Initial data and output of the differential equations integrating for Newtonian interaction of the Sun, the Moon, and the planets of the Solar System: folder OrbtData

1. Description of folders DbPr00-50M, DP-50-100M, ExPr20-26c, ExPr36-34c.

1.1. The integration results are presented in four folders DbPr00-50M, DP-50-100M, ExPr20-26c, and ExPr36-34c: at a double number length (accuracy) of 17 decimal digits at an integration step of $\Delta T = 10^{-4}$ yr in DbPr00-50M and DP-50-100M, and an extended one of 34 decimal digits at $\Delta T = 10^{-5}$ yr, in ExPr20-26c, and ExPr36-34c.

1.2. Folder DbPr00-50M contains computing results for a time span 0 ÷-50 Myr and consists of folder OrPr-50 with the orbits elements, file o1p17m with control results, and archived files RzIn-50M.part01.rar with coordinates, velocities and other parameters of the bodies of the Solar System at every 10 kyr.

Folder OrPr-50 includes nine files (O-50ml1d.prn and others) with Mercury to Pluto orbits elements (1 to 9, respectively, in the file names) and file sn49f.dat with the initial data of variant 1.

Archive RzIn-50M.part01.rar contains files 0, 1, 2, …, 5009 with the coordinates of the bodies and other parameters at every 10 kyr. The file 0 presents data for the initial epoch 30.12.1949. The numbers in file names correspond to the quantities of integrated 10 kyr intervals.

1.3. Folder DP-50-100M presents computing results for the time span -50 ÷ -100 Myr and consists of folder OrPr-100 with orbits elements, file o100p08m with control results and archived files RzI-100M.part01.rar with positions of the bodies at every 10 kyr.

Folder OrPr-100 includes nine files (O-100m1d.prn and others) with Mercury to Pluto orbits elements (1 to 9, respectively, in the file names). Archive RzI-100M.part01.rar contains files 5010, 5011, …, 10008 with the coordinates of the bodies and other parameters at every 10 kyr. The files contain results of continuously integrated differential equations of motion (3) after -50 Myr.

1.4. Folder ExPr20-26c contains files 1, 2, …, 24 with the positions of the bodies at every 200 years. The positions are result of integration of (3) with the extended number length at the initial data of variant 1. Four files (O1e-7Pl1.prn and others) contain orbits parameters from Mercury to Mars between +2 kyr and -2.6 kyr. Two other files contain initial data of variant 1 (sn49f.dat) and control results (op20-24c).

1.5. Folder ExPr36-34c consists of files 1, 2, …, 36 with positions of the bodies at every 200 years, from integration of (3) with the extended number length at the initial data of variant 2. Nine files (O2e-7cp1.prn and others) contain orbits from Mercury to Pluto from 3.6 kyr. Two other files contain initial data of variant 2 (sn49jplc.dat) and control results (op36-34c).

2. Description of files

2.1. Differential equations of motion of the Solar System bodies

The files with the initial data and integration results contain dimensionless values obtained by reducing the differential equations to the dimensionless form [1-7].

According to the universal law of gravitation, the body k attracts the body i and this attraction is described by Newton gravity force

$$\vec{F}_{ik} = -G\frac{m_i m_k}{r_{ik}^3}\vec{r}_{ik}\,, \tag{1}$$

where G is the gravitation constant and $\vec{r}_{ik}$ is the radius vector from the body of the mass m_k to the body of the mass m_i.

The i-th body experiences the total gravity pull from n bodies

$$\vec{F}_i = -Gm_i \sum_{k \neq i}^{n} \frac{m_k \vec{r}_{ik}}{r_{ik}^{\;3}}. \tag{2}$$

and moves, under this action (2) and according to the second law of mechanics $\vec{a} = \vec{F} / m$, with respect to the reference inertial (i.e. not-accelerated) system at the acceleration

$$\frac{d^2 \vec{r}_i}{dt^2} = -G \sum_{k \neq i}^{n} \frac{m_k \vec{r}_{ik}}{r_{ik}^{\;3}}, \qquad i = 1,2,\ldots,n, \tag{3}$$

where $\vec{r}_i$ is the radius vector of body m_i relative to some center in the inertial system (the barycenter of the Solar System in this case).

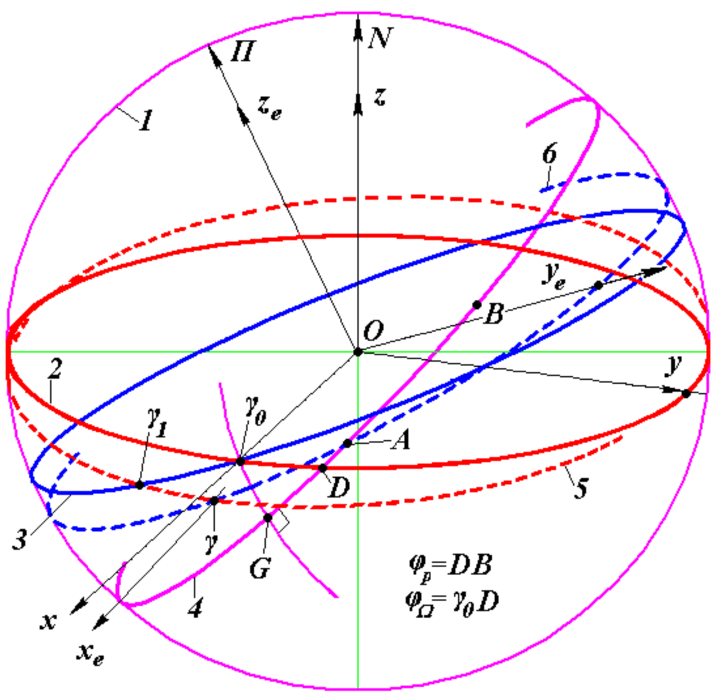

Fig. 1. Earth's orbital parameters in the fixed equatorial x,yz coordinate system.

1 = celestial sphere; *2* = equatorial plane of Earth at the epoch JD_s; *3* = orbital plane of Earth (ecliptic plane) at the epoch JD_s; 4 = orbital plane of planet at the epoch T; *5* = equatorial plane of Earth at the epoch T; 6 = Earth's orbital plane at the epoch T (tilt is exaggerated for better visualization); N = universal North pole (pole of fixed equator 2); Π = North pole of moving ecliptic 6; γ_0 = vernal equinox at JD_s; γ = vernal equinox at T (point at intersection of the moving equator at the epoch T with the moving ecliptic); $\gamma_0 G$ = great-circle arc normal to planet's orbital plane; B – heliocentric projection of planet's perihelion onto the celestial sphere; A = ascending node of planet's orbit in the moving ecliptic; D = ascending node of planet's orbit in the fixed equatorial plane; $\varphi_\Omega = \gamma_0 D$; $\varphi_p = DB$; $i = \angle\gamma_0 DG$ are the orbital elements of planet in the fixed heliocentric equatorial frame; $\Omega_a = \gamma A$; $\omega_a = AB$; $\pi_a = \gamma A + AB = \Omega_a + \omega_a$; $i_e = \angle\gamma AG$ are the orbital elements of planet in the moving ecliptic frame. The epoch JD_s of coordinate system is the Julian day number (e.g., at epoch JD_{S1} = 2433282.4234 on 1950.0.); the epoch T from time zero is counted in sidereal centuries from JD_0 (e.g., JD_0 = 2433280.5 on 30.12.1949).

System (3) consists of $3n$ nonlinear second order differential equations, where n = 11 (nine planets, the Moon, and the Sun). To solve it, we specify $3n$ coordinates and $3n$ components of the velocities at some epoch which we refer to hereafter as the initial epoch $T_0 = 0$, for example, 30.12.1949 corresponding to the Julian day JD_0 = 2433280.5. The problem is solved in a fixed barycentric equatorial frame (x,y,z) (Fig. 1) with the x axis pointing to the vernal equinox γ_0 of the fixed coordinate system, say, at epoch 1950.0 or JD_{S1} = 2433282.4234.

Differential equation (3) is in the non-accelerating (inertial) coordinate system. However, the motion of all bodies is accelerated, and only their barycenter C is fixed in the system of interacting bodies (assuming that the action of other bodies on the system is negligible). As a rule, the observed coordinates x_{Si}, y_{Si}, z_{Si} and velocities v_{xSi}, v_{ySi}, v_{zSi} relative to the Sun (S) are used to calculate the coordinates and the velocities of the barycenter, with the x components

$$X_C = \sum_{i=1}^{n} m_i \; x_{Si} / M_{Ss}; \qquad V_{xC} = \sum_{i=1}^{n} m_i \; Vx_{Si} / M_{Ss}, \tag{4}$$

where n is the number of bodies of the solar system and $M_{Ss} = \sum_{i=1}^{n} m_i$ is the mass of the solar system.

The x components of the coordinates and the velocities relative to the center C are:

$$x_{Ci} = x_{Si} - X_C; \qquad v_{xci} = v_{xsi} - V_{xc}. \tag{5}$$

Eq. (3) is integrated in the dimensionless form, which for the x component is [1]:

$$\frac{dv_{xi}}{dT} = -\sum_{k \neq i}^{n} \frac{m_{oi}(x_i - x_k)}{R_{ik}^{\;3}}, \tag{6}$$

where $x_i = x_{ci} / Am$ is the dimensionless coordinate of the i-th body; Am is the characteristic length of the solar system close to its half diameter. We define Am such that the dimensionless time T was expressed in sidereal centuries.

$m_{oi} = m_i / M_{Ss}$ is the dimensionless mass of the i-th body;

$R_{ik} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2}$ is the dimensionless distance between the i-th and k-th bodies;

$v_{xi} = v_{xci} \, k_v$ is the dimensionless velocity of the i-th body;

$k_v = \sqrt{\dfrac{Am}{G \cdot M_{Ss}}}$ is the velocity coefficient (sec/m);

$T = t \, k_t$ is the dimensionless time in sidereal centuries of 36525.636042 days in a century;

$k_t = \sqrt{\dfrac{G \cdot M_{Ss}}{Am^3}}$ is the time coefficient (sid.cent./sec).

In all calculations the time T was reckoned from the epoch 30.0 Dec. ET 1949, JD_0 = 2433280.5, and the gravitation constant $G = 6.67259 \cdot 10^{-11}$ $m^3/(s^2 \cdot kg)$. According to the initial conditions of variant 1 (file sn49f.dat), $M_{Ss} = 1.991787350282 \cdot 10^{30}$ kg, $Am = 1.09796077030958 \cdot 10^{13}$ m, $k_v = 2.874251102012487 \cdot 10^{-4}$ sec/m, in the equatorial frame at the epoch 1950.0, JD_{s1} = 2433282.4234. In the initial conditions of variant 2 (file sn49jplc.dat), M_{Ss} = 1.991588300600763E$\cdot 10^{30}$ kg, $Am = 1.097924194112168 \cdot 10^{13}$ m, $k_v = 2.874346854684925 \cdot 10^{-4}$ sec/m in the equatorial frame at the epoch 2000.0, JD_{S2} = 2451545.

2.2. Files of initial data and integration results

sn49f.dat, sn49jplc.dat and files: 1, 2, 3 etc.

The files are described using two different notations, one for program texts and the other for printed text. The files of input (sn49f.dat or sn49jplc.dat) and output (1, 2, 3 etc.) data contain 24 elements: T0, omm, Um, ΔTp, Px, Py, Pz, AMx, AMy, AMz, Spsx, Spsy, Spsz, E, Em, Ett, dT, i2b, j2b, k2b, A, B, C, Mu common for all interacting bodies. Then there follow fifteen elements for each k-th body: om(k), (X(k,q),q=1,3), (U(k;q),q=1,3), (dUp(k;q),q=1,3), (Sp(k;q),q=1,3), Ra(k), Et(k), where q marks enumeration of variables along the coordinates x, y, z.

The parameters common for all interacting bodies are:

T0 is the time in sidereal centuries of 36525.636042 days in a century;
omm is the maximum mass relative to the total mass of all bodies;
Um is the maximum velocity;
ΔTp is the time step in sidereal centuries at the previous iteration;
Px, Py, Pz are the components of the system's momentum;

AMx, AMy, AMz are the components of the system's angular momentum;
Spsx, Spsy, Spsz are the components of the total angular momentum due to the planets' spins (spin components);
E is the total kinetic energy of all bodies at moment T0;
Em is the maximum kinetic energy of all bodies from the beginning of integration;
Ett is the total kinetic energy of the bodies they acquired on collision and merging;
ΔT is the current time step in sidereal centuries;
i2b, j2b, k2b correspond to division of the mass of material along the axes x, y, z;
A, B, C are the dimensions of the mass of material along the axes x, y, z;
Mu is the number of segments the mass of material is divided into along the x axis.

The latter seven parameters are used to represent the interacting bodies as a mass of material distributed in space.

The parameters for each k-th body are:
om(k) ÷ m_{oi} is the dimensionless mass of the $k = i$-th body;
(X(k,q),q=1,3) ÷ x_i, y_i, z_i are the dimensionless barycentric equatorial coordinates of the i-th body;
(U(k,q),q=1,3) ÷ v_{xi}, v_{yi}, v_{zi} are the dimensionless velocities of the $k = i$-th body;
(dUp(k,q),q=1,3) ÷ $x_i^{(5)}$, $y_i^{(5)}$, $z_i^{(5)}$ are the dimensionless fifth-order derivatives for the $k = i$-th body;
(Sp(k,q),q=1,3) ÷ S_{pxi}, S_{pyi}, S_{pzi} – are the dimensionless spins of the rotating $k = i$-th body acquired by the body formed by merging of several interacting bodies; the initial rotations of the bodies do not be accounted, and their spins are $S_{pxi}=S_{pyi}=S_{pzi}=0$;
Ra(k) is the dimensionless equivalent radius of the $k = i$-th body;
Et(k) – E_{ti} is the dimensionless heat of the $k = i$-th body produced by merging of several bodies.

The initial heat energies of the bodies do not be accounted, and $E_{ti}=0$. The total heat of the body produced by merging of the bodies with the masses m_{oi} and m_{ok} is

$$E_{ti} = \frac{m_{ok} m_{oi} v^2{}_{rki}}{2(m_{ok} + m_{oi})}, \tag{7}$$

where $m_{oi} \geq m_{ok}$, and v_{rki} is the dimensionless radial velocity of the body with the mass m_{ok} relative to the body with the mass m_{oi}.

Note that the mass of the i-th body increases on merging: $m_{oi} + m_{ok}$, while the mass of the k-th body zeroes.

2.3. Files of orbits parameters

O2e-7cp1.prn, O1e-7Pl1.prn, O-50ml1d.prn и O-100m1d.prn

Most of files contain twelve elements, though some folders are slightly different. All files have the extension *.prn and their final form is made in MathCad. The twelve elements are T2, fi01, dl1, fip, fia, Rp, Ra, Tp, Ta, Year, AMSA, and zoa:

T2 = 0.5(T_s+T_f) is the time in sidereal centuries corresponding to the orbit mean point, where T_s and T_f are the times of the starting and final points, respectively.
fi01 = φ_Ω is the position of the orbit ascending node (Fig. 1) along the fixed equator circle relative to the vernal equinox γ_o;
dl1 = i is the inclination of the orbital plane relative to the fixed equatorial plane;
fip is the orbital angle of the shortest distance of a planet from the Sun;
fia is the orbital angle of the longest distance of a planet from the Sun.

First the program of definition of the trajectory parameters (DefTra) computes the angles of the shortest (fip0) and longest (fia0) distances for a cycle from 0 to 2π, and then fip0 and fia0 are converted in MathCad into a continuous series of fip and fia data.
Rp is the dimensionless perihelion radius;
Ra is the dimensionless aphelion radius;

Tp is the time required to reach the perihelion;
Ta is the time required to reach the aphelion;
Year = T_{rn} is the orbital period in sidereal years in the fixed frame, found as

$$T_{rn} = T_n - T_1 + \frac{2\pi - \varphi_n}{\varphi_{n+1} - \varphi_n}(T_{n+1} - T_n), \tag{8}$$

where T_1 is the time of the planet presence at the starting point of orbit; T_n is the time of the planet presence at one of final orbit points when the polar angle φ_n does not surpass 2π; where φ is the polar angle in the mean orbital plane from the starting point. The number of orbit points is specified such that they represent one full period with some safety margin.

Eq. (8) expresses the interpolation between the orbit points on passage of φ through 2π. A similar interpolation is applied to other parameters as well. As a rule, the computed orbits are represented by a few thousand points.

AMSA = $Mpf = \sqrt{Mpx^2 + Mpy^2 + Mpz^2}$ is the planet's dimensionless angular momentum averaged over the orbital period and *Mpx*, *Mpy*, and *Mpz* are its components along the coordinate axes.

zoa = z_0 is the relative width of the band of orbit points, with z_0 given by

$$z_0 = (z_{\max} - z_{\min})/(Ra + Rp), \tag{9}$$

where z_{max} is the maximum deviation of orbit points from its mean plane; z_{min} is the minimum deviation of orbit points from its mean plane, and $z_{min} < 0$.

The above parameters are used to obtain the mean perihelion angle

$$\varphi_p = (fip + fia \pm \pi)/2\,, \tag{10}$$

and the eccentricity

$$e = (Ra - Rp)/(Ra + Rp)\,. \tag{11}$$

The listed twelve parameters are available in files O2e-7cp1.prn, etc. of folder ExPr36-34c. Files O1e-7Pl1.prn, etc. of folder ExPr20-20c miss zoa data.

The files of 50 Myr and 100 Myr orbits (O-50ml1d.prn, etc. and O-100m1d.prn, etc.) in folders DbPr00-50M and DP-50-100M, respectively, include 14 parameters. The two additional parameters fip0 and fia0 are, respectively, the non-normalized perihelion and aphelion angles. The reason for including these parameters is that algorithm of their conversion into fip and fia is too sophisticated, because of retrograde motion of the perihelion and the cyclic form of the fip0 and fia0 data, which is fraught with failures hard to pick over the long time spans of 50 Myr or more.

2.4. Files of control results

op20-24c, op36-34c, o1p17m, and o100p08m

As the bodies converge to a distance equal to the sum of their radiuses, developed by us program Galactica merges them automatically into a single body with a total heat and intrinsic angular momentum. The monitored parameters are (Fig. 2) maximum mass (omm = m_{max}), momentum (P), angular momentum (M), sum of the intrinsic angular momentums (S), kinetic energy (E) and heat (E_t) of the whole system of bodies and the relative change (δM_z) of the z component of the angular momentum M. In the absence of external action, the angular momentum of the system, e.g., its z component

$$M_z = \sum_{i=1}^{n} m_i(v_{yi}\, x_i - v_{xi}\, y_i) = \text{const} \tag{12}$$

should not change. Therefore, the relative change

$$\delta M_z = (M_z - M_{z0})/M_{z0}, \tag{13}$$

where M_{z0} is the angular momentum at T_0 should be zero. If it is nonzero, the numerical solutions are inaccurate. Other integral parameters describe the dynamic evolution of the system.

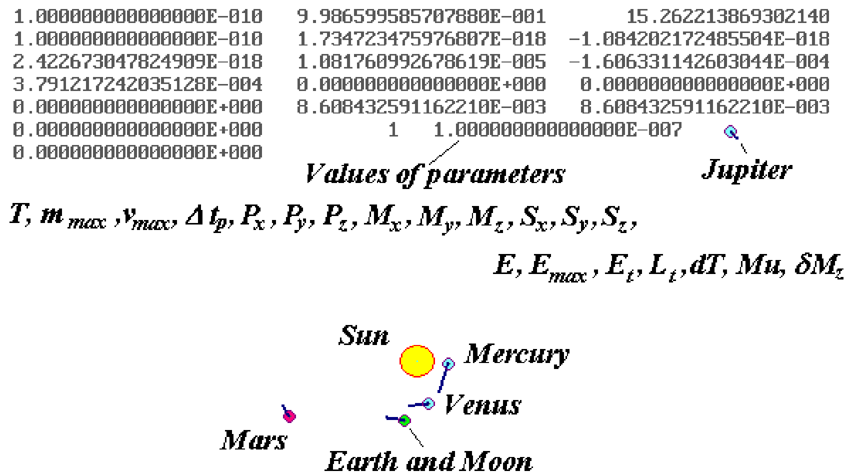

Fig. 2. Numerical solutions of system (3) displayed on the screen (South Pole view). The explanatory and notation of numerical parameters are given the italics.

At the solution of a task on the personal computer, the numerical information on process of integration and the positions and the velocities of the planets are displayed after a certain number of integration steps as in Fig. 2. This information allows to observe process of the solution and to determine qualitatively its reliability. The output file obtained on a supercomputer includes only numerical data. First 16 parameters repeat the common parameters:

TO, omm, Um, dTp, Px, Py, Pz, AMx, AMy, AMz, Spsx, Spsy, Spsz, E, Em, Ett,

from the files of initial data and integration results (see above, section 2.2). The three other parameters are the number (*Lt*) and size (*ΔT*) of the integration step and the change in the *z*-component of the angular momentum δM_z (note that the complete integration step number is the product *Lt·Lt2)*.

These elements, gathered in a single file of **control results**, facilitate monitoring the system evolution and the integration process. The files with the extended number length (op20-24c and op36-34c) include data over the whole computed time span, and the files with the double number length include data for some intervals at the beginning and at the end of the time span (first 1.17 Myr in o1p17m and last 1.5 Myr in o100p08m).

References

1. Smulsky, J.J. 2004. The Theory of Interaction. - Ekaterinburg, Russia: Publishing house "Cultural Information Bank". – 304 p. (In English). In Russian: Smulsky, J.J., 1999. The Theory of Interaction Izd. Novosib. Univ., NIC OIGGM SO RAN, Novosibirsk. http://www.ikz.ru/~smulski/TVfulA5_2.pdf.

2. Melnikov, V.P., Smulsky, J.J., Krotov, O.I., Smulsky, L.J., 2000. Orbits of the Earth and the Sun and possible effects on the Earth's cryosphere (problem formulation and preliminary results). Kriosfera Zemli IV (3), 3-13. http://www.ikz.ru/~smulski/smul1/Russian1/IntSunSyst/OrZS.pdf.

3. Melnikov, V.P., Smulsky, J.J., 2004. Orbital forcing of Earth's cryosphere and problems in related research. Kriosfera Zemli VIII (1), 3-14. http://www.ikz.ru/~smulski/smul1/Russian1/IntSunSyst/Astrfak7.htm.

4. Grebenikov, E.A., Smuslky, J.J., 2007. Evolution of Mars's orbit over 100 Myr. Papers on Applied Mathematics, Russian Acad. Sci., Dorodnitsyn Computing Center, Moscow. http://www.ikz.ru/~smulski/Papers/EvMa100m4t2.pdf.
5. Smulsky, J.J., 2003. New geometry of orbital evolution, in: New Geometry of Nature. Proc. Joint International Scientific Conference, August 25 — September 5, 2003. Book V. III. Astronomy. Education. Philosophy. Kazan State University, Kazan, pp. 192—195. http://www.ikz.ru/~smulski/smul1/Russian1/IntSunSyst/NeGeEv2.doc.
6. Smulsky, J.J., 2005. Modeling interaction in the Solar System for 50 Myr: Climate implications. "Bolshaya Medveditsa", Zhurnal Problem Zashchity Zemli, 1, 44-56. http://www.ikz.ru/~smulski/Papers/RasVSS2c.pdf.
7. Melnikov V.P., Smulsky J.J. Astronomical theory of ice ages: New approximations. Solutions and challenges. - Novosibirsk: Academic Publishing House "GEO", 2009. – 84 p. The book in two languages. On the back side: Мельников В.П., Смульский И.И. Астрономическая теория ледниковых периодов: Новые приближения. Решенные и нерешенные проблемы. – Новосибирск: Академическое изд-во «Гео», 2009. – 98 с. http://www.ikz.ru/~smulski/Papers/AsThAnE.pdf.

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