, the asteroids of prime interest have been two asteroids, Apophis and 1950 DA, the first predicted to approach the Earth in 2029, and the second, in 2880. Reported calculations revealed some probability of an impact of the asteroids on the Earth. Yet, by the end of the decade refined orbital-element values of the asteroids were obtained, and more precise algorithms for calculating the interactions among solar-system bodies were developed. Following this, in the present paper we consider the motion evolution of both asteroids. In addition, we discuss available possibilities for making the asteroids into the Earth-bound satellites. Initially, the analysis is applied to Apophis and, then, numerical data for 1950 DA obtained by the same method will be presented.

The background behind the problem we treat in the present study was recently outlined in Giorgini *et al.* 2008. On June 19-20, 2004, asteroid Apophis was discovered by astronomers at the Kitt Peak Observatory (Tucker *et al.* 2004), and on December 20, 2004 this asteroid was observed for the second time by astronomers from the Siding Spring Survey Observatory (Garradd 2004). Since then, the new asteroid has command international attention. First gained data on identification of Apophis' orbital elements were employed to predict the Apophis path. Following the first estimates, it was reported in Rykhlova *et al.* 2007 that on April 13, 2029 Apophis will approach the Earth center to a minimum distance of 38000 km. As a result of the Earth gravity, the Apophis orbit will alter appreciably. Unfortunately, presently available methods for predicting the travel path of extraterrestrial objects lack sufficient accuracy, and some authors have therefore delivered an opinion that the Apophis trajectory will for long remain unknown, indeterministic, and even chaotic (see Giorgini *et al.* 2008, Rykhlova *et al.* 2007, Emel'yanov *et al.* 2008a). Different statistical predictions points to some probability of Apophis' collision with the Earth on April 13, 2036. It is this aspect, the impact risk, which has attracted primary attention of workers dealing with the problem.

Rykhlova *et al.* 2007 have attempted an investigation into the possibility of an event that the Apophis will closely approach the Earth. They also tried to evaluate possible threats stemming from this event. Various means to resist the fall of the asteroid onto Earth were put forward, and proposals for tracking Apophis missions, made. Finally, the need for prognostication studies of the Apophis path accurate to a one-kilometer distance for a period till 2029 was pointed out.

Many points concerning the prospects for tracking the Apophis motion with ground- and space-based observing means were discussed in Giorgini *et al.* 2008, Rykhlova *et al.* 2007, Emel'yanov *et al.* 2008a, 2008b. Since the orbits of the asteroid and Earth pass close to each other, then over a considerable portion of the Apophis orbit the asteroid disc will only be partially shined or even hidden from view. That is why it seems highly desirable to identify those periods during which the asteroid will appear accessible for observations with ground means. In using space-based observation means, a most efficient orbital allocation of such means needs to be identified.

Prediction of an asteroid motion presents a most challenging problem in astrophysics. In Sokolov *et al.* 2008, the differential equations for the perturbed motion of the asteroid were integrated by the Everhart method (Everhart 1974); in those calculations, for the coordinates of perturbing bodies were used the JPL planetary ephemeris DE403 and DE405 issued by the

Jet Propulsion Laboratory, USA. Sufficient attention was paid to resonance phenomena that might cause the hypothetical 2036 Earth impact.

Bykova and Galushina 2008*a*, 2008*b* used 933 observations to improve the identification accuracy for initial Apophis orbital parameters. Yet, the routine analysis has showed that, as a result of the pass of the asteroid through several resonances with Earth and Mars, the motion of the asteroid will probably become chaotic. With the aim to evaluate the probability of an event that Apophis will impact the Earth in 2036, Bykova *et al.* 2008 have made about 10 thousand variations of initial conditions, 13 of which proved to inflict a fall of Apophis onto Earth.

Smirnov 2008 has attempted a test of various integration methods for evaluating their capabilities in predicting the motion of an asteroid that might impact the Earth. The Everhart method, the Runge-Kutta method of fourth order, the Yoshida methods of sixth and eighth orders, the Hermit method of fourth and sixth orders, the Multistep Predictor-Corrector (MS-PC) method of sixth and eighth orders, and the Parker-Sochacki method were analyzed. The Everhart and MS-PC methods proved to be less appropriate than the other methods. For example, at close Apophis-to-Earth distances E.A. Smirnov used, instead of the Everhart method, the Runge-Kutta method. He to the fact that, in the problems with singular points, finite-difference methods normally fail to accurately approximate higher-order derivatives. This conclusion is quite significant since below we will report on an integration method for motion equations free of such deficiencies.

In Ivashkin and Stikhno 2008 the mathematical problems on asteroid orbit prediction and modification were considered. Possibilities offered by the impact-kinetic and thermonuclear methods in correcting the Apophis trajectory were evaluated.

An in-depth study of the asteroid was reported in Giorgini *et al.* 2008. A chronologically arranged outline of observational history was given, and the trend with progressively reduced uncertainty region for Apophis' orbit-element values was traced. Much attention was paid to discussing the orbit prediction accuracy and the bias of various factors affecting this accuracy. The influence of uncertainty in planet coordinates and in the physical characteristics of the asteroid, and also the perturbing action of other asteroids, were analyzed. The effects on integration accuracy of digital length, non-spherical shape of Earth and Moon, solar-radiation-induced perturbations, non-uniform thermal heating, and other factors, were examined.

The equations of perturbed motion of the asteroid were integrated with the help of the Standard Dynamic Model (SDM), with the coordinates of other bodies taken from the JPL planetary ephemeris DE405. It is a well-known fact that the DE405 ephemerid was compiled as an approximation to some hundred thousand observations that were made till 1998. Following the passage to the ephemeris DE414, that approximates observational data till 2006, the error in predicting the Apophis trajectory on 2036 has decreased by 140000 km. According to Giorgini *et al.* 2008, this error proved to be ten times greater than the errors induced by minor perturbations. Note that this result points to the necessity of employing a more accurate method for predicting the asteroid path.

In Giorgini *et al.* 2008, prospects for further refinement of Apophis' trajectory were discussed at length. Time periods suitable for optical and radar measurements, and also observational programs for oppositions with Earth in 2021 and 2029 and spacecraft missions for 2018 and 2027 were scheduled. Future advances in error minimization for asteroid trajectory due to the above activities were evaluated.

It should be noted that the ephemerides generated as an approximation to observational data enable rather accurate determination of a body's coordinates in space within the approximation time interval. The prediction accuracy for the coordinates on a moment remote from this interval worsens, the worsening being the greater the more the moment is distant from the approximation interval. Therefore, the observations and the missions scheduled in Giorgini *et al.* 2008 will be used in refining future ephemerides.

In view of the afore-said, in calculating the Apophis trajectory the equation of perturbed motion were integrated (Giorgini *et al.* 2008, Sokolov *et al.* 2008, Ivashkin and Stikhno 2008), while the coordinates of other bodies were borrowed from the ephemerid. Difference integration methods were employed, which for closely spaced bodies yield considerable inaccuracies in calculating higher-order derivatives. Addition of minor interactions to the basic Newtonian gravitational action complicates the problem and enlarges the uncertainty region in predicting the asteroid trajectory. Many of the weak interactions lack sufficient quantitative substantiation. Moreover, the physical characteristics of the asteroid and the interaction constants are known to some accuracy. That is why in making allowance for minor interactions expert judgments were used. And, which is most significant, the error in solving the problem on asteroid motion with Newtonian interaction is several orders greater than the corrections due to weak additional interactions.

The researches, for example, Bykova and Galushina 2008*a*, 2008*b* apply a technique in Giorgini *et al.* 2008 to study of influence of the initial conditions on probability of collision Apophis with Earth. The initial conditions for asteroid are defined from elements of its orbit, which are known with some uncertainty. For example, eccentricity value $e=e_n\pm\sigma_e$, where e_n is nominal value of eccentricity, and σ_e is root-mean-square deviation at processing of several hundred observation of asteroid. The collision parameters are searched in the field of possible motions of asteroid, for example for eccentricitya, $3\sigma_e$, the initial conditions are calculated in area $e=e_n\pm\sigma_e$. From this area the 10 thousand, and in some works, the 100 thousand sets of the initial conditions are *chosen* by an accidental manner, i.e. instead of one asteroid it is considered movement 10 or 100 thousand asteroids. Some of them can come in collision with Earth. The probability of collision asteroid with the Earth is defined by their amount.

Such statistical direction is incorrect. If many measurement data for a parameter are available, then the nominal value of the parameter, say, eccentricity e_n, presents a most reliable value for it. That is why a trajectory calculated from nominal initial conditions can be regarded as a most reliable trajectory. A trajectory calculated with a small deviation from the nominal initial conditions is a less probable trajectory, whereas the probability of a trajectory calculated from the parameters taken at the boundary of the probability region (i.e. from $e = e_n \pm \sigma_e$) tends to zero. Next, a trajectory with initial conditions determined using parameter values trice greater than the probable deviations (i.e. $e = e_n \pm 3\sigma_e$) has an even lower, negative, probability. Since initial conditions are defined by six orbital elements, then simultaneous realization of extreme (boundary) values ($\pm 3\sigma$) for all elements is even a less probable event, i.e. the probability becomes of smaller zero.

That is why it seems that a reasonable strategy could consist in examining the effect due to initial conditions using such datasets that were obtained as a result of successive accumulation of observation data. Provided that the difference between the asteroid motions in the last two datasets is insignificant over some interval before some date, it can be concluded that until this date the asteroid motion with the initial conditions was determined quite reliably.

As it was shown in Giorgini *et al.* 2008, some additional activities are required, aimed at further refinement of Apophis' trajectory. In this connection, more accurate determination of Apophis' trajectory is of obvious interest since, following such a determination, the range of possible alternatives would diminish.

For integration of differential motion equations of solar-system bodies over an extended time interval, a program Galactica was developed (Grebenikov and Smulsky 2007, Melnikov and Smulsky 2009). In this program, only the Newtonian gravity force was taken into account, and no differences for calculating derivatives were used. In the problems for the compound model of Earth rotation (Mel'nikov *et al.* 2008) and for the gravity maneuver near Venus (Smulsky 2008), motion equations with small body-to-body distances, the order of planet radius, were integrated. Following the solution of those problems and subsequent numerous checks of numerical data, we have established that, with the program Galactica, we were able to rather accurately predict the Apophis motion over its travel path prior to and after the approach to the Earth. In view of this, in the present study we have attempted an investigation into orbit evolution of asteroids Apophis and 1950 DA; as a result of this investigation, some fresh prospects toward possible use of these asteroids have opened.

2. PROBLEM STATEMENT

For the asteroid, the Sun, the planets, and the Moon, all interacting with one another by the Newton law of gravity, the differential motion equations have the form (Smulsky 1999):

$$\frac{d^2\vec{r}_i}{dt^2} = -G\sum_{k\neq i}^{n}\frac{m_k\vec{r}_{ik}}{r_{ik}^{\ 3}}, \quad i = 1,2,\ldots, n, \tag{1}$$

where $\vec{r}_i$ is radius-vector of a body with mass m_i relatively Solar System barycenter; G is gravitational constant; $\vec{r}_{ik}$ is vector $\vec{r}_i - \vec{r}_k$ and r_{ik} is its module; $n = 12$.

As a result of numerical experiments and their analysis we came to a conclusion, that finite-difference methods of integration do not provide necessary accuracy. For the integration of Eq. (1) we have developed algorithm and program Galactica. The meaning of function at the following moment of time $t=t_0 + \Delta t$ is determined with the help of Taylor series, which, for example, for coordinate x looks like:

$$x = x_0 + \sum_{k=1}^{K}\frac{1}{k!}x_0^{\ (k)}(\Delta t)^k \ , \tag{2}$$

where $x_0^{(k)}$ is derivative of k order at the initial moment t_0.

The meaning of velocity x' is defined by the similar formula, and acceleration x_0'' by the Eq. (1). Higher derivatives $x_0^{(k)}$ are determined analytically as a result of differentiation of the Eq. (1). The calculation algorithm of the sixth order is now used, i.e. with $K = 6$.

3. PREPARATION OF INITIAL DATA

We consider the problem of interest in the barycentric coordinate system on epoch J2000.0, Julian day JD_s = 2451545. The orbital elements asteroids Apophis and 1950 DA, such as the eccentricity e, the semi-major axis a, the ecliptic obliquity i_e, the ascending node angle Ω, the ascending node-perihelion angle ω_e, etc., and asteroids position elements, such as the mean anomaly M, were borrowed from the JPL Small-Body database 2008 as specified on November 30.0, 2008. The data, represented to 16 decimal digits, are given in Table 1. For Apophis in Table 1 the three variants are given. The first variant is now considered. These elements correspond to the solution with number JPL sol. 140, which is received Otto Mattic at April 4, 2008. In Table 1 the uncertainties of these data are too given. The relative uncertainty value δ is in the range from $2.4 \cdot 10^{-8}$ to $8 \cdot 10^{-7}$. The same data are in the asteroid database by Edward Bowell 2008, although these data are represented only to 8 decimal digits, and they differ from the former data in the 7-th digit, i.e., within value δ. Giorgini *et al.* 2008 used the orbital elements of Apophis on epoch JD = 2453979.5 (September 01.0, 2006), which correspond to the solution JPL sol. 142. On publicly accessible JPL-system Horizons the solution sol. 142 can be prolonged till November 30.0, 2008. In this case it is seen, that difference of orbital elements of the solution 142 from the solution 140 does not exceed 0.5σ uncertainties of the orbit elements.

The element values in Table 1 were used to calculate the Cartesian coordinates of Apophis and the Apophis velocity in the barycentric equatorial system by the following algorithm (see Duboshin 1976, Smulsky 2007, Mel'nikov *et al.* 2008, Melnikov and Smulsky 2009).

From the Kepler equation

$$E - e \cdot \sin E = M, \tag{3}$$

we calculate the eccentric anomaly E and, then, from E, the true anomaly φ_0:

$$\varphi_0 = 2 \cdot \mathrm{arctg}\left[\sqrt{(1+e)/(1-e)} \cdot \mathrm{tg}(0.5 \cdot E)\right], \tag{4}$$

In subsequent calculations, we used results for the two-body interaction (the Sun and the asteroid) (Smulsky 2007, Smulsky 2008). The trajectory equation of the body in a polar coordinate system with origin at the Sun has the form:

$$r = \frac{R_p}{(\alpha_1 + 1)\cos\ \varphi - \alpha_1}, \tag{5}$$

where the polar angle φ, or, in astronomy, the true anomaly, is reckoned from the perihelion position $r = R_p$; $\alpha_1 = -1/(1+e)$ is the trajectory parameter; and $R_p = a \cdot (2\alpha_1 + 1)/\alpha_1$ is the perihelion radius. The expressions for the radial υ_r and transversal υ_t velocities are

$$\upsilon_r = \upsilon_p \sqrt{(\alpha_1 + 1)^2 - (\alpha_1 + 1/\bar{r})^2}\ , \quad \text{for } \varphi > \pi \text{ we have } \upsilon_r < 0;\ \upsilon_t = \upsilon_p / \bar{r}\ , \tag{6}$$

where $\bar{r} = r/R_p$ is the dimensionless radius, and the velocity at perihelion is

$$\upsilon_p = \sqrt{G(m_S + m_{As})/((-\alpha_1) R_p)}\ , \tag{7}$$

where $m_S = m_{11}$ is the Sun mass (the value of m_{11} is given in Table 2), and $m_{As} = m_{12}$ is the Apophis mass.

Table 1. Three variants of orbital elements of asteroids Apophis on two epochs and 1950 DA on one epoch in the heliocentric ecliptic coordinate system of 2000.0 with JD_S = 2451545 (see JPL Small-Body Database 2008).

Elements	Apophis				1950 DA	Units
	1-st variant November 30.0, 2008 JD_{01} = 2454800.5 JPL sol.140	Uncertainties $\pm\sigma$ 1-st var.	2-nd variant January 04.0 2010 JD_{02} = 2455200.5 JPL sol.144	3-rd variant November 30.0, 2008 JD_{01} = 2454800.5 JPL sol.144.	November 30.0, 2008 JD_0 = 2454800.5 JPL sol. 51	
	Magnitude					
e	.1912119299890948	7.6088e-08	.1912110604804485	.1912119566344382	0.507531465407232	
a	.9224221637574083	2.3583e-08	.9224192977379344	.9224221602386669	1.698749639795436	AU
q	.7460440415606373	8.6487e-08	.7460425256098334	.7460440141364661	0.836580745750051	AU
i_e	3.331425002325445	2.024e-06	3.331517779979046	3.331430909298658	12.18197361251942	deg
Ω	204.4451349657969	0.00010721	204.4393039605681	204.4453098275707	356.782588306221	deg
ω_e	126.4064496795719	0.00010632	126.4244705298442	126.4062862564680	224.5335527346193	deg
M	254.9635275775066	5.7035e-05	339.9486156711335	254.9635223452623	161.0594270670401	deg
t_p	2454894.912750123770 (2009-Mar-04.41275013)	5.4824e-05	2455218.523239657948 (2010-Jan-22.02323966)	2454894.912754286546 (2009-Mar-04. 41275429)	2.454438.693685309 (2007-Dec-12.0419368531	JD d
P	323.5884570441701 0.89	1.2409e-05 3.397e-08	323.5869489330219 0.89	323.5884551925927 0.89	808.7094041052905 2.21	D yr
n	1.112524233059586	4.2665e-08	1.112529418096263	1.112524239425464	0.445153720449539	deg/d
Q	1.098800285954179	2.8092e-08	1.098796069866035	1.098800306340868	2.560918533840822	AU

Table 2. The masses m_{bj} of the planets from Mercury to Pluto, the Moon, the Sun (1 – 11) and asteroids: Apophis (12a) and 1950 DA (12b), and the initial condition on epoch JD_0 = 2454800.5 (November 30.0, 2008) in the heliocentric equatorial coordinate system on epoch 2000.0 JD_S = 2451545. G = 6.67259E-11 m^3 s^{-2}·kg^{-1}.

Bodies, j	Bodies masses in kg, their coordinates in m and velocities in $m\ s^{-1}$			
	m_{bj}	x_{aj}, v_{xaj},	y_{aj}, v_{yaj}	z_{aj}, v_{zaj}
1	3.30187842779737E+23	-17405931955.9539	-60363374194.7243	-30439758390.4783
		37391.7107852059	-7234.98671125365	-7741.83625612424
2	4.86855338156022E+24	108403264168.357	-2376790191.8979	-7929035215.64079
		1566.99276862423	31791.7241663148	14204.3084779893
3	5.97369899544255E+24	55202505242.89	125531983622.895	54422116239.8628
		-28122.5041342966	10123.4145376039	4387.99294255716
4	6.4185444055007E+23	-73610014623.8562	-193252991786.298	-86651102485.4373
		23801.7499674501	-5108.24106287744	-2985.97021694235
5	1.89900429500553E+27	377656482631.376	-609966433011.489	-270644689692.231
		11218.8059775149	6590.8440254003	2551.89467211952
6	5.68604198798257E+26	-1350347198932.98	317157114908.705	189132963561.519
		-3037.18405985381	-8681.05223681593	-3454.56564456648
7	8.68410787490547E+25	2972478173505.71	-397521136876.741	-216133653111.407
		979.784896813787	5886.28982058747	2564.10192504801
8	1.02456980223201E+26	3605461581823.41	-2448747002812.46	-1092050644334.28
		3217.00932811768	4100.99137103454	1598.60907148943
9	1.65085753263927E+22	53511484421.7929	-4502082550790.57	-1421068197167.72
		5543.83894965145	-290.586427181992	-1757.70127979299
10	7.34767263035645E+22	55223150629.6233	125168933272.726	54240546975.7587
		-27156.1163326908	10140.7572420768	4468.97456956941
11	1.98891948976803E+30	0	0	0
		0	0	0
12a	30917984100.3039	-133726467471.667	-60670683449.3631	-26002486763.62
		16908.9331065445	-21759.6060221801	-7660.90393288287
12b	1570796326794.9	314388505090.346	171358408804.935	127272183810.191
		-5995.33838888362	9672.35319009371	6838.06006342785

The time during which the body moves along an elliptic orbit from the point of perihelion to an orbital position with radius $\bar{r}$ is given by

$$t = \frac{R_p}{\upsilon_p}\left[\frac{\bar{r}\,|\,\bar{\upsilon}_r\,|}{2\alpha_1+1} - \frac{\alpha_1\left(\pi/2+\arcsin\left\{\left[\left(2\alpha_1+1\right)\bar{r}-\alpha_1\right]/\left(-\alpha_1-1\right)\right\}\right)}{\left(-2\alpha_1-1\right)^{3/2}}\right], \qquad (8)$$

where $\bar{\upsilon}_r = \upsilon_r/\upsilon_p$ is the dimensionless radial velocity.

At the initial time $t_0 = 0$, which corresponds to epoch JD_0 (see Table 1), the polar radius of the asteroid r_0 as dependent on the initial polar angle, or the true anomaly φ_0, can be calculated by Eq. (5). The initial radial and initial transversal velocities as functions of r_0 can be found using Eq. (6).

The Cartesian coordinates and velocities in the orbit plane of the asteroid (the axis x_o goes through the perihelion) can be calculated by the formulas

$$x_o = r_0 \cdot \cos \varphi_0; \qquad y_o = r_0 \cdot \sin\varphi_0; \tag{9}$$

$$\upsilon_{xo} = \upsilon_r \cdot \cos\varphi_0 - \upsilon_t \cdot \sin\varphi_0; \qquad \upsilon_{yo} = \upsilon_r \cdot \sin\varphi_0 + \upsilon_t \cdot \cos\varphi_0 . \tag{10}$$

The coordinates of the asteroid in the heliocentric ecliptic coordinate system can be calculated as

$$x_e = x_o \cdot (\cos \omega_e \cdot \cos \Omega - \sin \omega_e \cdot \sin \Omega \cdot \cos i_e) -$$

$$- y_o \cdot (\sin \omega e \cdot \cos \Omega + \cos \omega e \cdot \sin \Omega \cdot \cos ie); \tag{11}$$

$$y_e = x_o \cdot (\cos \omega_e \cdot \sin \Omega - \sin \omega_e \cdot \cos \Omega \cdot \cos i_e) - y_o \cdot (\sin \omega_e \cdot \sin \Omega - \cos \omega_e \cdot \cos \Omega \cdot \cos i_e); \tag{12}$$

$$z_e = x_o \sin \omega_e \cdot \sin i_e + y_o \cdot \cos \omega_e \cdot \sin i_e. \tag{13}$$

The velocity components of the asteroid $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} in this coordinate system can be calculated by Equations analogous to (11)-(13).

Since Eq. (1) are considered in a motionless equatorial coordinate system, then elliptic coordinates (11) – (13) can be transformed into equatorial ones by the Equations

$$x_a = x_e; \qquad y_a = y_e \cdot \cos \varepsilon_0 \cdot - z_e \cdot \sin \varepsilon_0 ; \qquad z_a = y_e \cdot \sin \varepsilon_0 + z_e \cdot \sin \varepsilon_0, \tag{14}$$

where ε_0·is the angle between the ecliptic and the equator in epoch JD_S.

The velocity components $\upsilon_{xe}, \upsilon_{ye}$ and υ_{ze} can be transformed into the equatorial ones $\upsilon_{xa}, \upsilon_{ya}$ and υ_{za} by Equations analogous to (14). With known heliocentric equatorial coordinates of the Solar system n bodies x_{ai}, y_{ai}, z_{ai} $i = 1,2, \dots n$, the coordinates of Solar system barycentre, for example, along axis x will be:

$$X_c = \left(\sum_{i=1}^{n} m_i x_{ai} \right) / M_{Ss}, \text{ where } \quad M_{Ss} = \sum_{i=1}^{n} m_i \text{ is mass of Solar system bodies.}$$

Then barycentric equatorial coordinates x_i of asteroid and other bodies will be

$$x_i = x_{ai} - X_c.$$

Other coordinates y_i and z_i and components of velocity $\upsilon_{xi}, \upsilon_{yi}$ and υ_{zi} in barycentric equatorial system of coordinates are calculated by analogous Equations.

In the calculations, six orbital elements from Table 1, namely, e, a i_e, Ω, ω_e, and M, were used. Other orbital elements were used for testing the calculated data. The perihelion radius R_p and the aphelion radius $R_a = -R_p/(2\alpha_l+1)$ were compared to q and Q, respectively. The orbital period was calculated by Eq. (8) as twice the time of motion from perihelion to aphelion ($r = R_a$). The same Equation was used to calculate the moment at which the asteroid passes the perihelion ($r = r_0$). The calculated values of those quantities were compared to the values of P and t_p given in Table 1. The largest relative difference in terms of q and Q was within $1.9 \cdot 10^{-16}$, and in terms of P and t_p, within $8 \cdot 10^{-9}$.

The coordinates and velocities of the planets and the Moon on epoch JD_0 were calculated by the DE406/LE406 JPL-theory (Ephemerides 2008, Standish 1998). The masses of those bodies were modified in Grebenikov and Smulsky 2007, and the Apophis mass was calculated assuming the asteroid to be a ball of diameter d = 270 m and density ρ = 3000 kg/m^3. The masses of all bodies and the initial conditions are given in Table 2.

The starting-data preparation and testing algorithm (3)-(14) was embodied as a MathCad worksheet (program AstCoor2.mcd).

4. Apophis' Encounter with the Planets and the Moon

In the program Galactica, a possibility to determine the minimum distance R_{min} to which the asteroid approaches a celestial body over a given interval ΔT was provided. Here, we integrated Eq. (1) with the initial conditions indicated in Table 2. The integration was performed on the NKS-160 supercomputer at the Computing Center SB RAS, Novosibirsk. In the program Galactica, an extended digit length (34 decimal digits) was used, and for the time step a value $dT = 10^{-5}$ year was adopted. The computations were performed over three time intervals, 0 ÷ 100 years (Figure 1, *a*), 0 ÷ -100 years (Figure 1, *b*), and 0 ÷ 1000 years (Figure 1, *c*).

In the graphs of Figure 1 the points connected with the heavy broken line show the minimal distances R_{min} to which the asteroid approaches the bodies indicated by points embraced by the horizontal line. In other words, a point in the broken line denotes a minimal distance to which, over the time ΔT = 1 year, the asteroid will approach a body denoted by the point in the horizontal line at the same moment. It is seen from Figure 1, *a* that, starting from November 30.0, 2008, over the period of 100 years there will be only one Apophis' approach to the Earth (point A) at the moment T_A = 0.203693547133403 century to a minimum distance R_{minA} = 38907 km. A next approach (point B) will be to the Earth as well, but at the moment T_B = 0.583679164042455 century to a minimum distance R_{minB} = 622231 km, which is 16 times greater than the minimum distance at the first approach. Among all the other bodies, a closest approach with be to the Moon (point D) (see Figure 1, *b*) at T_D = - 0.106280550824626 century to a minimum distance R_{minD} =3545163 km.

In the graphs of Figs. 1, *a* and *b* considered above, the closest approaches of the asteroid to the bodies over time intervals ΔT = 1 year are shown. In integrating Eq. (1) over the 1000-year interval (see Figure 1, *c*), we considered the closest approaches of the asteroid to the bodies over time intervals ΔT = 10 years. Over those time intervals, no approaches to Mercury and Mars were identified; in other words, over the 10-year intervals the asteroid closes with other bodies. Like in Figure 1, *a*, there is an approach to the Earth at the moment T_A. A second closest approach is also an approach to the Earth at the point E at T_E = 5.778503 century to a minimum distance R_{minE} = 74002.9 km. During the latter approach, the asteroid will pass the Earth at a minimum distance almost twice that at the moment T_A.

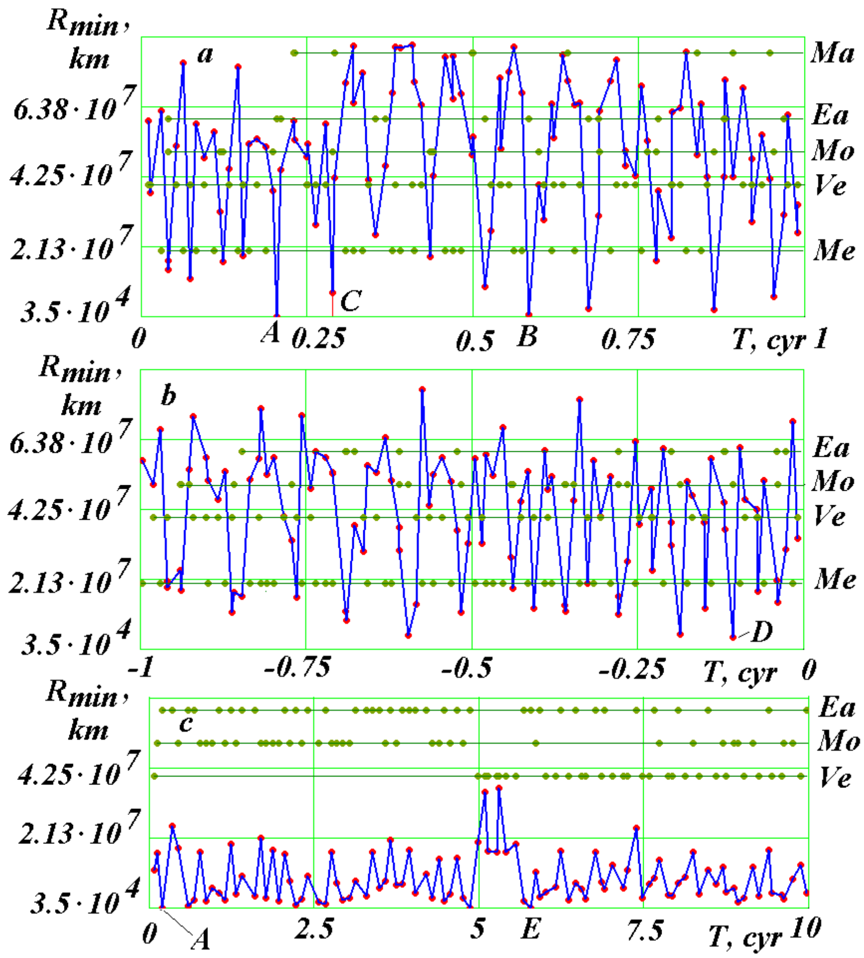

Figure 1. Apophis' encounters with celestial bodies during the time ΔT to a minimum distance R_{min}, km: Mars (Ma), Earth (Ea), Moon (Mo), Venus (Ve) and Mercury (Me); *a*, *b* – ΔT = 1 year; *c* – ΔT = 10 years. *T,* cyr (1 cyr = 100 yr) is the time in Julian centuries from epoch JD_0 (November 30.0, 2008). Calendar dates of approach in points: *A* – 13 April 2029; *B* – 13 April 2067; *C* – 5 September 2037; *E* – 10 October 2586.

With the aim to check the results, Eq. (1) were integrated over a period of 100 years with double digit length (17 decimal digits) and the same time step, and also with extended digit length and a time step $dT = 10^{-6}$ year. The integration accuracy (see Table 3) is defined (see Melnikov and Smulsky 2009) by the relative change of δM_z, the *z*-projection of the angular momentum of the whole solar system for the 100-year period. As it is seen from Table 3, the quantity δM_z varies from $-4.5 \cdot 10^{-14}$ to $1.47 \cdot 10^{-26}$, i.e., by 12 orders of magnitude. In the last two columns of Table 3, the difference between the moments at which the asteroid most closely approaches the Earth at point A (see Figure 1, *a*) and the difference between the approach distances relative to solution 1 are indicated. In solution 2, obtained with the short digit length, the approach moment has not changed, whereas the minimum distance has reduced by 2.7 m. In solution 3, obtained with ten times reduced integration step, the approach moment has changed by $-2\cdot 10^{-6}$ year, or by -1.052 minutes. This change being smaller than the step $dT = 1\cdot 10^{-5}$ for solution 1 and being equal twice the step for solution 3, the value of this change provides a refinement for the approach moment. Here, the refinement for the closest approach distance by -1.487 km is also obtained. On the refined calculations the Apophis approach to the Earth occurs at 21 hours 44 minutes 45 sec on distance of 38905 km. We emphasize here that the graphical data of Figure 1, *a* for solutions *1* and *3* are perfectly coincident. The slight differences of solution *2* from solutions *1* and *3* are observed for $T > 0.87$ century. Since all test calculations were performed considering the parameters of solution *1*, it follows from here that the data that will be presented below are accurate in terms of time within 1', and in terms of distance, within 1.5 km.

At integration on an interval of 1000 years the relative change of the angular momentum is $M_z = 1.45\cdot 10^{-20}$. How is seen from the solution 1 of Table 3 this value exceeds M_z at integration on an interval of 100 years in 10 times, i.e. the error at extended length of number is proportional to time. It allows to estimate the error of the second approach Apophis with the Earth in $T_E = 578$ years by results of integrations on an interval of 100 years of the solution with steps $dT = 1\cdot 10^{-5}$ years and $1\cdot 10^{-6}$ years. After 88 years from beginning of integration the relative difference of distances between Apophisом and Earth has become $\delta R_{88} = 1\cdot 10^{-4}$, that results in an error in distance of 48.7 km in $T_E = 578$ years.

So, during the forthcoming one-thousand-year period the asteroid Apophis will most closely approach the Earth only. This event will occur at the time T_A counted from epoch JD_0. The approach refers to the Julian day $JD_A = 2462240.406075$ and calendar date April 13, 2029, 21 hour 44'45'' GMT. The asteroid will pass at a minimum distance of 38905 km from the Earth center, i.e., at a distance of 6.1 of Earth radii. A next approach of Apophis to the Earth will be on the 578-th year from epoch JD_0; at that time, the asteroid will pass the Earth at an almost twice greater distance.

Table 3. Comparison between the data on Apophis' encounter with the Earth obtained with different integration accuracies: L_{nb} is the digit number in decimal digits.

№ solution	L_{nb}	dT, *yr*	δM_z	T_{Ai}-T_{A1}, *yr*	R_{minAi}-R_{minA1}, *km*
1	34	$1\cdot 10^{-5}$	$1.47\cdot 10^{-21}$	0	0
2	17	$1\cdot 10^{-5}$	$-4.5\cdot 10^{-14}$	0	$-2.7\cdot 10^{-3}$
3	34	$1\cdot 10^{-6}$	$1.47\cdot 10^{-26}$	$-2\cdot 10^{-6}$	-1.487

The calculated time at which Apophis will close with the Earth, April 13, 2029, coincides with the approach times that were obtained in other reported studies. For instance, in the recent publication Giorgini *et al.* 2008 this moment is given accurate to one minute: 21 hour 45' UTC, and the geocentric distance was reported to be in the range from 5.62 to 6.3 Earth radii, the distance of 6.1 Earth radii falling into the latter range. The good agreement between the data obtained by different methods proves the obtained data to be quite reliable.

As for the possible approach of Apophis to the Earth in 2036, there will be no such an approach (see Figure 1, *a*). A time-closest Apophis' approach at the point *C* to a minimum distance of 7.26 million km will be to the Moon, September 5, 2037.

5. APOPHIS ORBIT EVOLUTION

In integrating motion Eq. (1) over the interval –1 century ≤ T ≤ 1 century the coordinates and velocities of the bodies after a lapse of each one year were recorded in a file, so that a total of 200 files for a one-year time interval were obtained. Then, the data contained in each file were used to integrate Eq. (1) again over a time interval equal to the orbital period of Apophis and, following this, the coordinates and velocities of the asteroid, and those of Sun, were also saved in a new file. These data were used in the program DefTra to determine the parameters of Apophis' orbit relative to the Sun in the equatorial coordinate system. Such calculations were performed hands off for each of the 200 files under the control of the program PaOrb. Afterwards, the angular orbit parameters were recalculated into the ecliptic coordinate system (see Figure 2).

As it is seen from Figure 2, the eccentricity e of the Apophis orbit varies non-uniformly. It shows jumps or breaks. A most pronounced break is observed at the moment T_A, at which Apophis most closely approaches the Earth. A second most pronounced break is observed when Apophis approaches the Earth at the moment T_B.

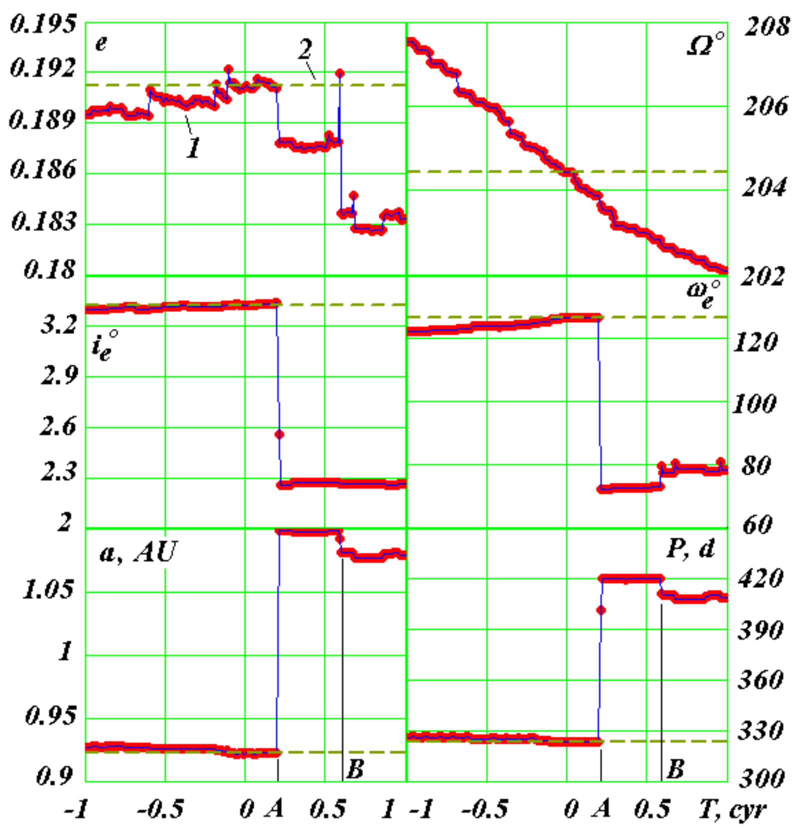

Figure 2. Evolution of Apophis' orbital parameters under the action of the planets, the Moon and the Sun over the time interval -100 years ÷ +100 years from epoch November 30.0, 2008: *1* – as revealed through integration of motion Eq. (1); *2* – initial values according to Table 1. The angular quantities: Ω, i_e, and ω_e are given in degrees; the major semi-axis a in AU; and the orbital period P in days.

The longitude of ascending node Ω shows less breaks, exhibiting instead rather monotonic a decrease (see Figure 2). Other orbital elements, namely, i_e, ω_e, a, and P, exhibit pronounced breaks at the moment of Apophis' closest pass near the Earth (at the moment T_A).

The dashed line in Figure 2 indicates the orbit-element values at the initial time, also indicated in Table 1. As it is seen from the graphs, those values coincide with the values obtained by integration of Eq. (1), the relative difference of e, Ω, i_e, ω_e, a, and P from the initial values at the moment $T=0$ (see Table 1) being respectively $9.4{\cdot}10^{-6}$, $-1.1{\cdot}10^{-6}$, $3.7{\cdot}10^{-6}$, $-8.5{\cdot}10^{-6}$, $1.7{\cdot}10^{-5}$, and $3.1{\cdot}10^{-5}$. This coincidence testifies the reliability of computed data at all calculation stages, including the determination of initial conditions, integration of equations, determination of orbital parameters, and transformations between the different coordinate systems.

As it was mentioned in Introduction, apart from non-simplified differential Eq. (1) for the motion of celestial bodies, other equations were also used. It is a well-known fact (see Duboshin 1976) that in perturbed-motion equations orbit-element values are used. For this reason, such equations will yield appreciable errors in determination of orbital-parameter breaks similar to those shown in Figure 2. Also, other solution methods for differential equations exist, including those in which expansions with respect to orbital elements or difference quotients are used. As it was already mentioned in Introduction, these methods proved to be sensitive to various resonance phenomena and sudden orbit changes observed on the approaches between bodies. Eq. (1) and method (2) used in the present study are free of such shortcomings. This suggests that the results reported in the present paper will receive no notable corrections in the future.

6. Influence of Initial Conditions

With the purpose of check of influence of the initial conditions (IC) on Apophis trajectory the Eq. (1) were else integrated on an interval 100 years with two variants of the initial conditions. The second of variant IC is given on January 04.0, 2010 (see Table 1). They are taken from the JPL Small-Body database 2008 and correspond to the solution with number JPL sol. 144, received Steven R. Chesley on October 23, 2009. In Figure 3 the results of two solutions with various IC are submitted. The line *1* shows the change in time of distance R between Apophis and Earth for 100 years at the first variant IC. As it is seen from the graphs, the distance R changes with oscillations, thus it is possible to determine two periods: the short period T_{R1} = 0.87 years and long period T_{R2}. The amplitude of the short period R_{a1} = 29.3 million km, and long is R_{a2} = 117.6 million km. The value of the long oscillation period up to T ~ 70 years is equal T_{R20} = 7.8 years, and further it is slightly increased. After approach of April 13, 2029 (point A in Figure 3) the amplitude of the second oscillations is slightly increased. Both short and the long oscillations are not regular; therefore their average characteristics are above given.

Let's note also on the second minimal distance of Apophis approach with the Earth on interval 100 years. It occurs at the time T_{F1} = 58.37 years (point F_1 in Figure 3) on distance R_{F1} = 622 thousand km. In April 13, 2036 (point H in Figure 3) Apophis passes at the Earth on distance R_{H1} = 86 million km. The above-mentioned characteristics of the solution are submitted in Table 4.

The line 2 in Figure 3gives the solution with the second of variant IC with step of integration $dT = 1{\cdot}10^{-5}$ years. The time of approach has coincided to within 1 minutes, and distance of approach with the second of IC became R_{A2} = 37886 km, i.e. has decreased on 1021 km. To determine more accurate these parameters the Eq. (1) near to point of approach were integrated with a step $dT = 1{\cdot}10^{-6}$ years. On the refined calculations Apophis approaches with the Earth at 21 hours 44 minutes 53 second on distance R_{A2} = 37880 km. As it is seen from Table 4, this moment of approach differs from the moment of approach at the first of IC on 8 second. As at a step $dT = 1{\cdot}10^{-6}$ years the accuracy of determination of time is 16 second, it is follows, that the moments of approach coincide within the bounds of accuracy of their calculation.

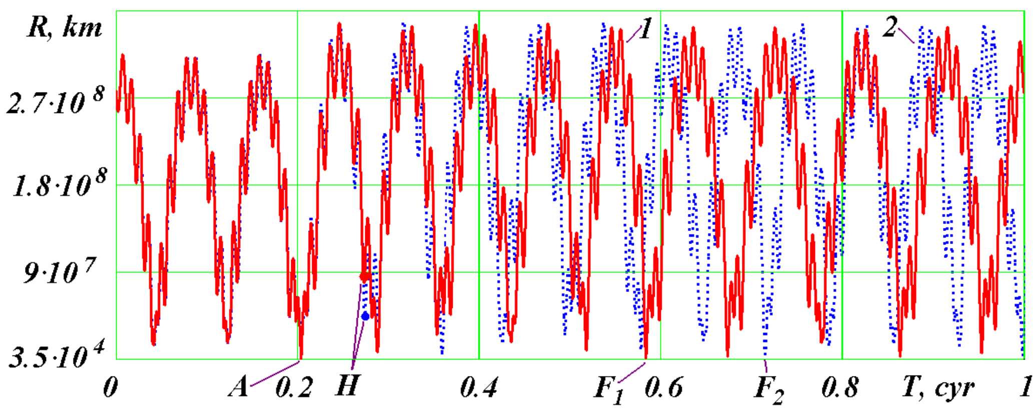

Figure 3. Evolution of distance R between Apophis and Earth for 100 years. Influence of the initial conditions (IC): *1* - IC from November 30.0, 2008; *2* - IC from January 04.0, 2010. Calendar dates of approach in points: A – 13 April 2029; F_1– 13 April 2067; F_2 – 14 April 2080.

The short and long oscillations at two variants IC also have coincided up to the moment of approach. After approach in point A the period of long oscillations has decreased up to T_{R22} = 7.15 years, i.e. became less than period T_{R20} at the first variant IC. The second approach on an interval 100 years occurs at the moment T_{F2} = 70.28 years on distance R_{F2} = 1.663 million km. In 2036 г (point H) Apophis passes on distance R_{H2} = 43.8 million km.

At the second variant of the initial conditions on January 04.0, 2010 in comparison with the first of variant the initial conditions of Apophis and of acting bodies are changed. To reveal only errors influence of Apophis IC, the third variant of IC is given (see Table 1) as first of IC on November 30.0, 2008, but the Apophis IC are calculated in system Horizons according to JPL sol. 144. How follows from Table 1, from six elements of an orbit e, a, i_e, Ω, ω_e and M the differences of three ones: i_e, Ω и ω_e from similar elements of the first variant of IC are 2.9, 1.6 and 1.5 appropriate uncertainties. The difference of other elements does not exceed their uncertainties.

At the third variant of IC with step of integration $dT = 1{\cdot}10^{-5}$ year the moment of approach has coincided with that at the first variant of IC. The distance of approach became R_{A3} = 38814 km, i.e. has decreased on 93 km. For more accurate determination of these parameters the Eq. (1) near to a point of approach were also integrated with a step $dT = 1 \cdot 10^{-6}$ year. On the refined calculations at the third variant of IC Apophis approaches with the Earth at 21 hours 44 minutes 45 second on distance R_{A3} = 38813 km. These and other characteristics of the solution are given in Table 4. In comparison with the first variant IC it is seen, that distance of approach in 2036 and parameters of the second approach in point F_1 are slightly changed. The evolution of distance R in a Figure 3 up to T = 0.6 centuries practically coincides with the first variant (line *1*).

Table 4. Influence of the initial conditions on results of integration of the Eq. (1) by program Galactica and of the equations of Apophis motion by system Horizons: Time$_A$ and R_{minA} are time and distance of Apophis approach with the Earth in April 13, 2029, accordingly; R_H is distance of passage Apophis with the Earth in April 13, 2036; T_F and R_F are time and distance of the second approach (point F on Figure 3).

Parameters	Solutions at different variants of initial conditions					
	Galactica			Horizons		
	1	2	3	1	2	3
	30.11.2008 JPL sol.140	04.01.2010 JPL sol.144	30.11.2008 JPL sol.144	18.07.2006 JPL sol.144	30.11.2008 JPL sol.140	04.01.2010 JPL sol.144
Time$_A$	21:44:45	21:44:53	21:44:45	21:46:47	21:45:47	21:44:45
R_{minA}, km	38905	37880	38813	38068	38161	38068
R_H, 10^6 km	86.0	43.8	81.9	51.9	55.9	51.8
T_F, cyr from 30.11.08	0.5837	0.7138	0.6537	0.4237	0.9437	0.4238
R_F, 10^3 km	622	1663	585	1515	684	1541

It is seen (Table 4) that the results of the third variant differ from the first one much less than from the second variant. In the second variant the change of positions and velocities of acting bodies since November 30, 2008 for 04.01.2010 is computed under DE406, and in the third variant it does under the program Galactica. The initial conditions for Apophis in two variants are determined according to alike JPL sol. 144, i.e. in these solutions the IC differ for acting bodies. As it is seen from Table 4, the moment of approach in solutions *2* and *3* differs on 8 seconds, and the approach distance differs on 933 km. Other results of the third solution also differ in the greater degree with second ones, in comparison of the third solution with first one. It testifies that the differences IC for Apophis are less essential in comparison with differences of results of calculations under two programs: Galactica and DE406 (or Horizons).

So, the above-mentioned difference of the initial conditions (variants *1* and *3* tab. *4*) do not change the time of approach of April 13, 2029, and the distance of approach in these solutions differ on 102 km. Other characteristics: R_H, T_F and R_F also change a little. Therefore it is possible to make a conclusion, that the further refinement of Apophis IC will not essentially change its trajectory.

The same researches on influence of the initial conditions we have carried out with the integrator of NASA. In system Horizons (the JPL Horizons On-Line Ephemeris System, manual look on a site http://ssd.jpl.nasa.gov/?horizons_doc) there is opportunity to calculate asteroid motion on the same standard dynamic model (SDM), on which the calculations in

Giorgini *et al.* 2008 are executed. Except considered two IC we used one more IC for Apophis at date of July 12, 2006, which is close to date of September 01, 2006 in Giorgini *et al.* 2008. The characteristics and basic results of all solutions are given in Table 4. In these solutions the similar results are received. For example, for *3*-rd variant of Horizons the graphs *R* in a Figure 3 up to *T* = 0.45 centuries practically has coincided with *2*-nd variant of Galactica. The time of approach in April 13, 2029 changes within the bounds of 2 minutes, and the distance is close to 38000 km. The distance of approach in April 13, 2036 changes from 52 up to 56 million km. The characteristics of second approach for 100 years changes in the same bounds, as for the solutions on the program Galactica. The above-mentioned other relations about IC influence have also repeated for the NASA integrator.

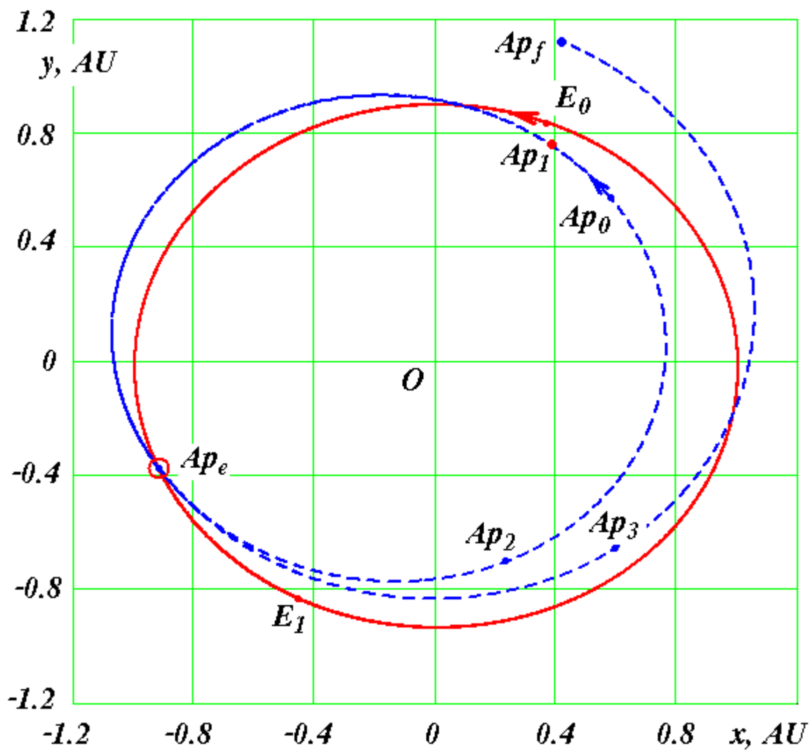

Figure 4. The trajectories of Apophis (*Ap*) and Earth (*E*) in the barycentric equatorial coordinate system *xOy* over a two-year period: Ap_0 and E_0 are the initial position of Apophis and Earth; Ap_f is the end point of the Apophis trajectory; Ap_e is the point at which Apophis most closely approaches the Earth; the coordinates *x* and *y* are given in AU.

So, the calculations at the different initial conditions have shown that Apophis in 2029 will be approached with the Earth on distance 38÷39 thousand km, and in nearest 100 years it once again will approach with the Earth on distance not closer 600 thousand km.

7. Examination of Apophis' Trajectory in the Vicinity of Earth

In order to examine the Apophis trajectory in the vicinity of Earth, we integrated Eq. (1) over a two-year period starting from T_1 = 0.19 century. Following each 50 integration steps, the coordinate and velocity values of Apophis and Earth were recorded in a file. The moment T_A at which Apophis will most closely approach the Earth falls into this two-year period. The ellipse E_0E_1 in Figure 4 shows the projection of the two-year Earth's trajectory onto the equatorial plane *xOy*. Along this trajectory, starting from the point E_0, the Earth will make two turns. The two-year Apophis trajectory in the same coordinates is indicated by points denoted with the letters *Ap*. Starting from the point Ap_0, Apophis will travel the way $Ap_0Ap_1Ap_eAp_2Ap_0Ap_1$ to most closely approach the Earth at the point Ap_e at the time T_A. After that, the asteroid will follow another path, namely, the path $Ap_eAp_3Ap_f$.

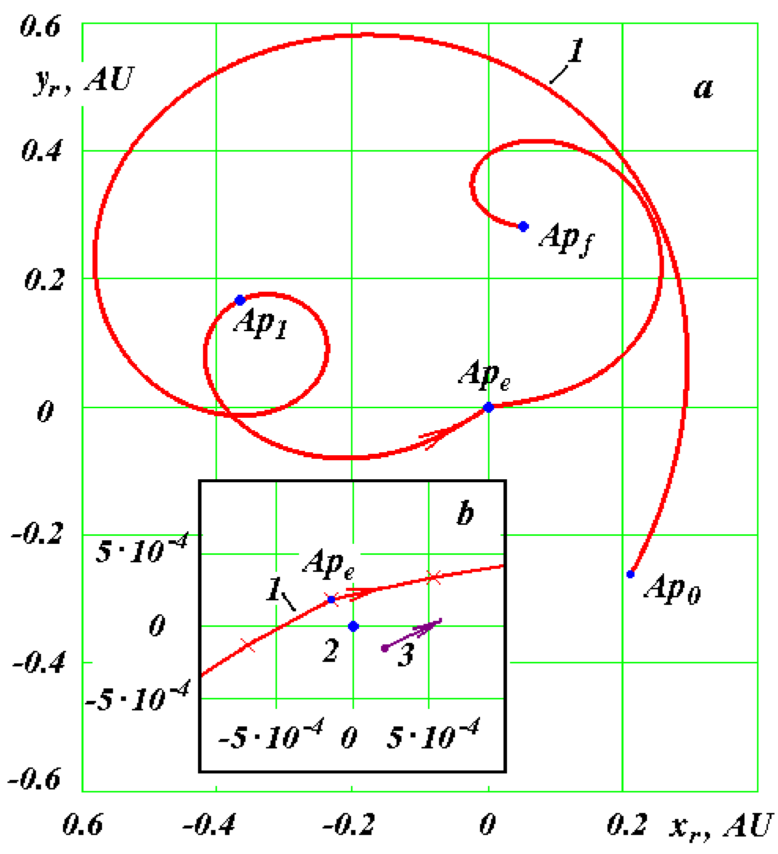

Figure 5. Apophis' trajectory (1) in the geocentric equatorial coordinate system x_rOy_r: *a* – on the normal scale, *b* – on magnified scale on the moment of Apophis' closest approaches to the Earth (2); *3* – Apophis' position at the moment of its closest approach to the Earth following the correction of its trajectory with factor k = 0.9992 at the point Ap_1; the coordinates x_r and y_r are given in AU.

Figure 5, *a* shows the trajectory of Apophis relative to the Earth. Here, the relative coordinates are determined as the difference between the Apophis (*Ap*) and Earth (*E*) coordinates:

$$y_r = y_{Ap} - y_E; \qquad x_r = x_{Ap} - x_E. \tag{15}$$

Along trajectory 1, starting from the point Ap_0, Apophis will travel to the Earth-closest point Ap_e, the trajectory ending at the point Ap_f. The loops in the Apophis trajectory represent a reverse motion of Apophis with respect to Earth. Such loops are made by all planets when observed from the Earth (Smulsky 2007).

At the Earth-closest point Ap_e the Apophis trajectory shows a break. In Figure 5, *b* this break is shown on a larger scale. Here, the Earth is located at the origin, point 2. The Sun (see Figure 4) is located in the vicinity of the barycenter *O*, i.e., in the upper right quadrant of the Earth-closest point Ap_e. Hence, the Earth-closest point will be passed by Apophis as the latter will move in between the Earth and the Sun (see Figure 5, *b*). As it will be shown below, this circumstance will present certain difficulties for possible use of the asteroid.

8. Possible Use of Asteroid Apophis

So, on April 13, 2029, we will become witnesses of a unique phenomenon, the pass of a body 31 million tons in mass near the Earth at a minimum distance of 6 Earth radii from the center of Earth. Over subsequent 1000 years, Apophis will never approach our planet closer.

Many pioneers of cosmonautics, for instance, K.E. Tsiolkovsky, Yu.A. Kondratyuk, D.V. Cole etc. believed that the near-Earth space will be explored using large manned orbital stations. Yet, delivering heavy masses from Earth into orbit presents a difficult engineering and ecological problem. For this reason, the lucky chance to turn the asteroid Apophis into an Earth bound satellite and, then, into a habited station presents obvious interest.

Among the possible applications of a satellite, the following two will be discussed here. First, a satellite can be used to create a space lift. It is known that a space lift consists of a cable tied with one of its ends to a point at the Earth equator and, with the other end, to a massive body turning round the Earth in the equatorial plane in a 24-hour period, P_d = 24·3600 sec. The radius of the satellite geostationary orbit is

$$R_{gs} = \sqrt[3]{P_d^2 G\left(m_A + m_E\right)/4\pi^2} = 42241 \text{ km} = 6.62\ R_{Ee}\ . \tag{16}$$

In order to provide for a sufficient cable tension, the massive body needs to be spaced from the Earth center a distance greater than R_{gs}. The cable, or several such cables, can be used to convey various goods into space while other goods can be transported back to the Earth out of space.

If the mankind will become able to make Apophis an Earth bound satellite and, then, deflect the Apophis orbit into the equatorial plane, then the new satellite would suit the purpose of creating a space lift.

A second application of an asteroid implies its use as a “shuttle” for transporting goods to the Moon. Here, the asteroid is to have an elongated orbit with a perihelion radius close to that of a geostationary orbit and an apogee radius approaching the perigee radius of the lunar orbit. In the latter case, at the geostationary-orbit perigee goods would be transferred onto the satellite Apophis and then, at the apogee, those goods would arrive at the Moon.

The two applications will entail the necessity of solving many difficult problems which now can seem even unsolvable. On the other hand, none of those problems will be solved at all without making asteroid an Earth satellite. Consider now the possibilities available here.

The velocity of the asteroid relative to the Earth at the Earth-closest point Ap_e is v_{AE} = 7.39 km s^{-1}. The velocity of an Earth bound satellite orbiting at a fixed distance R_{minA} from the Earth (circular orbit) is

$$v_{CE} = \sqrt{G\left(m_A + m_E\right) / R_{\min A}} = 3.2 \text{ km s}^{-1}. \tag{17}$$

For the asteroid to be made an Earth-bound satellite, its velocity v_{AE} should be brought close to v_{CE}. We performed integration of Eq. (1) assuming the Apophis velocity at the moment T_A to be reduced by a factor of 1.9, i.e., the velocity v_{AE}=7.39 km s^{-1} at the moment T_A was decreased to 3.89 km s^{-1}. In the later case, Apophis becomes an Earth bound satellite with the following orbit characteristics: eccentricity e_{s1} = 0.476, equator-plane inclination angle i_{s1} = 39.2°, major semi-axis a_{s1} = 74540 km, and sidereal orbital period P_{s1} = 2.344 days.

We examined the path evolution of the satellite for a period of 100 years. In spite of more pronounced oscillations of the orbital elements of the satellite in comparison with those of planetary orbit elements, the satellite's major semi-axis and orbital period proved to fall close to the indicated values. For the relative variations of the two quantities, the following estimates were obtained: $|\delta a| < \pm 2.75 \cdot 10^{-4}$ and $|\delta P| < \pm 4.46 \cdot 10^{-4}$. Yet, the satellite orbits in a direction opposite both to the Earth rotation direction and the direction of Moon's orbital motion. That is why the two discussed applications of such a satellite turn to be impossible.

Thus, the satellite has to orbit in the same direction in which the Earth rotates. Provided that Apophis (see Figure 5, *b*) will round the Earth from the night-side (see point *3*) and not from the day-side (see line *1*), then, on a decrease of its velocity the satellite will be made a satellite orbiting in the required direction.

For this matter to be clarified, we have integrated Eq. (1) assuming different values of the asteroid velocity at the point Ap_1 (see Figure 5). This point, located at half the turn from the Earth-closest point Ap_e, will be passed by Apophis at the time T_{Ap1}=0.149263369488169 century. At the point Ap_1 the projections of the Apophis velocity in the barycentric equatorial coordinate system are v_{Ap1x} = -25.6136689 km s^{-1}, v_{Ap1y} = 17.75185451 km s^{-1}, and v_{Ap1z} = 5.95159206 km s^{-1}. In the numerical experiments, the component values of the satellite velocity were varied to one and the same proportion by multiplying all them by a single factor *k*, and then Eq. (1) were integrated to determine the trajectory of the asteroid. Figure 6 shows the minimum distance to which Apophis will approach the Earth versus the value of *k* by which the satellite velocity at the point Ap_1 was reduced.

We found that, on decreasing the value of *k* (see Figure 6), the asteroid will more closely approach the Earth, and at *k* = 0.9999564 Apophis will collide with the Earth. On further decrease of asteroid velocity the asteroid will close with the Earth on the Sun-opposite side, and at *k* = 0.9992 the asteroid will approach the Earth center (point 3 in Figure 5, *b*) to a minimum distance R_{min3} = 39157 km at the time T_3 = 0.2036882 century. This distance R_{min3}

roughly equals the distance R_{minA} to which the asteroid was found to approach the Earth center while moving in between the Earth and the Sun.

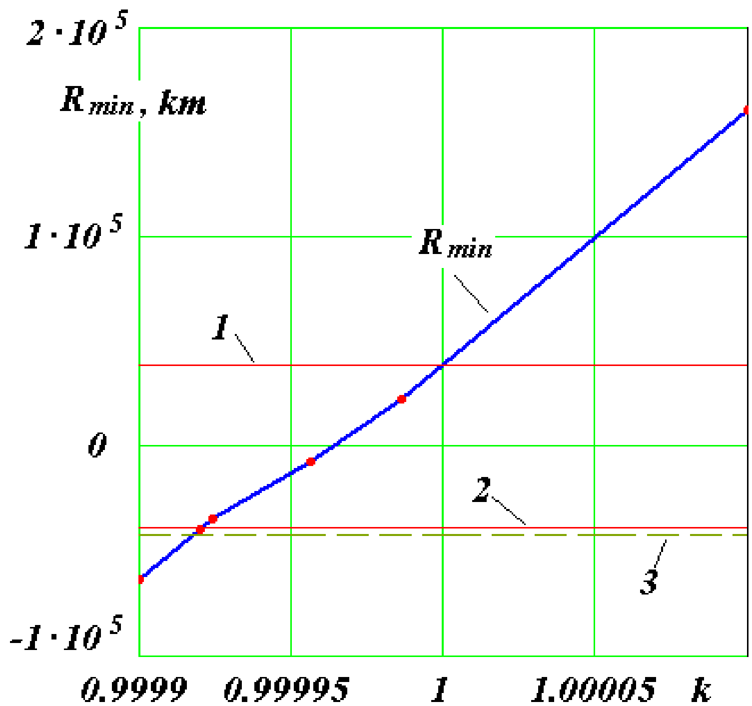

Figure 6. The minimum distance R_{min} to which Apophis will approach the Earth center versus the value of k (k is the velocity reduction factor at the point A_{p1} (see Figure 4)). The positive values of R_{min} refer to the day-side: the values of R_{min} are given in km; *1* – the minimum distance to which Apophis will approach the Earth center on April 13, 2029 (day-side); *2* – the minimum distance to which Apophis will approach the Earth center after the orbit correction (night-side); *3* – geostationary orbit radius R_{gs}.

In this case, the asteroid velocity relative to the Earth is also v_{AE} =7.39 km s^{-1}. On further decrease of this velocity by a factor of 1.9, i.e., down to 3.89 km s^{-1} Apophis will become an Earth bound satellite with the following orbit parameters: eccentricity e_{s2} = 0.486, equator plane inclination angle i_{s2} = 36°, major semi-axis a_{s2} = 76480 km, and sidereal period P_{s2} = 2.436 day. In addition, we investigated into the path evolution of the Earth bound satellite over a 100-year period. The orbit of the satellite proved to be stable, the satellite orbiting in the same direction as the Moon does.

Thus, for Apophis to be made a near-Earth satellite orbiting in the required direction, two decelerations of its velocity need to be implemented. The first deceleration is to be effected prior to the Apophis approach to the Earth, for instance, at the point Ap_1 (see Figure 4), 0.443 year before the Apophis approach to the Earth. Here, the Apophis velocity needs to be decreased by 2.54 m/s. A second deceleration is to be effected at the moment the asteroid closes with the Earth. In the case under consideration, in which the asteroid moves in an elliptic orbit, the asteroid velocity needs to be decreased by 3.5 km s^{-1}.

Slowing down a body weighing 30 million tons by 3.5 km s^{-1} is presently a difficult scientific and engineering problem. For instance, in Rykhlova *et al.* 2007 imparting Apophis with a velocity of 10^{-6} m/s was believed to be a problem solvable with presently available engineering means. On the other hand, Rykhlova *et al.* 2007 consider increasing the velocity of such a body by about 1-2 cm/s a difficult problem. Yet, with Apophis being on its way to the Earth, we still have a twenty-year leeway. After the World War II, even more difficult a problem, that on injection of the first artificial satellite in near-Earth orbit and, later, the launch of manned space vehicles, was successfully solved in a period of ten years. That is why we believe that, with consolidated efforts of mankind, the objective under discussion will definitely be achieved.

It should be emphasized that the authors of Giorgini *et al.* 2008 considered the possibility of modifying the Apophis orbit for organizing its impact onto asteroid (144898) 2004 VD17. There exists a small probability of the asteroid's impact onto the Earth in 2102. Yet, the problem on reaching a required degree of coordination between the motions of the two satellites presently seems to be hardly solvable. This and some other examples show that many workers share an opinion that substantial actions on the asteroid are necessary for making the solution of the various space tasks a realistic program.

9. ASTEROID 1950 DA APPROACHES TO THE EARTH

The distances to which the asteroid 1950 DA will approach solar-system bodies are shown versus time in Figure 7. It is seen from Figure 7, *a*, that, following November 30.0, 2008, during the subsequent 100-year period the asteroid will most closely approach the Moon: at the point A (T_A = 0.232532 cyr and R_{min} = 11.09 million km) and at the point B (T_B = 0.962689 cyr and R_{min} = 5.42 million km). The encounters with solar-system bodies the asteroid had over the period of 100 past years are shown in Figure 7, *b*. The asteroid most closely approached the Earth twice: at the point C (T_C = -0.077395 cyr and R_{min} = 7.79 million km), and at the point D (T_D = -0.58716 cyr and R_{min} = 8.87 million km).

Over the interval of forthcoming 1000 years, the minimal distances to which the asteroid will approach solar-system bodies on time span ΔT = 10 years are indicated in Figure 7, *c*. The closest approach of 1950 DA will be to the Earth: at the point E (T_E = 6.322500 cyr and R_{min} = 2.254 million km), and at the point F (T_F = 9.532484 cyr and R_{min} = 2.248 million km).

To summarize, over the 1000-year time interval the asteroid 1950 DA will most closely approach the Earth twice, at the times T_E and T_F, to a minimum distance of 2.25 million km in both cases. The time T_E refers to the date March 6, 2641, and the time T_F, to the date March 7, 2962.

Giorgini et al. 2002 calculated the nominal 1950 DA trajectory using earlier estimates for the orbit-element values of the asteroid, namely, the values by the epoch of March 10.0, 2001 (JPL sol. 37). In Giorgini et al. 2002, as the variation of initial conditions for the asteroid, ranges were set three times wider than the uncertainty in element values. For the extreme points of the adopted ranges, in the calculations 33 collision events were registered. In this connection, Giorgini et al. 2002 have entitled their publication «Asteroid 1950 DA Encounter with Earth in 2880…».

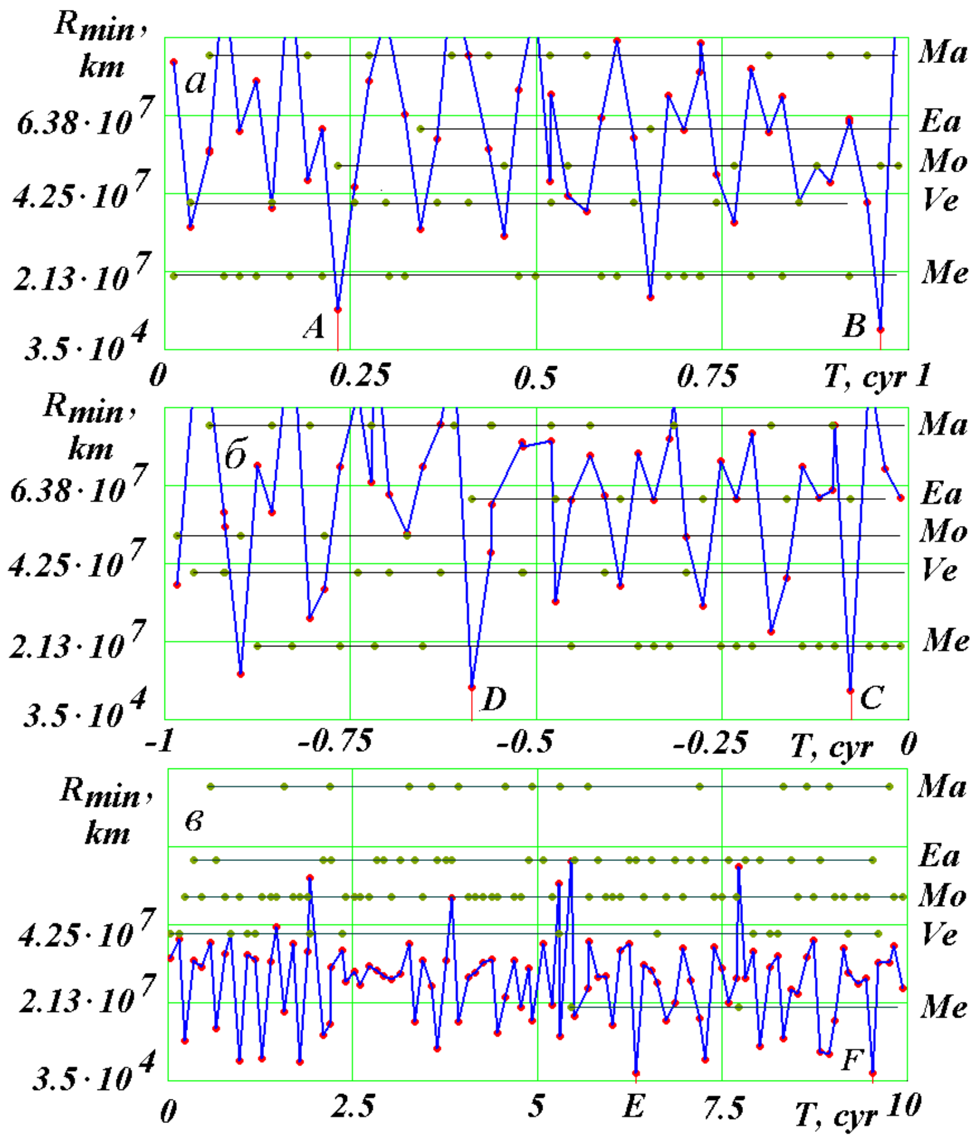

Figure 7. Approach of the asteroid 1950 DA to solar-system bodies. The approach distances are calculated with time interval ΔT: *a*, *b* – ΔT=1 year; *c* – ΔT = 10 years. R_{min}, km is the closest approach distance. Calendar dates of approach in points see Table 5. For other designations, see Figure 1.

We made our calculations using the orbit-element values of 1950 DA by the epoch of November 30.0, 2008 (JPL sol. 51) (see Table 1). By system Horizons the JPL sol. 37 can be prolonged till November 30.0, 2008. As it is seen in this case, the difference of orbital elements of the solution 37 from the solution 51 on two - three order is less, than uncertainties of orbit elements, i.e. the orbital elements practically coincide.

With the aim to trace how the difference methods of calculation has affected the 1950 DA motion, in Table 5 we give a comparison of the approach times of Figure 7 with the time-closest approaches predicted in Giorgini et al. 2002. According to Table 5, the shorter the separation between the approach times (see points *C* and *A*) and the start time of calculation (2008-11-30), the better is the coincidence in terms of approach dates and minimal approach distances R_{min}. For more remote times (see points *D* and *B*) the approach times differ already by 1 day. At the point *E*, remote from the start time of calculation by 680 year, the approach times differ already by eight days, the approach distances still differing little. At the most remote point *F*, according to our calculations, the asteroid will approach the Earth in 2962 to a distance of 0.015 AU, whereas, according to the data of Giorgini et al. 2002, a most close approach to the Earth, to a shorter distance, will be in 2880.

Table 5. Comparison between the data on asteroid 1950 DA encounters with the Earth and Moon: our data are denoted with characters A, B, C, D, E, F, as in Figure 7, and the data by Giorgini et al. [24] are denoted as Giorg.

Source	JD, days	Date	Time, days	Body	R_{min}, AU
D	2433354	1950-03-13	0.730	Earth	0.059273
Giorg.	-	1950-03-12	0.983	Earth	0.059286
C	2451973	2001-03-05	0.157	Earth	0.052075
Giorg.	-	2001-03-05	0.058	Earth	0.052073
A	2463293	2032-03-02	0.222	Moon	0.074158
Giorg.	-	2032-03-02	0.281	Earth	0.075751
B	2489962	2105-03-09	0.224	Moon	0.036260
Giorg.	-	2105-03-10	0.069	Earth	0.036316
E	2685729	2641-03-06	0.338	Earth	0.015070
Giorg.	-	2641-03-14	0.330	Earth	0.015634
F	2802974	2962-03-07	0.985	Earth	0.015030
Giorg.	-	2880-03-16	0.836	Earth	0.001954

So, our calculations show that the asteroid 1950 DA will not closely approach the Earth. It should be noted that our calculation algorithm for predicting the motion of the asteroid differs substantially from that of Giorgini et al. 2002. We solve non-simplified Eq. (1) by a high-precision numerical method. In doing so, we take into account the Newtonian gravitational interaction only. In Giorgini et al. 2002, additional weak actions on the asteroid were taken into account. Yet, the position of celestial bodies acting on the asteroid is calculated from the ephemerides of DE-series. Those ephemeredes approximate observational data and, hence, they describe those data to good precision. Yet, the extent to which the predicted motion of celestial bodies deviates from the actual motion of these bodies is the greater the farther the moment of interest is remote from the time interval during which the observations were made. We therefore believe that the difference between the present calculation data for the times 600 and 900 years (points *E* and *F* in Table 5) and the data of Giorgini et al. 2002 results from the indicated circumstance.

10. Evolution of the 1950 DA Orbit

Figure 8 shows the evolution of 1950 DA orbital elements over a 1000-year time interval as revealed in calculations made with time span ΔT=10 years. With the passage of time, the orbit eccentricity e non-monotonically increases. The angle of longitude of ascending node Ω_e, the angle of inclination i_e to the ecliptic plane, and the angle of perihelion argument ω_e show more monotonic variations. The semi-axis a and the orbital period P both oscillate about some mean values. As it is seen from Figure 8, at the moments of encounter with the Earth, T_E and T_F, the semi-axis a and the period P show jumps. At the same moments, all the other orbit elements exhibit less pronounced jumps.

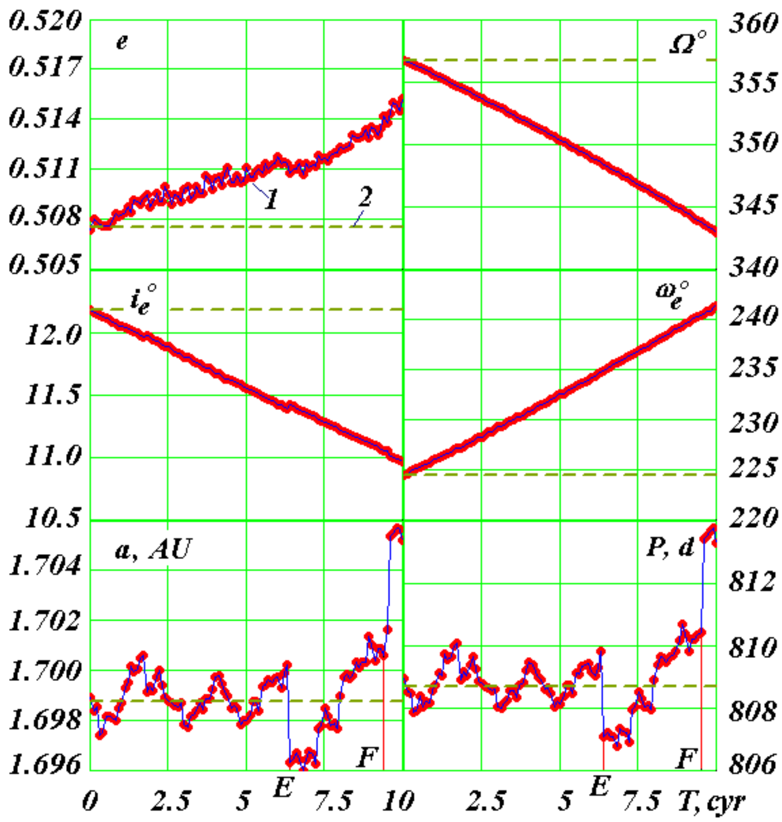

Figure 8. Evolution of 1950 DA orbital parameters under the action of the planets, the Moon, and the Sun over the time interval 0 ÷ 1000 from the epoch November 30.0, 2008: *1*- as revealed through integration of motion equation (1) obtained with the time interval ΔT = 10 years: *2* – initial values according to Table 1. The angular quantities, Ω, i_e, and ω_e, are given in degrees, the major semi-axis a – in AU, and the orbital period P, in days.

The dashed line in Figure 8 indicates the initial-time values of orbital elements presented in Table 1. As it is seen from the graphs, these values are perfectly coincident with the values for T = 0 obtained by integration of Eq. (1). The relative differences between the values of e,

Ω, i_e, ω_e, a, and P and the initial values of these parameters given in Table 1 are -3.1·10^{-4}, -1.6·10^{-5}, -6.2·10^{-5}, -1.5·10^{-5}, -1.5·10^{-5}, -1.0·10^{-4}, and -3.0·10^{-4}, respectively. Such a coincidence validates the calculations at all stages, including the determination of initial conditions, integration of Eq. (1), determination of orbital-parameter values, and the transformation between different coordinate systems.

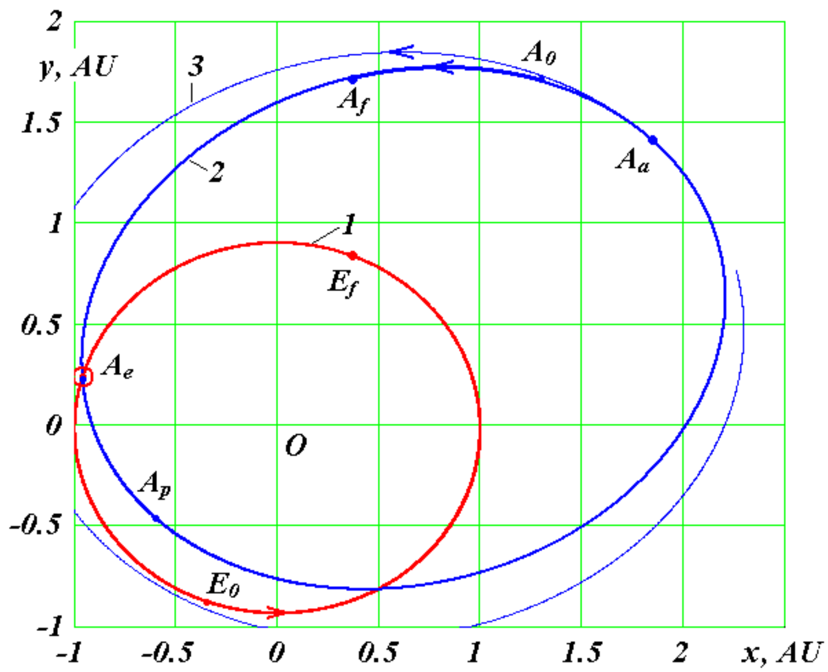

Figure 9. The trajectories of Earth (*1*) and 1950 DA (*2*) in the barycentric equatorial coordinate system *xOy* over 2.5 years in the encounter epoch of March 6, 2641 (point A_e): A_0 and E_0 are the starting points of the 1950 DA and Earth trajectories; A_f and E_f are the end points of the 1950 DA and Earth trajectories; *3* - 1950 DA trajectory after the correction applied at the point A_a is shown arbitrarily; the coordinates *x* and *y* are given in AU.

It should be noted that the relative difference for the same elements of Apophis is one order of magnitude smaller. The cause for the latter can be explained as follows. Using the data obtained by integrating Eq. (1), we determine the orbit elements at the time equal to half the orbital period. Hence, our elements are remote from the time of determination of the initial conditions by that time interval. Since the orbital period of Apophis is shorter than that of 1950 DA, the time of determination of Apophis' elements is 0.66 year closer in time to the time of determination of initial conditions than the same time for 1950 DA.

11. Study of the 1950 DA Trajectory in the Encounter Epoch of March 6, 2641

Since the distances to which the asteroid will approach the Earth at the times T_E and T_F differ little, consider the trajectories of the asteroid and the Earth at the nearest approach time T_E, March 6, 2641. The ellipse E_0E_f in Figure 9 shows the projection of the Earth trajectory over a 2.5-year period onto the equatorial plane *xOy*. This projection shows that, moving from the point E_0 the Earth will make 2.5 orbital turns. The trajectory of 1950 DA starts at the point A_0. At the point A_e the asteroid will approach the Earth in 2641 to a distance of 0.01507 AU. The post-encounter trajectory of the asteroid remains roughly unchanged. Then, the asteroid will pass through the perihelion point A_p and aphelion point A_a, and the trajectory finally ends at the point A_f.

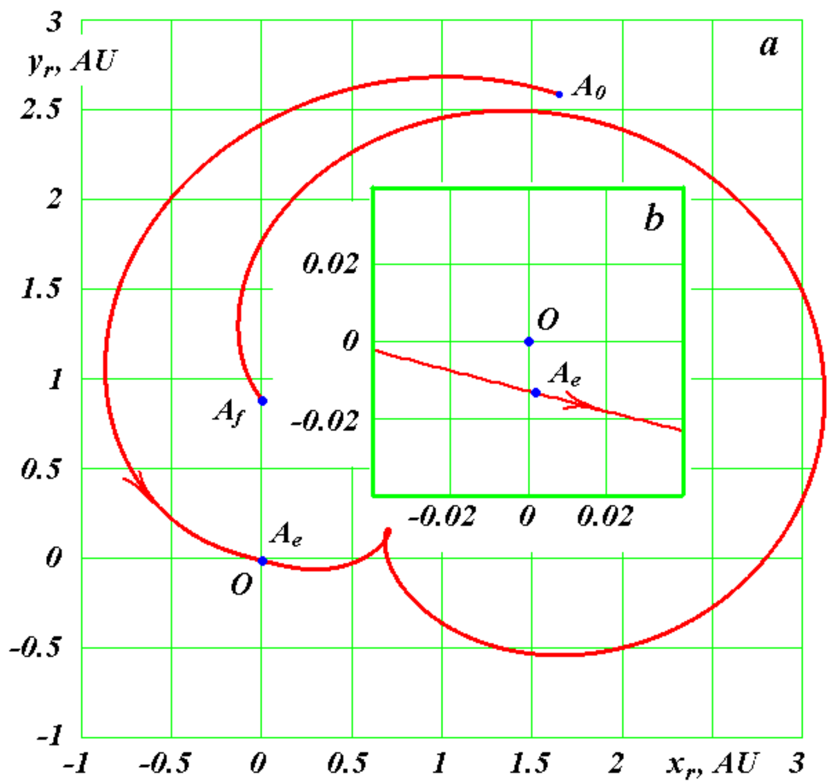

Figure 10. The 1950 DA trajectory in the geocentric equatorial coordinate system x_rOy_r: *a* – on ordinary scale; *b* – on an enlarged scale by the moment of 1950 DA encounter with the Earth: point *O* – the Earth, point A_e – the asteroid at the moment of its closest approach to the Earth; the coordinates x_r and y_r are given in AU.

Figure 10, *a* shows the trajectory of the asteroid relative to the Earth. The relative coordinates x_r and y_r were calculated by a Equation analogous to (15). Starting at the point A_0, the asteroid 1950 DA will move to the point A_e, where it will most closely approach the Earth, the end point of the trajectory being the point A_f. The loop in the 1950 DA trajectory represents a reverse motion of the asteroid relative to the Earth.

On an enlarged scale, the encounter of the asteroid with the Earth is illustrated by Figure 10, *b*. The Sun is in the right upper quadrant. The velocity of the asteroid relative to the Earth at the closing point A_e is v_{AE} = 14.3 km s^{-1}.

12. Making the Asteroid 1950 DA an Earth-Bound Satellite

Following a deceleration at the point A_e (see Figure 10, *b*), the asteroid 1950 DA can become a satellite orbiting around the Earth in the same direction as the Moon does. At this point *E* (see Table 5) the distance from the asteroid to the Earth's center is R_{minE} = 2.25 million km, the mass of the asteroid being m_A = 1.57 milliard ton. According to (17), the velocity of a satellite moving in a circular orbit of radius R_{minE} is v_{CE} = 0.421 km s^{-1}. For the asteroid 1950 DA to be made a satellite, its velocity needs to be brought close to the value v_{CE} or, in other words, the velocity of the asteroid has to be decreased by $\Delta V \approx 13.9$ km s^{-1}. In this situation, the asteroid's momentum will become decreased by a value $m_a\Delta V = 2.18\cdot10^{16}$ kg·m/s, for Apophis the same decrease amounts to $m_a\cdot\Delta V = 1.08\ 10^{14}$ kg·m s^{-1}, a 200 times greater value. Very probably, satellites with an orbital radius of 2.25 million km will not find a wide use. In this connection, consider another strategy for making the asteroid an Earth-bound satellite. Suppose that the velocity of the asteroid at the aphelion of its orbit (point A_a in Figure 9) was increased so that the asteroid at the orbit perihelion has rounded the Earth orbit on the outside of it passing by the orbit at a distance R_1. To simplify calculations, we assume the Earth's orbit to be a circular one with a radius equals the semi-axis of the Earth orbit a_E = 1 AU. So, in the corrected orbit of the asteroid the perihelion radius will be

$$R_{pc} = a_E + R_1. \tag{18}$$

Then, let us decrease the velocity of the asteroid at the perihelion of the corrected orbit to a value such that to make the asteroid an Earth-bound satellite. To check efficiency of this strategy, perform required calculations based on the two-body interaction model for the asteroid and the Sun (Smulsky 2007, Smulsky 2008). We write the expression for the parameter of trajectory in three forms:

$$\alpha_1 = -0.5\left(1 + R_p/R_a\right) = \frac{\mu_1}{R_p\cdot v_p^2} = \frac{R_p\mu_1}{R_a^2\cdot v_a^2}, \tag{19}$$

where

$$\mu_1 = -G\,(m_s + m_{As}) \tag{20}$$

is the interaction parameter of the Sun and the asteroid, m_S is the Sun mass, m_{As} is the asteroid mass, and α_1 = -0.6625 is the 1950 DA trajectory parameter.

Then, using (19), for the corrected orbit of the asteroid with parameters R_{pc} and v_{ac} we obtain:

$$-0.5\left(1 + R_{pc}/R_a\right) = \frac{R_{pc}\mu_1}{R_a^2 v_{ac}^2}. \tag{21}$$

From (21), we obtain the corrected velocity of the asteroid at aphelion:

$$v_{ac} = \sqrt{\frac{2 \cdot R_{pc}\left(-\mu_1\right)}{R_a^2\left(R_a + R_{pc}\right)}} . \tag{22}$$

Using (19), we express μ_1 in terms of α_1 and v_a, and after substitution of this expression into (22) we obtain the corrected velocity at aphelion:

$$v_{ac} = v_a\sqrt{\frac{2\left(-\alpha_1\right) R_{pc} \cdot R_a}{\left(R_a + R_{pc}\right) \cdot R_p}} . \tag{23}$$

From the second Kepler law, $R_a \cdot v_{ac} = R_{pc} \cdot v_{pc}$, we determine the velocity at the perihelion of the corrected orbit:

$$v_{pc} = v_{ac} \cdot R_a / R_{pc} . \tag{24}$$

As a numerical example, consider the problem on making the asteroid 1950 DA an Earth-bound satellite with a perihelion radius equal to the geostationary orbit radius $R_1 = R_{gs}$ = 42241 km. Prior to the correction, the aphelion velocity of the asteroid is v_a = 13.001 km s^{-1}, whereas the post-correction velocity calculated by Equation (23) is v_{ac} = 13.912 km s^{-1}. Thus, for making the asteroid a body rounding the Earth orbit it is required to increase its velocity at the point A_a in Figure 9 by 0.911 km s^{-1}. The corrected orbit is shown in Figure 9 with line *3*.

According to (24), the velocity of the asteroid at the perihelion of the corrected orbit is v_{pc} = 35.622 km s^{-1}. Using Eq. (7), for a circular Earth orbit with α_1 = -1 and $R_p = a_E$, and with the asteroid mass m_{AS} replaced with the Earth mass m_E, for the orbital velocity of the Earth we obtain a value v_{OE} = 29.785 km s^{-1}. According to (17), the velocity of the satellite in the geostationary orbit is v_{gs} = 3.072 km s^{-1}. Since those velocities add up, for the asteroid to be made an Earth satellite, its velocity has to be decreased to the value $v_{OE} + v_{CE}$ = 32.857 km s^{-1}. Thus, the asteroid 1950 DA will become a geostationary satellite following a decrease of its velocity at the perihelion of the corrected orbit by v_{pc} - ($v_{OE} + v_{CE}$) = 2.765 km s^{-1}.

We have performed the calculations for the epoch of 2641. Those calculations are, however, valid for any epoch. Our only concern is to choose the time of 1950 DA orbit correction such that at the perihelion of the corrected orbit the asteroid would approach the Earth. Such a problem was previously considered in Smulsky 2008, where a launch time of a space vehicle intended to pass near the Venus was calculated. The calculations by Eq. (18)-(24) were carried out on the assumption that the orbit planes of the asteroid and the Earth, and the Earth equator plane, are coincident. The calculation method of Smulsky 2008 allows the calculations to be performed at an arbitrary orientation of the planes. In the same publication it was shown that, following the determination of the nearest time suitable for correction, such moments in subsequent epochs can also be calculated. They follow at a certain period.

In the latter strategy for making the asteroid 1950 DA a near-Earth satellite, a total momentum $m_a \cdot \Delta V = m_a \cdot (0.911 + 2.765) \cdot 10^3 = 5.77 \cdot 10^{15}$ kg·m/s needs to be applied. This value is 4.8 times smaller than that in the former strategy and 53 times greater than the momentum required for making Apophis an Earth satellite. It seems more appropriate to start the creation of such Earth satellites with Apophis. In Corliss 1970, page 189, it is reported that an American astronaut Dandridge Cole and his co-author (Cole and Cox 1964) advanced a proposal to capture planetoids in between the Mars and Jupiter and bring them close to the Earth. Following this, mankind will be able to excavate rock from the interior of the planetoids and, in this way, produce in the cavities thus formed artificial conditions suitable for habitation. Note that another possible use of such satellites mentioned in Cole and Cox 1964 is the use of ores taken from them at the Earth.

Although the problem on making an asteroid an Earth satellite is a problem much easier to solve than the problem on planetoid capture, this former problem is nonetheless also a problem unprecedented in its difficulty. Yet, with this problem solved, our potential in preventing the serious asteroid danger will become many times enhanced. That is why, mankind getting down to tackling the problem, this will show that we have definitely passed from pure theoretical speculations in this field to practical activities on Earth protection of the asteroid hazard.

CONCLUSIONS

1. Through an analysis of literature sources, deficiencies of the previous calculation methods for asteroid motion were revealed.
2. The new method was used to numerically integrate non-simplified motion Equations of asteroid, the planets, the Moon, and the Sun over a 1000-year period.
3. On 21 hour 45' GMT, April 13, 2029 Apophis will pass close to the Earth, at a minimum distance of 6 Earth radii from Earth's center. This will be the closest pass of Apophis near the Earth in the forthcoming one thousand years.
4. Calculations on making Apophis an Earth bound satellite appropriate for solving various space exploration tasks were performed.
5. The asteroid 1950 DA will twice approach the Earth to a minimal distance of 2.25 million km, in 2641 and in 2962.
6. At any epoch, the asteroid 1950 DA can be made an Earth-bound satellite by increasing its aphelion velocity by ~ 1 km s^{-1} and by decreasing its perihelion velocity by ~ 2.5 km s^{-1}.

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